DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

PRODUCT RELIABILITY

by
Umut GÜREL

September, 2008
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PRODUCT RELIABILITY

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in Industrial Engineering, Industrial Engineering Program

by
Umut GÜREL

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"PRODUCT RELIABILITY"** completed by **UMUT GÜREL** under supervision of **ASST. PROF. DR. MEHMET ÇAKMAKÇI** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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PRODUCT RELIABILITY

ABSTRACT

This study presents product reliability on component basis and investigates the possible effects on warranty which constitutes a non-technical issue. Product reliability is a key factor which is used in considering the warranty period. It plays a significant role that a mistake in warranty forecasting costs a lot for companies. The objective is to empirically examine the nature of general reliability of manufactured goods and define a statement about them, based on findings of practicing in an electronics company. In this regard reliability will be stated from the actual manufacturing point of view. A case study was conducted as an application to consider how product reliability results in manufacturing industry. LCD TVs were undertaken to examine. To test reliability, a parametric Weibull model was exploited and hazard rates of products were estimated with linear regression method. For this research, the lifetime data obtained by service departments, censored both left and right, were used in MINITAB14 to produce the reliability results. The results of the analysis build up the basis for evaluating the performance of LCDs in means of service. By the help of it, upcoming failures were forecasted and defined when and how many of them could occur in say six months, a year or two years. The time at which a particular percentage of the production will have failed can be determined

Keywords: Product reliability, Weibull distribution, Censored lifetime data.

ÜRÜN GÜVENİLİRLİĞİ

ÖZ

Bu çalışma komponent bazlı ürün güvenilirliği ve teknik bir konu olmayan garanti üzerindeki etkilerini incelemektedir. Ürün güvenilirliği, garanti süresinin belirlenmesinde kullanılan kilit bir faktördür. Bu açıdan güvenilirlik önemli bir rol oynar ki garantiye ilişkin yanlış tahminlemeler firmalara çok büyük maliyetler getirir. Çalışmanın amacı, imalat sektöründeki ürünlerin genel güvenilirliğine ilişkin yapılarını ampirik olarak incelemek ve bir elektronik fabrikasında vaka incelemesi olarak yapılan çalışmanın sonucuna göre çıkarsamalarda bulunmaktır. Bu yüzden güvenilirlik kavramı üretim bakış açısı ile verilecektir. Bir vaka çalışması da uygulama olarak ele alınmış olup güvenilirliğin üretim endüstrisinde nasıl sonuçlandığı incelenmiştir. LCD televizyonlar araştırmaya tabi tutulmuştur. Güvenilirliği test etmek için parametrik Weibull modeli kullanılmış ve bozulma oranları doğrusal regresyon yolu ile tahminlenmiştir. Bu çalışma için servis departmanından ürünlerin yaşamlarına ilişkin veriler toplanmış ve bunlar sensörlü bir şekilde MINITAB14 yazılımında değerlendirilmiştir. Analiz sonuçları LCD televizyonların operasyonel çalışma sürelerine ilişkin performanslarını değerlendirmede referans oluşturmuştur. Bunun yardımıyla bozulmaların aylık, yıllık veya belirli bir zaman dilimine ilişkin dağılımları tahmin edilmiştir. Böylece ürünlerin yüzdesel olarak ne kadarının bu süreler içerisinde bozulduğu öngörülebilir.

Anahtar Kelimeler: Ürün Güvenilirliği, Weibull Dağılımı, Sensörlü Yaşam Verileri.

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CHAPTER ONE
INTRODUCTION TO RELIABILITY

1.1 Introduction

For both manufacturers and consumers reliability is one of the most significant characteristics defining quality of products, or large and complex systems. The role and effects of reliability can be observed in daily life, such as attempting to use computer, television, or in an industrial manner screw driver while assembling a screw. In those situations users expect the products to perform the desired function, which is inherently existed, when they are requested. If the products do not deliver these functions by the time of usage, then reliability becomes a question of matter. The reliability characteristic of a product today represents one of the essential demands of buyers. Consumers would like to buy a product that works perfectly whenever its button is pushed.

It is likely to happen a failure when products can not perform the expected quality of service. Every step realizing to manufacture a unit of product requires a specific amount of labor force integrated with the ease of technological advancement. Since no human activity contains zero risk and no equipment a zero rate of failure, it is common to come across flaws and defects in products. Failures can be emerged from software elements, resulted due to human or environmental factors. That leads some sort of business sorrows within reliability problems.

Reliability plays a fundamental role in assessing and predicting the quality of products. It enables manufacturers to become a proactive leader of their quality from the time they decide to manufacture products to the time they introduce to the market. In the lightening of reliability analysis they tend to predict the product durability and maintenance which constitutes one of the values given by consumers. One of the benefits of this prediction is to make the service departments function more accurately by improving the ability to assess different repair times, design configurations and failure rates. Even a simple reliability plan and model is far more valuable than any pure forecast.

The reliability studies can be implemented in three different stages of product manufacture. Figure 1.1 illustrates these stages which are design, manufacture and field. In design stage reliability analysis is conducted to correct design problems. The design qualifications are verified and validated. These improvements are set to increase design reliability. That is the reliability on component basis. In manufacture stage the processes in which products are started to be manufactured are monitored and controlled. Mandatory method studies are developed to improve these process outcomes. The final stage is field which clarifies the reliability of products by evaluating failure feedbacks and carrying out preventive maintenance. At the end of this stage achieved reliability would be stick on the product which means the reliability demonstrated by the physical product (Feigenbaum, 1991). The products evolve through these stages and finally get their achieved reliability which is commonly less than design reliability. As passing through stages the first predictions made in the design are tend to depart from the values of design reliability. Because there are many sources of failures in processes which make the design reliability values cut down to achieved reliability. But on the other hand, hypothetically if the final products are tested for a long and sufficient time, then it is possible to reach the design reliability, as the failures can be weeded out.

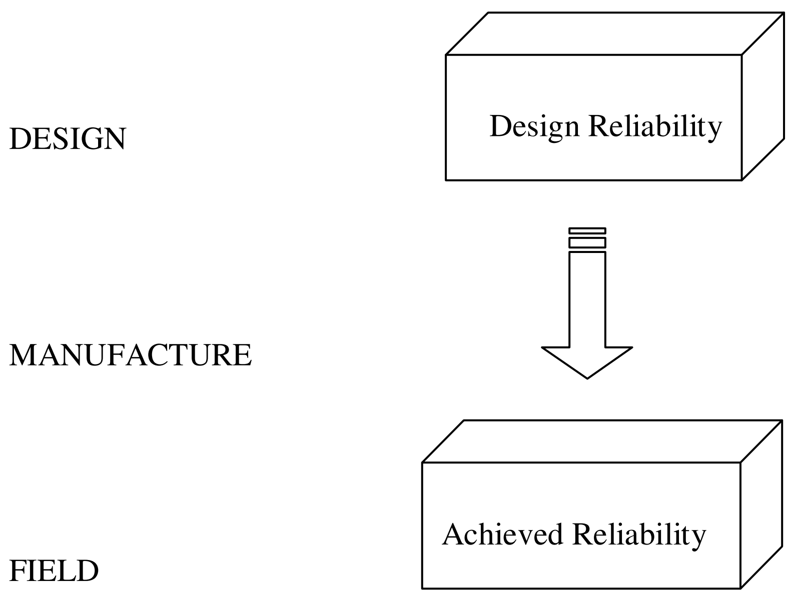

Figure 1.1 Different reliability studies in different stages of manufacture

Consequently this figure summarizes the evolution of the product reliability through the stages of manufacturing and gives a perspective of an idea that how the reliability subject to change in the manufacture flow.

1.2 System and Component Reliability

To mention a little bit about numerical reliability, the notions of system and component must be explained. Component is the single unit located in any system. It can stand alone and have an independent nature of function or operate with the other components one after one. The latter notion, system is the collection of components which are organized for the same purpose. Since every component has a numerical reliability, so the system reliability is computed through the reliability values of components. The numerical value of the reliability stands for the probability that an item performs a required function under stated conditions. For now let the definition of reliability stick here and explain in detail later.

As products become more complex, that is have more components, the chance of failure increases. The reliability of a system depends on its components. Simply increasing the reliability of each component and decreasing the number of components will raise the system reliability up. Moreover, the method of arranging the components affects the entire system. Components can be placed in series, parallel, or a combination of both (Besterfield, 1994).

1.2.1 Reliability of a Series System

A series system is the one whose components are arranged to operate dependently. If one of the components fails, then the system fails. As more components are added, it is likely to decrease the system reliability. Thus system will be reliable as long as the possible defective component operates. Reliability of a series system can easily be calculated by multiplication. Let R_i designates the ith component reliability in the system and R_s the system reliability. The system reliability is calculated as follows:

$$R_s = \prod_{i=1}^{n} R_i$$

Here is a simple example of a series system

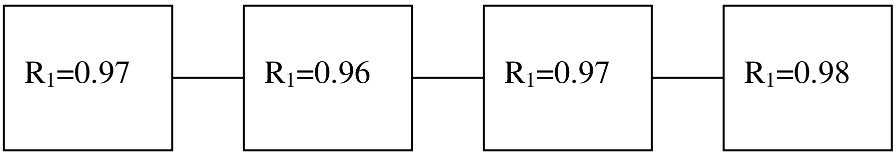

Figure 1.2 A series system

Above there is a series system that is composed of four components whose reliabilities are 0.97, 0.96, 0.97, 0.98 respectively. On the basis of multiplication rule the system reliability is calculated as:

R_s=0.97 x 0.96 x 0.97 x 0.98=0.8852

Although the component reliabilities are relatively of high values, the system value is under 90%. So it is always risky to have a system that has many components. By the influence of multiplication, a system, that is made up of 100 components each having a reliability of 0.99, will have a reliability value of 0.99^{100}, which is 36.6%. It is hard to sustain a high level of reliability in those systems. Such effect of increasing the number of components in series arrangement on reliability is illustrated in Table 1.1 below.

Table 1.1 How complexity affects system reliability in series arrangement (Kececioglu, 2002)

Number of Components	Individual Component Reliability			
	99.999%	99.99%	99.9%	99.0%
	System Reliability			
10	99.99%	99.90%	99.00%	90.44%
100	99.90%	99.01%	90.48%	36.60%
250	99.75%	97.53%	77.87%	8.11%
500	99.50%	95.12%	60.64%	0.66%
1000	99.01%	90.48%	36.77%	<0.1%
10000	90.48%	36.79%	<0.1%	<0.1%
100000	36.79%	<0.1%	<0.1%	<0.1%

1.2.2 Reliability of a Parallel System

In this type of system, components are arranged in parallel. The system operates until all branches of parallel arranged components fail. Parallel systems have two properties: The more components in parallel the more reliable the system is. The reliability of parallel arranged system is greater than the reliability of the individual component. The reliability of a parallel system is calculated by subtracting the total probability of failure, which is the unreliability, of all components from 1. If R_i is the reliability of the ith component, then F_i is said to be the unreliability of that component. The system reliability R_s is calculated as follows:

$$R_s = 1 - \prod_{i=1}^{n} F_i$$

Let have a look at parallel system in a small example below:

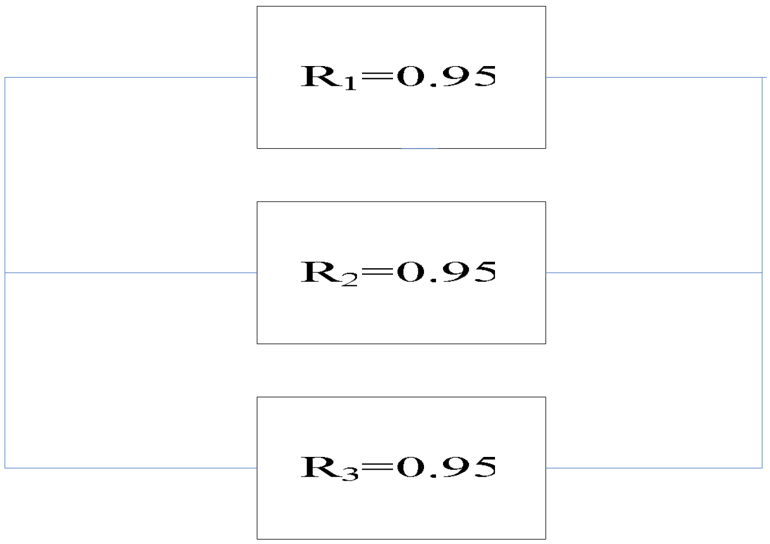

Figure 1.3 A parallel system

The system above consists of three identical components that have a reliability value of 0.95. The unreliability of each branches or components is 0.05. Hence F_i values are equal since the components are same. So the total unreliability of all branches or the system is equal to the multiplication of each F_i values. The reliability of the system can be found as easily as follows:

$$R_s = 1 - (0.05x0.05x0.05) = 0.999875$$

It is verified that adding identical components increases the system reliability and the reliability has a greater value than any of the components.

The most products are designed with the combination of both parallel and series arrangements. Firstly each of the parallel branch reliabilities is calculated and then multiplied by component reliabilities in series.

1.3 What is Reliability?

So far it has been mentioned what reliability looks like but not explained how it is told to be one of the building blocks of quality. It is time to look over the exact definitions of product reliability.

Bazovsky (1961) simply stated the reliability as *the capability of an item not to break down in operation*. When an item works well, and works whenever called to do the job for which it was designed, such item is said to be reliable. This expression takes place in qualitative definition of reliability. One of broad qualitative definitions was brought up by BS 4778 that it is *the ability of an item to perform a required function under stated conditions for a stated period of time* (Ahmed, 1996). Since reliability is an engineering discipline, statistical tools and methods as statistical efforts play a significant role in conducting reliability studies. Therefore there is another definition of reliability in statistical terms. From quantitative point of view, reliability is expressed by *the probability that the item will perform its required function under given conditions for a stated time interval* (Birolini, 1999). Barringer (2000) has also given a definition as a probabilistic statement that reliability is concerned with the probability of future events based on past observations.

Reliability has a solid relation with failure because a product remains reliable as long as it does not fail. So it is essential to fully understand what a failure is. A failure occurs whenever the item stops performing the intended function. Generally it is unknown how much operating time goes on because of its randomness. An item can fail whenever it is started off or after a certain time. This depends on both manufacturing and design properties of the item. On the other hand when an item fails, it is often possible to restore to its original performance.

Smith (2000) briefly explains that a failure is nonconformance to some defined performance criteria. It is important to define these criteria in order to verbalize what it is meant from an intended function of an item. Because one may state an outcome of a performance as adequate and one may perceive insufficient which leads to a failure. Consequently failures are designated through diversions based on the specifications.

Product reliability definition consists of four main elements (Feigenbaum, 1991):

- Probability
- Performance
- Time
- Conditions

Probability is the numerical value in the reliability concept. Each identical product does not have the same reliability characteristic. Some may have a relatively longer life and some not. In this manner a group of products have a statistical probability of survival which identifies the distribution of failures.

Performance deals with the quality characteristic. In order to ensure a product to be reliable, it must perform a certain function when called upon.

The third element in reliability is time. Product's intended and required function must be identified for a stated period of time. That is lifespan of a product is determined.

The last element conditions in which the application and operating circumstances under which the product is put to use is a critical factor in evaluating reliability. These conditions establish the stresses that will be imposed upon the product. They need to be viewed broadly because they can have significant effects on product reliability.

The following section includes short explanations of mostly used reliability related terms (Blischke & Murthy, 2003).

Reliability theory deals with the interdisciplinary use of probability, statistics, and stochastic modeling, combined with engineering insights into the design and the scientific understanding of the failure mechanisms, to study the various aspects of reliability. As such, it encompasses issues such as reliability modeling, reliability analysis and optimization, reliability engineering, reliability science, reliability technology, and reliability management.

Reliability modeling deals with model building to obtain solutions to problems in predicting, estimating, and optimizing the survival or performance of an unreliable system, the impact of unreliability, and actions to mitigate this impact.

Reliability analysis can be divided into two broad categories: qualitative and quantitative. The former is intended to verify the various failure modes and causes that contribute to the unreliability of a product or system. The latter uses real failure data (obtained, for example, from a test program or from field operations) in conjunction with suitable mathematical models to produce quantitative estimates of product or system reliability.

Reliability engineering deals with the design and construction of systems and products, taking into account the unreliability of its parts and components. It also includes testing and programs to improve reliability. Good engineering results in a more reliable end product.

Reliability science is concerned with the properties of materials and the causes for deterioration leading to part and component failures. It also deals with the effect of manufacturing processes (e.g., casting, annealing, and assembly) on the reliability of the part or component produced.

Reliability management deals with the various management issues in the context of managing the design, manufacture, and/or operation and maintenance of reliable products and systems. Here the emphasis is on the business viewpoint, because unreliability has consequences in cost, time wasted, and, in certain cases, the welfare of an individual or even the security of a nation.

Reliability prediction deals basically with the use of models, past history regarding similar products, engineering judgment, and so forth, in an attempt to predict the reliability of a product at the design stage. The process may be updated in later stages as well, in an effort to predict ultimate reliability.

Reliability assessment is concerned with the estimation of reliability based on actual data, which may be test data, operational data, and so forth. It involves system modeling, goodness-of-fit to probability distributions, and related analyses.

Reliability optimization covers many areas and is concerned with achieving suitable trade-offs between different competing objectives such as performance, cost, and so on.

Reliability test design deals with methods of obtaining valid, reliable, and accurate data, and doing so in an efficient and effective manner.

Reliability data analysis deals with estimation of parameters, selection of distributions, and many of the aspects discussed above.

1.4 Why is Reliability Important?

Rapidly increasing concept of reliability has an impact on product manufacturing in several ways. The benefits and the need for reliability can be classified as follows (Feigenbaum, 1991; Smith, 2000; Kececioglu, 2002; Dhillon, 2005):

- The products are reviewed under reliability studies in different stages of development along time such as in design, manufacture, and post sale periods and are followed from birth to death.
- Reliability provides an early indication of the products' inadequacy or nonconformance to specifications.
- Reliability studies reveal the types of failures experienced by components and systems and recommend design, research, and development efforts to minimize these failures.

- Reliability studies establish what failures occur at what time in the life of a product and prepare to cope with them.
- As the complexity increases and more sophisticated products are launched to market it becomes inevitable to maintain a desired quality by building high reliability levels. For instance a typical Boeing 747 is composed of approximately 4.5 million parts in which it is hard to sustain each of these components' reliability to a required value.
- The costs because of low reliability (e.g. design changes, vendor rejects, rework, scrap, warranty) may be excessive because of too many premature product failures.
- Maintenance and repair costs during the expected life of the product may be excessively high.
- Consequences of product failure may be serious (e.g. loss of human, damage to the environment) and those, which are publicized, have unfavorable effects such as in Chernobyl Nuclear Reactor explosion which occurred in April 1986.
- Competitive products may be pushing to higher reliability values since many products are advertised by their reliability ratings and thus business forces companies to make them fully control of their reliabilities. To be ahead of the competition companies need to gain the knowledge of reliability and its practices.
- Expectations of consumers may not be fulfilled unless higher reliability values are achieved because today consumers are conscious of how unreliability is more costly. Otherwise companies are faced with the loss of goodwill.

1.5 Interrelationships between Reliability, Quality, and Warranty

Reliability is the fundamental base of the warranty concept. It may be considered as the technical side of the warranty which is a commercial issue. Warranty concept is defined by Murthy as manufacturer's assurance to the buyer that a product or service is or shall be as represented (Murthy & Djamaladin, 2002). Several warranty strategies are driven with the help of the product's availability, safety, maintenance and reliability. Consumers believe that warranty terms are an important source of information regarding brand reliability. The reason why reliability is used in warranty studies is that warranty period is determined according to reliability tests, mostly accelerated life testing, and most of the claims are used as feedbacks to reduce the unreliability. Warranty is one of the most important ways of promoting and marketing products that better warranty signals of better quality product. Today warranty strategies and terms are hardly defined and determined although they are basic problems in theory but the optimal warranty period and terms are affected by different factors. It is important to find out the root causes of the problems associated with the product claims in order to reassess and evaluate the warranty. Companies are enthusiastic in increasing the length of the warranty in electronic devices used at home due to the market pressure, but the costs could be unexpected when they offer better warranty than the products.

Therefore product reliability is a key factor which is used in considering the warranty period. It plays a significant role that a mistake in warranty forecasting costs a lot for companies. During the warranty period, companies apply some sort of service strategies. Every service department means cost for maintaining or repairing product, hiring and training personnel for service and supplying and holding spare parts in stock. That is why reliability is a major economic factor in product's success. Failures over the warranty period are linked to product reliability which is determined by decisions made during design, development, manufacturing (Murthy, 2006). The cost of servicing warranty claims are expected to be much lower for the reliable rather than the unreliable company because the latter producer will be faced with higher rates of product failure and subsequent need for repair (Agrawal & Richardson & Grimm, 1996).

Today warranty terms are defined by both consumer and manufacturer sides. As long as the market pressure is on one side, the other side is obligated to determine the warrant terms due to the other. To gain competitive advantage, one should offer a better warranty to capture the interest of consumers.

There is also a strong relation among warranty, reliability and quality. A longer warranty period cause companies to incur more cost, but if the product is of better quality, then the reliability will be satisfactory that there will be less claims and costs associated with the warranty claims will reduce (Murthy & Djamaladin, 2002). Therefore reliability is not an independent factor about the warranty, since it is the consequence of the quality politics. If reliability is improved, then warranty costs will be reduced. Better reliability is achieved by better quality that is controlled in both design and manufacturing phase of product development. The cost of improving quality must be less than the cost reduction of the expected warranty. Longer warranty is used as a marketing tool as it reflects the quality of product and the commitment of the company. At the same time it brings forth higher costs, the manufacturer should minimize the conditions that will cause extra cost during warranty period and calculate the correct price of products not to make any lose because of this long warranty. See Figure 1.4 for the relationship inbetween.

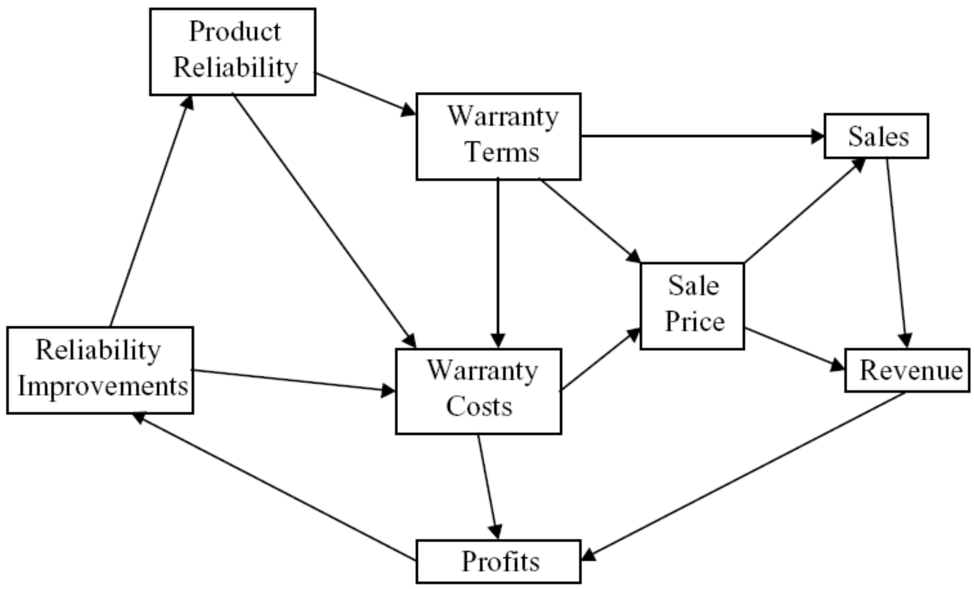

Figure 1.4 Relationship between product reliability and warranty concept (Murthy, 2006)

The main difference between quality and reliability is defined by National Institute of Standards and Technology [NIST] (2003) that quality is a snapshot at the start of life and reliability is a motion picture of the day by day operation. Time zero defects are manufacturing mistakes that escaped final test. The additional defects that appear over time are reliability defects or fallout.

On the other hand reliability deals with getting over the issue of preventing and controlling failures, because a consumer wants a product which performs its expected functions in a predefined period without any quality loss. If any failures or unexpected stoppages occur and the product continues to fail or frequency of the failures is close, then downtime will be high which can not be easily tolerated from the consumer point of view. From the micro standpoint, it constitutes an economical problem and causes an extra cost to recover and maintain the failure. However from the macro standpoint, this may lead the consumer not to rely on the product again and may choose to use an equivalent product of another company. So reliability deals with the long term strategy, drives the consumer satisfaction, and defines the operational life as a measure of quality. One important fact is it takes a long time to make a product reliable whereas it takes a short time to call it unreliable product.

Improving reliability is an important part of the overall goal of improving product quality. Reliability was stated as quality over time by Condra (1993). This implies that good quality is necessary but not sufficient. An unreliable product is not a high quality product. One major difficulty and difference between quality and reliability is that reliability can be assessed directly only after a product has been in field for some time (Meeker & Escobar, 2003).

1.6 Reliability and Cost

A fully controlled product reveals more confidence and accurate functions. But it is hard to control all the factors affecting the functioning. This makes reliability studies challenging and costly. Therefore an optimal point between cost and reliability must be defined. After a point, at which the cost increases, contribution to the reliability will be in slightly increments. For this reason it is irrational to spend more money on reliability studies after a good and enough reliability value is attained.

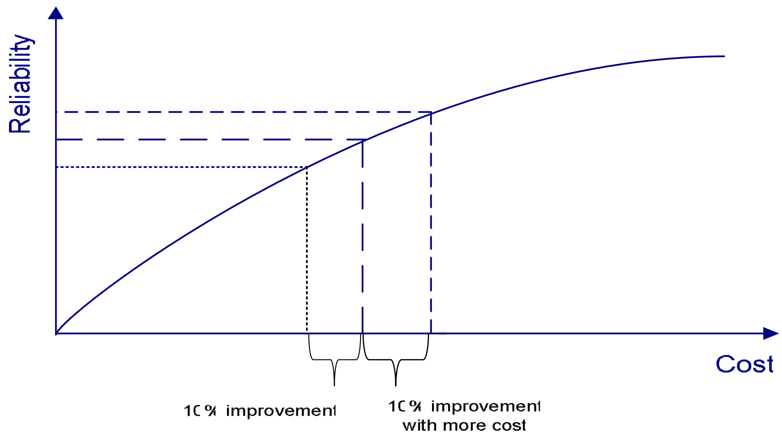

Figure 1.5 Improvement in reliability along cost

The optimum reliability is the level at which the cost to operate and maintain the product for its desired life is the minimum. For every product there is a certain reliability level at which the total cost of the product is minimum. Simply the total cost consists of product failure cost and the investment in reliability. Prevention costs which are cost of preventing failures, appraisal costs which are related to measurement of products quality make up the investment in reliability.

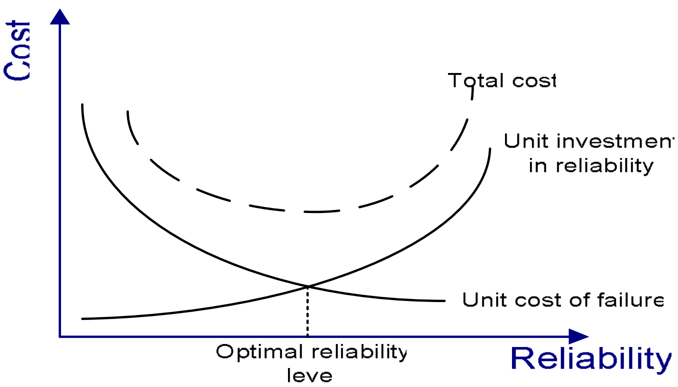

Figure 1.6 Optimal reliability level

An improvement in product reliability will not only bring about a reduction in warranty parts and labor cost, the impacts of this improvement will also cascade down to support groups, being very substantial in areas such as spares inventory both in physical and monetary terms, product engineering changes and rework both during use, manufacture, and development, cost of reworking expensive parts to be used as spare buffers stocks. The benefits are depicted in Figure 1.7 (Ahmed, 1996).

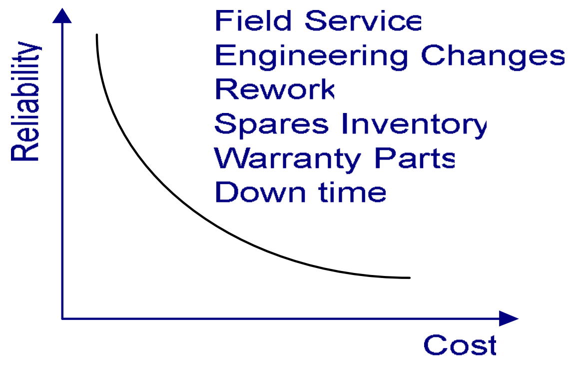

Figure 1.7 Benefits of product reliability in terms of cost

1.7 Reliability Growth

One of the important subjects in reliability is the reliability growth. It is generally used for new design products and briefly means improvement or deterioration in reliability. A product's reliability evolves during its design, development, testing, manufacture, and field use. This ongoing change is referred to reliability growth (Juran, 1999).

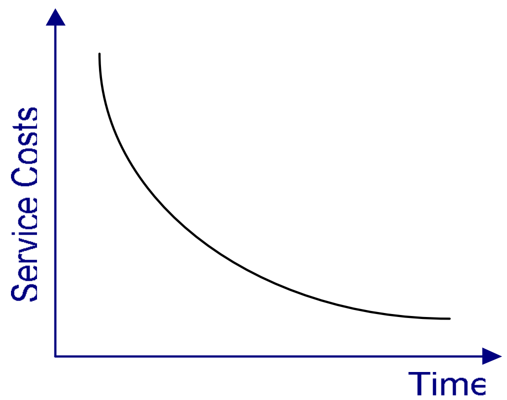

Figure 1.8 A positive reliability growth along time

As the initial production finishes, some flaws and drawbacks could be observed and experienced. In respect to the complexity of the product the learning factor draws the duration of how the problems are solved on the basis of those experiences. In the course of time reliability related service cost can be declined as seen in Figure 1.8. By improving reliability the failure rate function will be shifted downward on the bathtub curve. (Bathtub curve will be explained in the next chapter, but as an overview bathtub points out typical life history of a population of products.)

Reliability growth involves pinpointing the flaws during production, analyzing these problems, developing solution to them and implementing the changes generally concerning the design. The next production will have the probability of owning a higher rate of reliability and lower associated costs. Therefore, a certain period of time is required to develop the necessary infrastructure to achieve and maintain targeted levels of reliability.

1.8 Evolution of Reliability in History

The notion of reliability came out in 1940s. During the World War II (WW2) several military problems were emerged. Such vital problems can be set examples such as electronic gear on bombers gave less trouble free operation. 60% to 75% of radio vacuum tubes in communications equipments were failing. To cope with these problems some mathematical techniques which were quite new were applied to the operational and strategical problems of WW2. Later statistical models and techniques composed the basis of reliability.

In 1941 Robert Lusser, an electrical engineer, who worked on German missile testing program in Germany became one of the first men to recognize the need for reliability as a separate discipline. He came to USA after WW2 and joined research and development division in US Army. Reliability studies of V-1 rockets were carried on by his efforts. Later his studies contributed to the development of V-2 rockets. He also wrote numerous papers about reliability theory and application.

During the Korean War studies showed that military maintenance costs were computed as high levels. These high costs motivated to establish reliability requirements for procurement of military equipment and new military standard (MIL-STD) documents.

One of the milestones in reliability evolution is the establishment of a group on reliability by US Department of Defense in 1950s. In 1952 this group started to be evolved and permanently called as AGREE, that is Advisory Group on the Reliability of Electronic Equipment. This group's objective is to monitor and stimulate interest in reliability matters and recommend measures (Kececioglu, 2002). Later it shows a common set of assumptions that seem to give fairly accurate description of pattern of failures of certain types of electronic components as well as complex systems. In 1957 the group put forward the well know failure rate versus time curve, the bathtub (Grant, Leavenwarth, 1980).

The needs of modern technology, especially the complex systems used in the military and in space programs, led to the quantitative approach, based on mathematical modeling and analysis. In space applications, high reliability is especially essential because of the high level of complexity of the systems and the inability to make repairs of, or changes to, most systems once they are deployed in an outer space mission. This gave impetus to the rapid development of reliability theory and methodology beginning in the 1950s. As the space program evolved and the success of the quantitative approach became apparent, the analysis was applied in many non-defense/space applications as well. Important newer areas of application are biomedical devices and equipment, aviation, consumer electronics, communications, and transportation (Blischke & Murthy, 2003).

Finally product reliability's evolution ensured fully effective and fully economic operation and utilization of the mathematical and statistical techniques applied in reliability activities, not as ends in themselves, but as integral parts of the complete company program for quality. These reliability activities are thus significant components of modern total quality systems which assure all aspects of customer quality satisfaction for a company (Feigenbaum, 1991).

In the late 1990s, the largest number of reliability engineers in the world is concentrated in the automotive industry. Some automotive companies estimate warranty cost represents 1/3 the cost for a new automobile. This cost pressure results in reliability engineers working to reduce the cost of unreliability in the automotive industry for one reason-prevent loss of money (Barringer, 1998).

1.9 Reliability Analyses Classification

According to the acquisition of reliability data, modeling the failure mechanism and reliability is obtained primarily in two ways: Either developing an accelerated model, or life data model.

1.9.1 Acceleration Models

To derive most profound findings from reliability studies, it is necessary to run the products until they fail. In this way the failure mechanism can be determined. It is hard to obtain time to fail values of each product in the population. If each item were tested to fail, then a relatively long period of time would go by until all items incurred to a failure. In addition to that, the reliability tests are destructive and usually expensive to conduct because of the appraisal cost of measuring reliability. As a result, testing under normal operating conditions and using extensive number of items is impractical. This led to the purpose of reliability studies testing as few samples as possible over a short period of time.

To overcome this problem accelerated life tests and acceleration models -a.k.a. true acceleration models- were developed where products are subjected to more severe environment (increased or decreased stress levels) than the normal operating environment (Pham, 2003). Acceleration models thus produce the same failures that would occur at typical use conditions, except that they happen much quicker. The trick is that time is being accelerated.

When there is acceleration, changing stress is equivalent to transforming the time axis which is used to plot failures. The transformations are commonly linear which implies that time to fail values are multiplied by an acceleration factor to obtain the equivalent time to fail at use conditions. In other words, when every time of failure is multiplied by the same constant value to get the results at another operating stress, it is called linear acceleration. Acceleration modeling is represented in Figure 1.9.

On the other hand many other products can not be accelerated because the increased stresses create additional failure mechanisms. They severely speed up the failures compared to normal operating conditions.

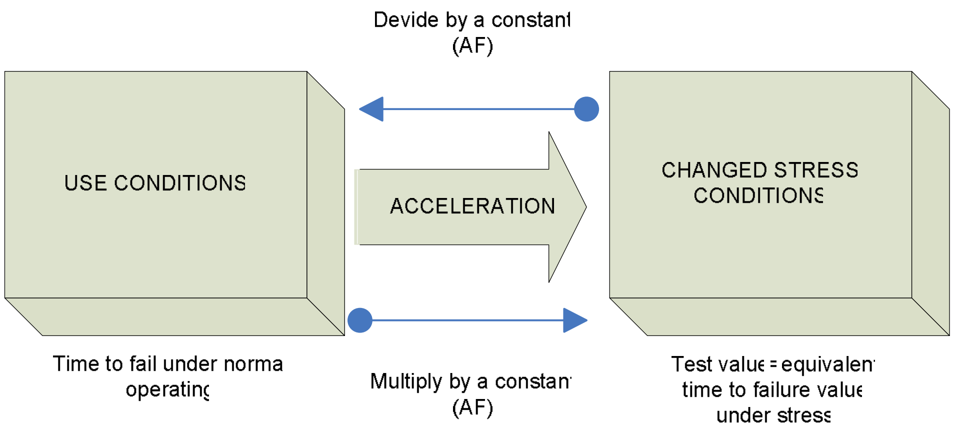

Figure 1.9 Acceleration model

1.9.2 Life Data Models

Life data models -sometimes called lifetime models- are based on actual life times of products. These models consist of general life tests. The tests begin when first prototype products are manufactured and they continue within the lifespan of the products. Potential reliability problems are defined with these tests in design and production processes. There are generally three types of general life tests (Feigenbaum, 1991):

Design Test

It is a reliability test aiming to identify and correct design problems. After manufacturing of prototypes of the product, they are put on a test to understand the primary reliability requirements. If the prototypes can not qualify to reliability requirements, the design is improved.

Process Test

After completion of manufacturing of the products, they are operated for a given period and their performances are measured. Therefore the flawed products after the first production are cleaned off from the population. The failures observed during this period are analyzed to solve problems.

Life Test

Time to failures is measured on a number of samples and distribution of the failure mechanism is determined. The life test ensures that wear out starts beyond a desired life time. The distribution of failure mechanism is analyzed to find out if the failure is caused by wear out or is truly random.

In the following chapter frequently used reliability terms and concepts are provided with details and acceleration models are beyond the scope. Further information about acceleration models such as Arrhenius, Eyring can be found Pham's Handbook of Reliability Engineering.

CHAPTER TWO

BASIC RELIABILITY CONCEPTS AND TERMS

2.1 Measures for Reliability

In this chapter the yardsticks for evaluating reliability of products are briefly introduced. Concepts and terms which are necessary to describe, estimate and predict reliability are defined. As it is mentioned earlier, the reliability theory was derived from probability and statistics. In this way a fundamental statistical knowledge is required to understand reliability.

2.1.1 *The Reliability Function*

The models used to describe lifetimes of items are known as lifetime distribution models. These models consist of a collection of lifetimes of all items in a population. Lifetimes of items are treated as random variables and they form the statistical distribution. As all distributions have properties, lifetime distribution models have their own properties in means of reliability.

The cumulative distribution function (CDF), which is symbolized by F(t), has two important meanings:

- The probability a random item chosen from the population fails by time t
- The fraction of all items in the population which fail by time t

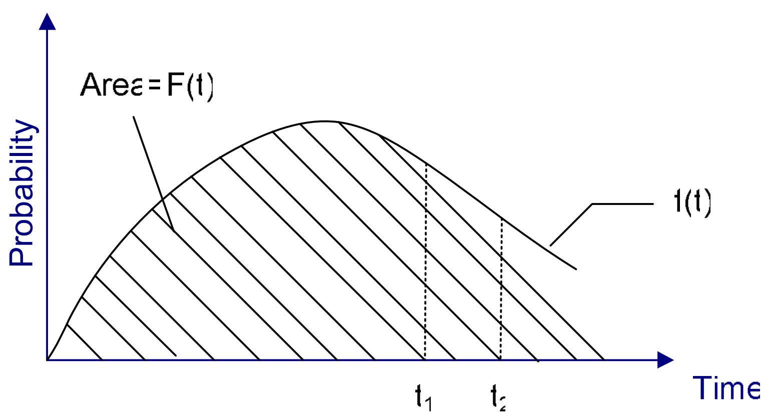

Figure 2.1 Cumulative distribution function

The CDF or the unreliability function F(t) can be plotted on time versus probability like the figure above. The area under the probability density function f(t) expresses the unreliability function F(t). CDF has nonnegative values. It starts from 0 and goes to 1 as time approaches to infinity.

The area between time t_1 and t_2 corresponds to the probability of an item surviving to time t_1 and then failing before time t_2. Secondly, another meaning is that the area is the fraction of the population which fails in that time interval.

From the complement rule that either an item fails or survives, the reliability and unreliability functions are mutually exclusive. So the reliability (survival) function is defined by:

$$R(t) = 1-F(t) \tag{2.1}$$

The reliability function conveys the probability of survival to time t and it is a non-increasing function of time.

Likewise the unreliability function the reliability function has also two important inferences:

- The probability a random item chosen from the population survives to time t
- The fraction of all items in the population which survives to at least time t

2.1.2 *Failure Rate Function*

Distribution of lifetime data can be modeled with the help of probability density function (PDF), cumulative density function (CDF), reliability function and failure (hazard) rate function. PDF and CDF are very known terms since they are broadly used in statistical manner but failure rate function has a particular property in reliability study and because of this it is not widely known.

PDF or f(t) stands for failure probability density function in reliability. From statistics it is familiar that f(t) is the derivative of F(t) with respect to t. It is *the probability of failure in the interval t to t+dt in which dt is an instant of time*.

Failure or hazard rate function is *the instantaneous rate of failure for the survivors to time t during the next instant of time* (Tobias & Trindade, 1995). Failure rate function is expressed as units of failures per unit time. It is not a probability and can take values greater than 1. Failure rate is denoted by either z(t) or h(t). To see how failure rate is calculated, a little probability statistics must be used:

$$\text{P(fail in next dt | survive to t)}=\frac{F(t+dt)-F(t)}{R(t)} \tag{2.2}$$

The equation is divided by dt to convert it to a rate:

$$\frac{F(t+dt)-F(t)}{R(t).dt} \tag{2.3}$$

If dt let approach zero, derivative of F(t) is obtained:

$$\frac{F'(t)}{R(t)} \tag{2.4}$$

Since f(t) is the derivative of F(t) with respect to t, the following rate is derived:

$$\frac{f(t)}{R(t)} \tag{2.5}$$

So this *instantaneous* rate is called failure rate h(t).

It is also expressed in terms of negative derivative of lnR(t):

$$h(t)=-\frac{d\ln R(t)}{dt} \tag{2.6}$$

Failure rate sometimes called conditional failure rate because the denominator R(t) makes it conditional.

Failure rate can be expressed variations of failure per unit time. Below a small example of how the description of failure rate varies is given. As an example let a product has a failure rate of 0.000000286 failures/hours. This rate simply converted to the followings:

h=0.000000286 failures/hour=0.000286 failures/1000 hours
=0.000286K failures/hour=0.0286%/1000 hours
And since there are 8760 hours in a year
h=0.25%/year

Moreover, by integrating the failure rate function h(t), the cumulative failure rate function H(t) is obtained:

$$H(t) = \int_0^t h(t)dt \tag{2.7}$$

The integral can also be expressed in closed form as:

$$H(t) = -\ln R(t) \tag{2.8}$$

2.1.3 *Average Failure Rate Function*

It is sometimes useful to define average rate over an interval of time that averages the failure rates in that interval. AFR(t_1, t_2) stands for the average failure rate between time t_1 to time t_2. The simplest way to specify AFR is to integrate the failure rate over the internal and divide by duration of the interval.

$$AFR(t_1,t_2) = \frac{\int_{t_1}^{t_2} h(t)dt}{t_2 - t_1} \tag{2.9}$$

$$AFR(t_1,t_2) = \frac{H(t_2) - H(t_1)}{t_2 - t_1} \tag{2.10}$$

$$AFR(t_1,t_2) = \frac{\ln R(t_1) - \ln R(t_2)}{t_2 - t_1} \tag{2.11}$$

If the time interval is from 0 to T, then AFR simplifies to:

$$AFR(T) = \frac{H(T)}{T} = -\frac{\ln R(T)}{T} \tag{2.12}$$

2.1.4 *Other Measures for Reliability*

MTTF, MTBF, MTTR:

The expected value or the mean of the lifetimes of items is called mean time to failure (MTTF) or expected life of item. MTTF describes the average time to failure of an item and can be obtained by using these formulae:

$$MTTF = \int_{o}^{\infty} tf(t)dt \tag{2.13}$$

$$MTTF = \int_{o}^{\infty} R(t)dt \tag{2.14}$$

MTTF is used for the items which are not repairable because for repairable items it is said to be the time to the first failure. Many items can fail more than once and after repairing they continue to operate. For these repairable items mean time between failures (MTBF) is used instead of MTTF. For instance a product having a MTTF of 45000 hours implies that some units will actually operate longer than 45000, others shorter than 45000. But on the average the expected lifetime of the units will be 45000 hours.

The MTBF represents the average operating time from the point that a failed device is restored to operation to the point of time that it becomes failed again. It does not include the amount of time needed to repair the failed item. If each repair restores the device to as good as new condition, it is said that the repair is perfect. Under perfect repairs, MTBF is equal to MTTF. Since there is usually an aging effect in most products, very often it is seen a decreasing MTBF as more failures are experienced by the product. The average amount of time needed to repair a failed item is called mean time to repair (MTTR) (Kuo & Zuo, 2003).

Sometimes MTBF stands for mean time before failure which is same as MTTF and sometimes minimum time before failure is used in place of MTTF. Minimum time before failure is completely non-statistical and nonsensical that includes no concept of reliability.

MDT, Availability:

Except for MTTR, there is also a term called mean down time (MDT), which is confused over MTTR. Some literature use MTTR in place of MDT or vice versa but there is a fine distinction between these two terms and they are not identical. Down time is the period in which the item is in failed state. Whenever an item fails, it is not always common that the item is immediately settled to repair. So once the item becomes idle, it does not count the repair time, rather the down time commence. For instance a cutting tool may fail after its usage and are not operated until the next task. After end of the usage the tool become defective but up to the time of the next task it can keep its defect so the down time starts to tick. When the failure is realized, the repair process starts so the repair time begins. A snapshot of the elements of both down and repair time is illustrated in Figure 2.2.

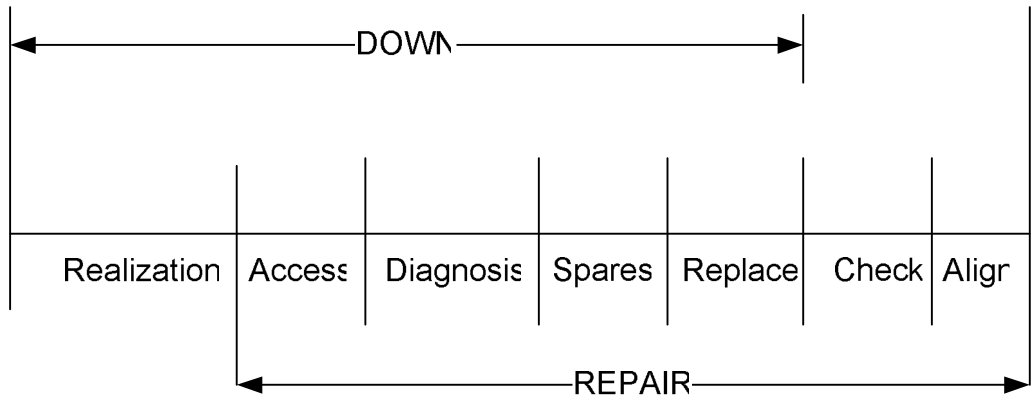

Figure 2.2 Elements of down and repair time

For repairable items, another frequently mentioned term availability is used as a measure of its performance. The availability of an item is defined to be the probability that the item is available whenever needed. For a repairable item with perfect repair on any failure, its availability can be expressed as:

$$Availability = \frac{\text{Up time}}{\text{Total time}} = \frac{\text{Up time}}{\text{Up time} + \text{Down time}}$$

The exact ratio is shown below:

$$Availability = \frac{\text{MTBF}}{\text{MTBF} + \text{MDT}} \tag{2.15}$$

Lastly the figure below summarizes the most important formulae which are widely used in reliability and their relationships amongst and gives a snapshot of their conversion.

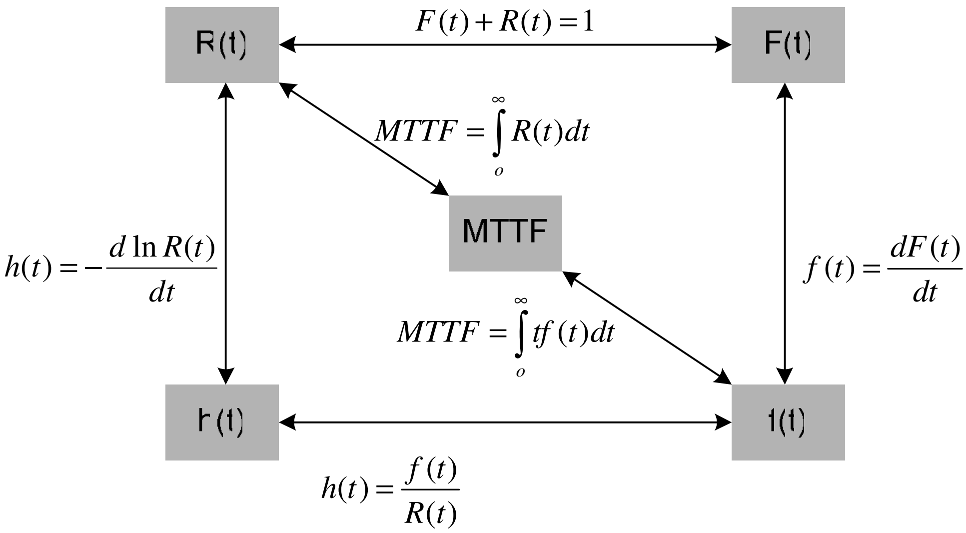

Figure 2.3 Relationship between R(t), F(t), h(t), f(t), and MTTF

2.2 Bathtub Curve

The graph on which the failure rate is depicted over time is called the bathtub curve. It is also named as common life characteristic curve. Its name was given as bathtub curve because of the resemblance to bathtub. This graphical representation describes lifetime of a population of products and is used to show accurate description of product failure and failure patterns. Because the failure rate of products can change with time. So the curve does not display failure mechanism of a single unit of product, contrarily depicts an entire population. Some products in population fail early and some last longer. The all failures form the bathtub curve. As a visual model failure rate versus time illustrates the key periods of product failures. Bathtub curve generally looks like the curve in Figure 2.4.

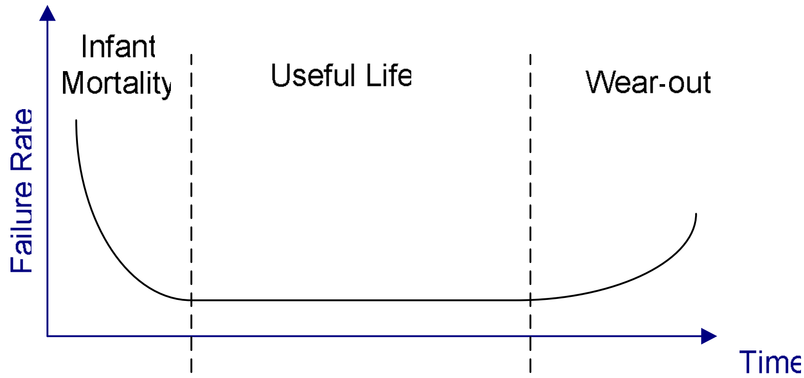

Figure 2.4 Bathtub curve

There are three distinct periods in product failures as they are represented above figure. The failures in population start under infant mortality period with a decreasing failure rate but a high rate in the beginning. This period is also called early failure or debugging period. After the first runs of products, infant mortality comes to an end and useful life period starts. Alternative terms such as normal, intrinsic period are also mentioned in literature. The last period is wear-out period with an increasing failure rate under which the effects of aging is mostly seen.

It should be noted that the shapes of bathtub curves of different devices may be dramatically different. For example, electronic devices have a very long useful life period. Computer softwares generally have a decreasing failure rate. Mechanical devices have a long wear-out period where preventive maintenance measures are used to extend the lives of these devices. Stresses applied on the devices often shift the bathtub curve upward. Figure 2.5 illustrates these different bathtub curves (Kuo & Zuo). According to the figure it can be realized that software products have relatively longer life than hardware products. Software reliability is related with the operational behavior of software based systems with respect to user requirements. Since software development process is more likely to under control of failures, they tend to be more reliable. Software validation and verification are precisely carried out and most of the bugs are eliminated before releasing the product. As the algorithm behind the execution works systematically, the product lasts long until new requirements emerge. But big failures can also be observed in softwares such case called Y2K which occurred in 2000 and affected lots of computers in business. The softwares are therefore continued to improve their bugs.

Consequently actual time periods in the bathtub curve can vary greatly. Some products may have longer periods under which failure rate has a stabilized character and some may immediately wear out or age. In such case it could be a disaster from the warranty standpoint.

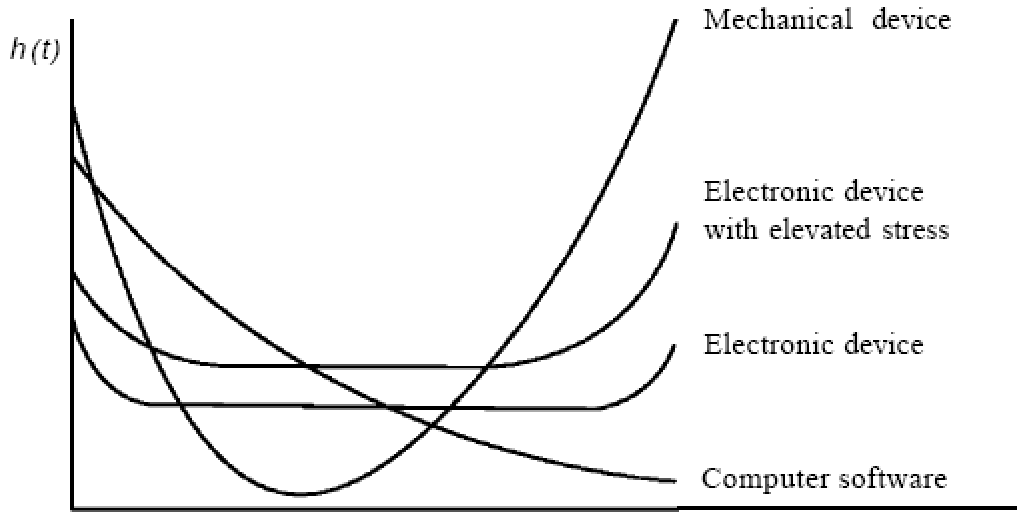

Figure 2.5 Variations of bathtub curve

For a product on the bathtub curve, only one or at most two regions match with the failure distributions of it. Lifetime distribution generally mirrors one part of the curve. For infant mortality period Weibull and Gamma, for useful life period, Weibull, Gamma, and Exponential, for wear-out period, Weibull, Gamma and Normal distributions are applicable to derive the lifetime distribution and failure pattern of product. It should be realized that Weibull and Gamma distributions can be used to describe all failure types occurring in three regions.

As referred to reliability growth which is mentioned in Chapter 1, it is described in means of improvement on the bathtub curve. By eliminating the failures the growth in reliability can be seen on Figure 2.6. The figure illustrates reliability growth by showing two bathtub curves for the same product. Both curves display three periods of bathtub but the below one demonstrates the improvement in reliability after eliminating the errors and defects.

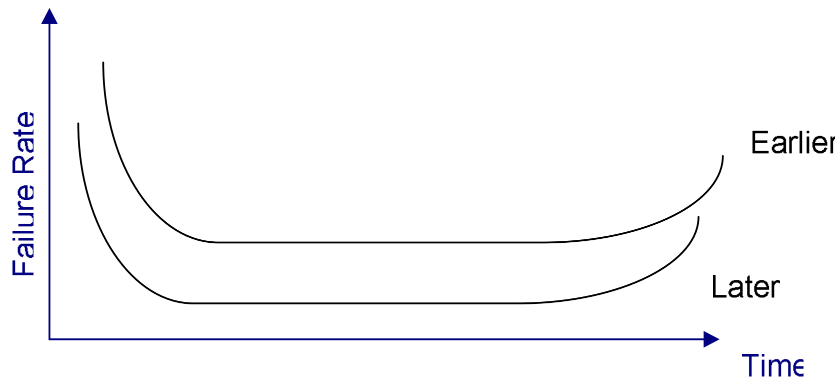

Figure 2.6 Reliability growth

Now let us have a look these three periods of bathtub curve

2.2.1 Infant Mortality Period

Infant mortality is the first period on the bathtub curve. During this period, failure rate starts with a high value and drops as time elapses. Failures can not be tolerated from the customer satisfaction viewpoint since the products that arrive to customers specified as faulty. That is why this period is also called early failure because the failures happen earlier than expected and do not have a random characteristic. Besides products can be defected during their transportation and they become dead before arriving to customer although they were manufactured without any early flaw.

After the first runs of products, the defectively manufactured units appear to be failing first. Hence at the beginning of infant mortality the failure rate is high and after the defective units fail, it owns a decreasing trend and reaches a low level. The decreasing failure rate typically lasts several weeks to a few months. Therefore during the period weak products are weeded out.

The early failures are caused by weakness in materials, components, production processes. That is the defects in design and production constitute infant mortalities. Numerous early failure causes are listed in Table 2.1. To avoid these early failures manufacturer must find out how to eliminate the defects.

To prevent early failures before products are released to customers, appropriate specifications, adequate design tolerances can help but even the best design intent can fail to cover all possible interactions of components in operation. In addition to best design approaches stress testing should be started at the earliest development phases and used to evaluate design weaknesses and uncover specific assembly and material problems. Stress tests like these are called highly accelerated life test (HALT) or highly accelerated stress test (HAST). These tests are applied with increasing stress levels until failures are separated. The Failures should be investigated and design improvements should be made to improve product robustness. Such an approach can help to eliminate design and material defects that would otherwise show up with product failures in the field (Wilkins, 2002).

These stress tests are generally applied only for early production, and then they are reduced to audits as root causes of failures are identified, process design errors are corrected, and significant problems are removed.

Table 2.1 Early failure causes (Kececioglu, 2002)

- Poor manufacturing techniques, including processes, handling, and assembly practices
- Poor quality control
- Poor workmanship
- Substandard materials
- Substandard parts
- Replacing failed components with non-screened ones
- Parts that failed in storage or transit due to improper storage, packaging, and transportation practices
- Parts failing when energized for the first time due to sudden surges of power
- Human error
- Improper installation
- Improper start-up

Nevertheless stress tests can be used in an ongoing manner where the root causes of failures can not be eliminated. These tests and screening is called burn-in. Burn-in tests can be viewed as a type of 100 percent inspection or screening of the product population. All units are run for a period of time before shipment or installation. To accelerate the process components may be run at high levels of temperature or other stresses (Juran, 1999). However some manufacturers tend to carry out fully and long burn-in processes and keep continuously using them. In this respects they rework over the same defects and this is not a cost effective way to improve reliability.

2.2.1.1 Burn-in Process

To see the effect of burn-in process, let us take a small example. Suppose that there is a product population which has an infant mortality period that lasts months and follows a distribution just like Figure 2.7.

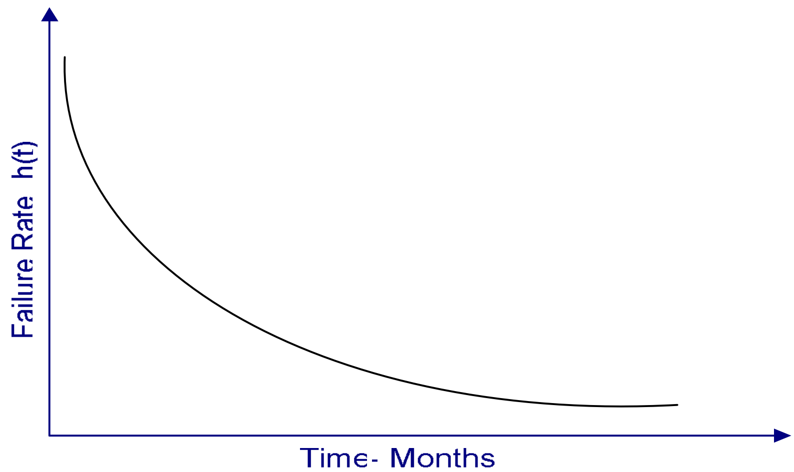

Figure 2.7 Infant mortality and decreasing failure rate

To see how burn-in can improve reliability of the population, survival or reliability plot will be used to represent how many units from the population have survived to a given time.

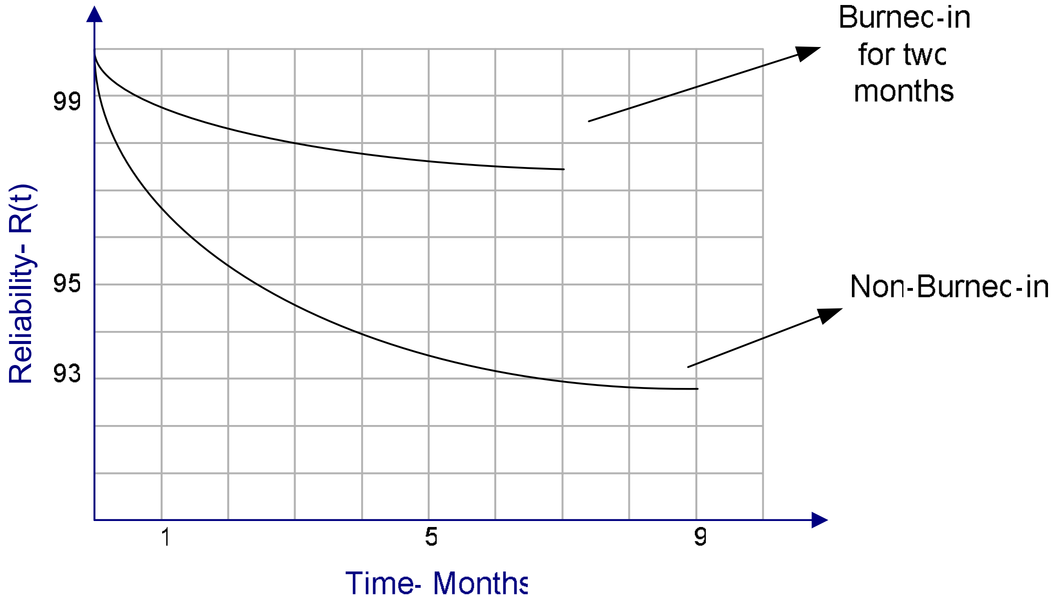

Figure 2.8 Burn-in effects

If failures are driven by defects during infant mortality such the case above, then burn-in process can help. If two months of time the products have operated before they were shipped and were screened during these months, the defective products would fail. If products were burned-in, in seven months their reliability would fall by just around %2.5 as seen on Figure 2.8. Thus by month seven, %97.5 of the products would survive to function properly. On the other hand, if the population were not burned-in, by month seven %7 of them would fail and by month nine the reliability would drop under %93. Consequently if most of the infant mortalities were eliminated, the remaining products would be more reliable than the original population. Of course the products that go through the two months burn-in process would last more in the field. So what is the effect of the two month burn-in? From the Figure 2.8 it can be stated that if no burn-in was applied to population, by month two approximately %4.5 of products would fail. Likewise between month two and month nine-that is a seven month period- the reliability would fall from %95.5 to around %93. Therefore the probability of products which survive to month two but fail before month nine is around %2.5. So there is a decrement of %2.5 in reliability between those months. What if the population were run for two months and later released to customers? This constitutes the effect of burn-in process. By burning-in the population for two months the reliability would only drop by 2.5%. The burned-in population would have a reliability value of %97.5 after seven months. It can also be deduced why burn-in process is applicable for only infant mortality period since there is no advantage to burn-in a population during useful life period under which the failure rate is constant. Likewise burn-in process is not applied in wear-out period because of the increasing failure rate. If it was applied, it would yield worse results as the probability of failure is increasing along time.

As it is mentioned before manufacturers do not have sufficient time to actually burn-in their product populations. They tend to accelerate the stresses during the burn-in process to get results more quickly. For instance take a look at manufacture of conductors. These electronic products can be accelerated by altering the temperature and voltage conditions.

2.2.2 Useful Life Period

The next region on the bathtub curve is the useful life period. During this period failure rate reaches its lowest value and remains fairly constant. After the elimination of defective units in infant mortality, the population incurs useful life period. The failures that occur in this period are called chance or random failures. The names take after causes by chance events which occur unexpectedly in time at random, irregular intervals. Again Kececioglu (2002) has also listed the causes of chance failures which are tabulated in Table 2.2. It should be noted that most products spend their lifetime in this flat portion of the bathtub curve (NIST, 2002).

All the failures during useful life are not chance failures. After some time, failures from infant mortality defects can spread out that they appear to be approximately random in time. Combination of low level infant mortality failures and random failures results in a product failure distribution of useful life period.

Table 2.2 Chance failure causes (Kececioglu, 2002)

• Interference or overlap of designed in strength and experienced stress during operation • Insufficient designed in safety factors • Occurrence of higher than expected random loads • Defects which escape even the best available detection techniques • Human errors in usage • Misapplication • Abuse • Those failures that neither through burn-in nor the best preventive maintenance practices can eliminate • Unexplainable causes • Act of natural failures due to storms, lighting, earthquakes, floods, etc.

2.2.3 Wear-out Period

Wear-out is located on the last region of the bathtub curve under which the failure rate tends to increase since population starts to have degradation and fatigue due to aging. In the long run, everything malfunctions and wear-out occurs after a reasonable useful life. In Table 2.3 causes of wear-out failures which occur late in lifetime of products are given. It is a normal routine to replace the units which are worn out with the new ones so this exchange of components in a system increases the possibility of its service life.

Table 2.3 Wear-out failure causes (Kececioglu, 2002)

- Aging
- Wear
- Degradation in strength
- Fatigue
- Creep
- Corrosion
- Mechanical, electrical, chemical deterioration
- Poor service, maintenance, repair, replacement
- Short designed in life

2.3 Reliability Data

To achieve results from reliability studies for considering product performance in the field, it is necessary to obtain facts from nature to understand failure pattern of products. Reliability data constitute these facts and come from either testing before product release which means testing of prototype or production models or they are obtained from field studies.

The data can be collected from various sources. Subsequent product reliability study can be based on the reliability data of similar existing product. In such cases historical data are used to predict the new product's reliability. Warranty claims which are based on customer dissatisfaction under warranty period usually obtained from dealers or service centers. Operational data are collected from customers as field data. Production and sampling data are collected from in-house in order to evaluate performance before release. These sources of data are displayed below in Figure 2.9.

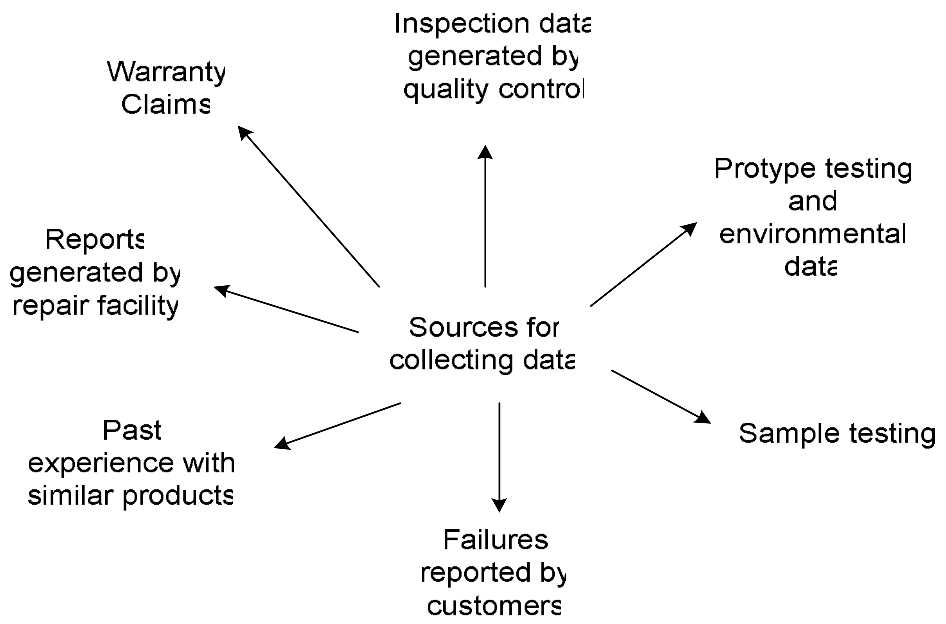

Figure 2.9 Various reliability data sources

2.3.1 Types of Data

The reliability data are classified as complete and incomplete. If the exact values of failure times –a.k.a. lifetimes- are known for each observation in the data set, it is called complete data. Incomplete data are used when exact values are not known for some or all of the observations (Blischke & Murthy, 2003). Incomplete data are frequently encountered in studies. Because they are obtained in a less costly way than complete data and people do not have enough time to observe the failures of all units. Another name for the incomplete data type is censored data. Incomplete data consists of time censored data (Type 1) and failure censored data (Type 2). Moreover grouped data such as readout (Interval censored) and multicensored data are also used as censored data in reliability studies.

2.3.1.1 Censored Type 1 Data

With this type of data, the study is conducted for a fixed duration. The failure times are recorded up to the time in which the censorship takes place. At the end of the study the failure times of the survivors can not be known. Another name for type 1 data is right censored data since the times of failures to the right (larger than the fixed test duration) are missing.

For instance suppose that n numbers of items are put on a test for a planned period of T. During T, r numbers of items fail and their lifetimes are recorded as t_1, t_2, t_3…The failure time of the last item is said to be t_r. At the end of the test (n-r) numbers of items survive. As a result all the information gathered is up to T. The possible failures beyond T are unknown. Let us say that 150 items are put on a test for 100 hours operation. The test would stop by time reaches 100 hours and the remaining unfailed items would not give any information about their lifetimes. Hence lifetimes of 80 items would be evaluated.

Bottom line under censor type 1 testing, the numbers of failures are random since testing time is limited and it is unknown how many items would fail in that duration. Estimating the parameters of lifetime distributions and failure rate depends on the number of failures so inappropriate test conditions would give insufficient information.

2.3.1.2 Censored Type 2 Data

These data are used when the study is terminated after reaching predetermined number of failures. This leads better data but it is less popular because of the open-ended nature of testing duration.

Again n items are put on test and failures times are recorded. Failure times are again recorded as t_1, t_2, t_3 … The test does not end after a predetermined duration, rather is terminated until a number of failures is reached which is planned before. Test lasts until r numbers of items fail so t_r which is the failure time of rth item determines the length of test, T. Since how much data is obtained in the test is specified by the researchers, that is the test has failure terminated characteristic, so-called sufficient data are collected. On the other hand duration of censor type 2 testing is random and that is why it is open-ended.

2.3.1.3 Readout Data

Because of the difficulty in defining the exact failure times readout data is sometimes used instead of censored data. To collect exact failure times, instruments that can record lifetimes and continuously monitoring of test items are needed. This kind of testing setup appears to be impractical in many cases. If exact values are not known readout –a.k.a. interval- data are used.

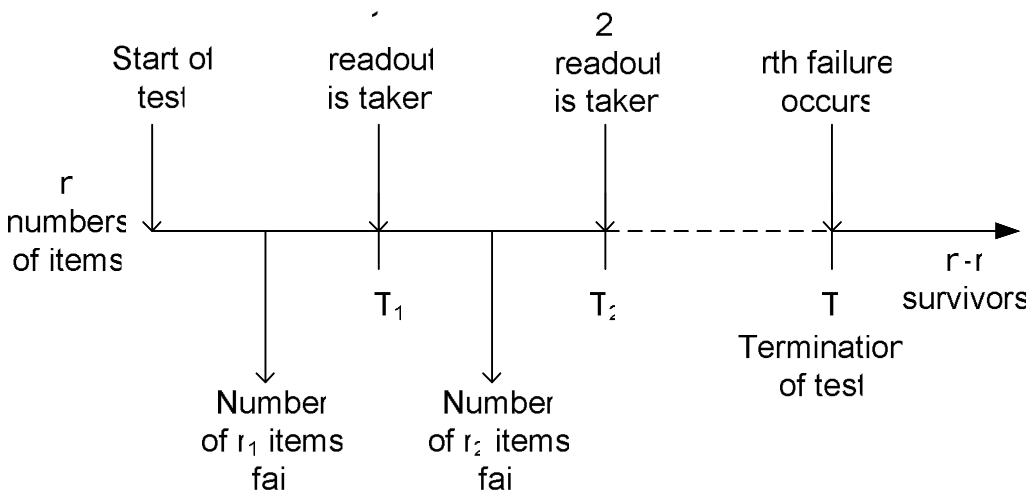

Figure 2.10 Readout data

In readout data testing n numbers of items are put on test. At time T_1 first readout takes place where all items are examined and numbers of r_1 failed items are recorded and removed from sample. Numbers of $(n-r_1)$ items go back to test. At time T_2 second readout takes place and numbers of r_2 failed items are further recorded. This procedure lasts until predetermined numbers of readouts are taken so test duration is known in this type of data. Figure 2.10 illustrates how readout data is tested.

2.3.1.4 Multicensored Data

This type of data is combination of the former types. To sum up every item yields three kind of information (NIST, 2002):

- A runtime of unit that does not fail while under observation
- An exact failure time
- An interval of time during which unit fails

Multicensored data cover all these information listed above.

All in all associated with the reliability data there are two important facts that should be considered. One of them is censorship of the data and lack of failures.

Since collecting complete data consumes a lot of effort and resources, censorship becomes a must in conducting reliability studies. But greatness of the censorship delimits the characteristic of study. If much censorship is applied, then even in a large sample size insufficient data would be obtained and many actual failure times would be undermined. These two facts causing serious problems in practice must be assessed while planning reliability tests and analyzing data.

When fitting models and estimating failure rates from reliability data, the precision of the estimates, which is measured by the width of the confidence intervals, tends to vary not the number of items on test or the length of test. In other words, a test where 5 fail out of a total of 10 on test gives more information than a test with 1000 items but only 2 failures.

Since the number of failures r is critical, not the sample size n on test, it becomes difficult to assess the failure rates of high reliable products. Products that have failure rates measured in units per million per thousand hours will have few or no failures when tested for reasonable time periods with affordable sample sizes (NIST, 2002). So there is the obstacle of how to test products of highly reliable. At this point accelerated life tests come out where these highly reliable products are tested under higher stress levels than their use conditions. If no acceleration is applied it would take many years to get adequate data.

2.4 Review of Reliability Terms

Unreliability function or CDF- F(t): The probability a random item chosen from the population fails by time t and the fraction of all items in the population which fail by time t.

The reliability function- R(t): The probability a random item chosen from the population survives to time t and the fraction of all items in the population which survives to at least time t.

Probability density function (PDF)- f(t): The probability of failure in the interval t to t+dt in which dt is an instant of time.

Failure or hazard rate function- h(t): The instantaneous rate of failure for the survivors to time t during the next instant of time.

Average failure rate function- AFR(t_1, t_2): The average failure rate between time t_1 to time t_2.

Mean time to failure (MTTF): The average time to failure of an item.

Mean time between failures (MTBF): The average operating time from the point that a failed device is restored to operation to the point of time that it becomes failed again.

Mean time to repair (MTTR): The average amount of time needed to repair a failed item.

Mean down time (MDT): The average amount of time in which the item is in failed state.

Availability: The probability that the item is available whenever needed.

Bathtub curve: The graph on which the failure rate is depicted over time.

Burn-in: The ongoing stress test under infant mortality period of the bathtub curve to eliminate root causes of failures by screening.

Censored Data: Data type in which exact values of failures are not known for some or all of the observations.

CHAPTER THREE
RELIABILITY MODELING

3.1 Introduction to Modeling

In this chapter how to model the lifetimes of failures are presented. Common lifetime distributions and methods of estimating the parameters of these distributions are explained.

Two approaches for modeling failure times are as follows (Murthy & Bulmer & Eccleston, 2004):

- Theory based modeling: The modeling based on the established theories for failures. This kind of model is also called white-box model.
- Empirical modeling: The failure data forms the basis for the model building. This kind of model is also called black-box model.

Empirical modeling involves selection of model and estimation of model parameters. Life data can be described using a variety of distributions. In model selection stage the distribution which fits best to the failure time data is defined. This is a parametric distribution selection. If a distribution that fits data can not be found, then nonparametric estimation is used to estimate parameters. The data are fit to a curve to interpret failure. This fitting can be provided by statistical viewpoint that are either analytical or graphical methods.

On the whole the aim in distribution analysis of reliability data is to estimate the lifetime distribution of products. Either parametric or nonparametric estimates are used. Parametric estimates are based on an assumed parametric distribution whereas nonparametric estimates assume no parametric distribution.

3.2 Common Lifetime Distribution Models

There are handy parametric lifetime distribution models which are successfully used to model failure times of products. These parametric distribution models are preferred over nonparametric modeling as they generally match with a failure mechanism. It should be pointed out that the chosen model must make sense. That is data that constitutes an exponential distribution should not be modeled by a normal distribution. Secondly the model of course must pass goodness of fit test.

Weibull, exponential, normal, gamma, lognormal, extreme value are used in modeling failure times. Weibull and exponential distributions will be broadly explained and the others will be behind the scope.

3.2.1 Weibull Distribution

Weibull distribution invented by Waloddi Weibull is the leading distribution in the world for fitting and analyzing life data. It proved to be a successful model for many product failure mechanisms because it is a flexible distribution with a wide variety of failure rate curve shapes. Hence it is capable of describing various failure rate conditions by adjustment of parameters and so different models can be derived from Weibull distribution.

The advantages of the distribution were stated by Abernethy (2006) in following ways:

- The primary advantage is the ability to provide reasonably accurate failure analysis and failure forecast with extremely small samples. Small samples thus allow the study to be in a cost effective manner.

- Another advantage of Weibull distribution is that it provides a simple and useful graphical plot of the failure data as the plot is extremely important to engineers and managers.

- It exhibits wide range of distribution shapes which makes Weibull the leading distribution.

Weibull distribution has either two or three parameters. The two parameter Weibull distribution consists of beta (β) and eta (η). β is the shape or sometimes called Weibull gradient whereas η is the scale, feature or characteristic life parameter. Shape parameter controls the shape of the distribution while scale parameter fixes one point of the CDF F(t), the 63.2 percentile or characteristic life point. 63.2% of the population fails by the characteristic life point, independent of the value of the shape parameter. This is expressed in hours, cycles, etc. if it is known that an item will not fail until a specific time in service, then a third parameter can be added to Weibull distribution. This parameter is called location parameter and is symbolized by gamma (γ). It should be noted that there are several ways of symbolizing these parameters but here β, η and γ are fixed. Shape, scale, and location parameters must be greater than zero and the distribution is defined for only positive times.

Associated with the bathtub curve, shape parameter is the most important issue because all the three distinct region of the bathtub curve can be modeled with Weibull distribution. If shape parameter is wrongly estimated, the lifetime model will be useless to find out the reliability or survival at one point of time. The parameter takes three different types of values. The infant mortality period of lifetime is modeled with a $\beta<1$. Failure rate is decreasing during infant mortality and Weibull distribution with a shape parameter of $\beta<1$ also indicates the same curve of the failure rate function. In the same way Weibull distribution with a shape parameter of $\beta=1$ is used to model useful life period failures, because if $\beta=1$, Weibull reduces to exponential distribution with a constant failure rate of λ. For modeling wear-out period, Weibull's shape parameter takes a value of $\beta>1$ since the failure rate is increasing.

As a result, the wrong estimation of the shape parameter affects results of the Weibull model, because wrong lifetime period of population could be taken up. The actual values could show different periods but the model could point out values of different period. That is why any reliability estimation at any time would be incorrect.

3.2.1.1 Properties of Weibull Distribution

The two parameter Weibull distribution has a PDF given below.

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \tag{3.1}$$

Where

$f(t) \geq 0$

β= shape parameter

η= scale parameter

If the three parameter Weibull is used, then location parameter (γ) will be added by replacing t by (t- γ). If the location parameter is known, it can be stated that no failure can occur before time γ, so the time scale starts at γ not zero. In three parameter Weibull, PDF will be:

$$f(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right] \tag{3.2}$$

A snapshot of Weibull distribution graphs are given as example in Figure 3.1 in which it has parameter values of β=1, η=1 and γ=0.

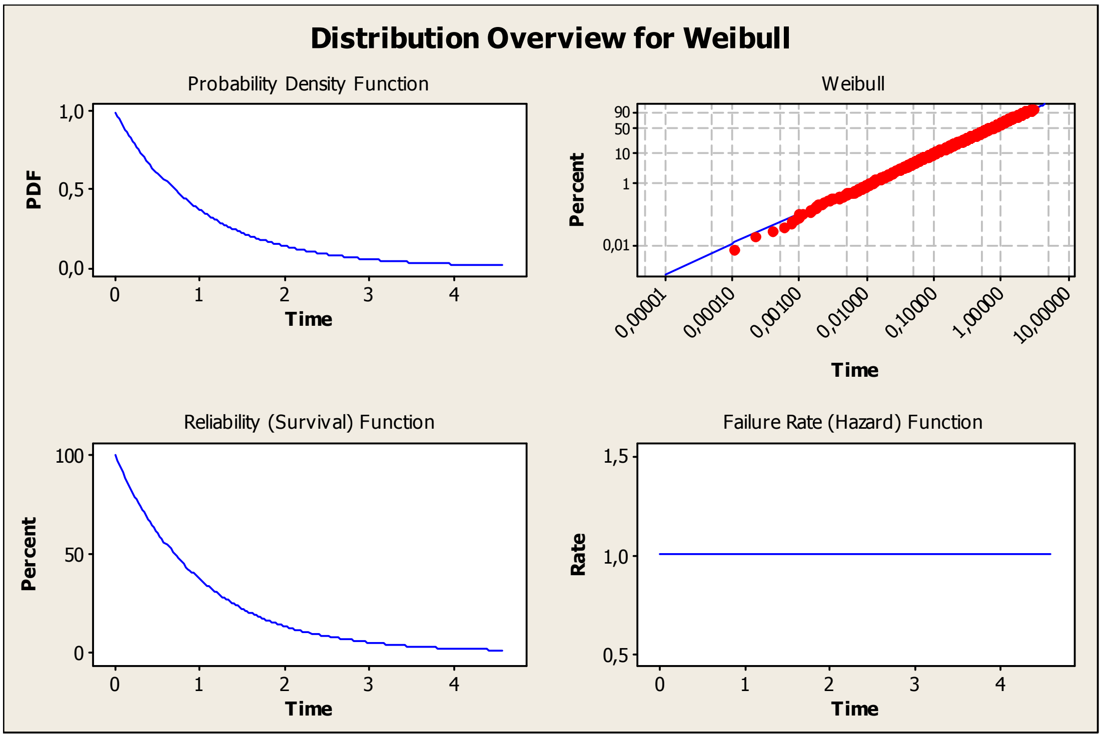

Figure 3.1 Overview of Weibull distribution graphs with β=1, η=1 and γ=0

By changing the value of the shape parameter, the distribution has different shapes as it is shown in Figure 3.2 and Figure 3.3.

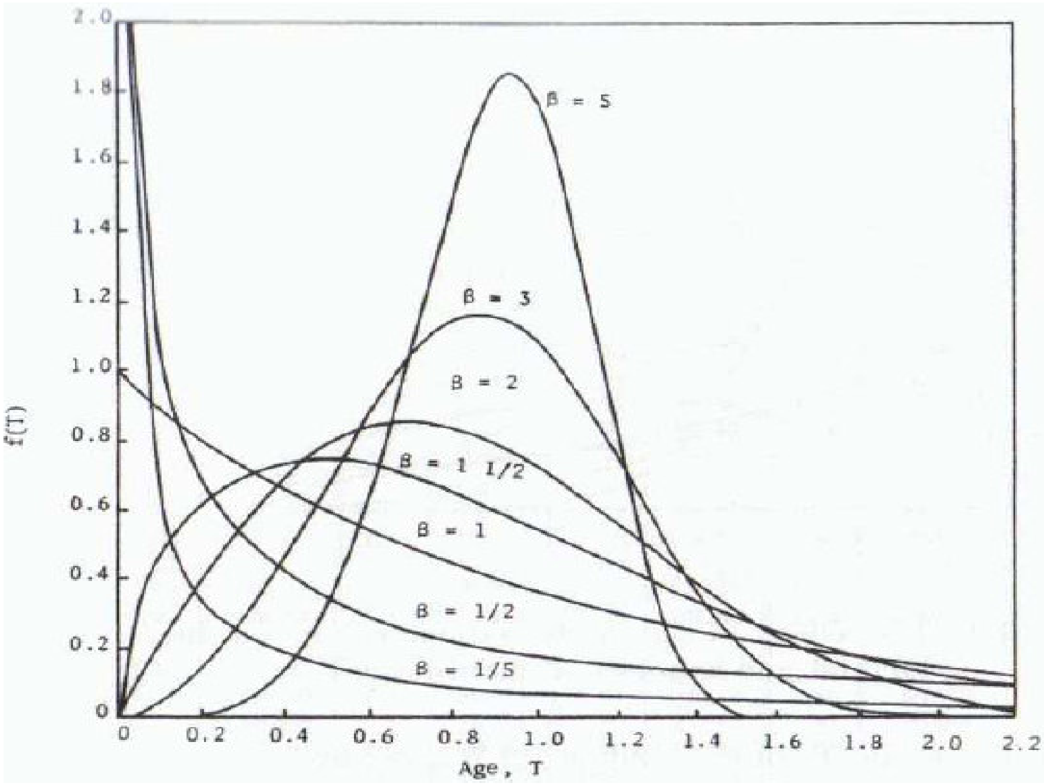

Figure 3.2 PDF versus different values of β (Kececioglu, 2002)

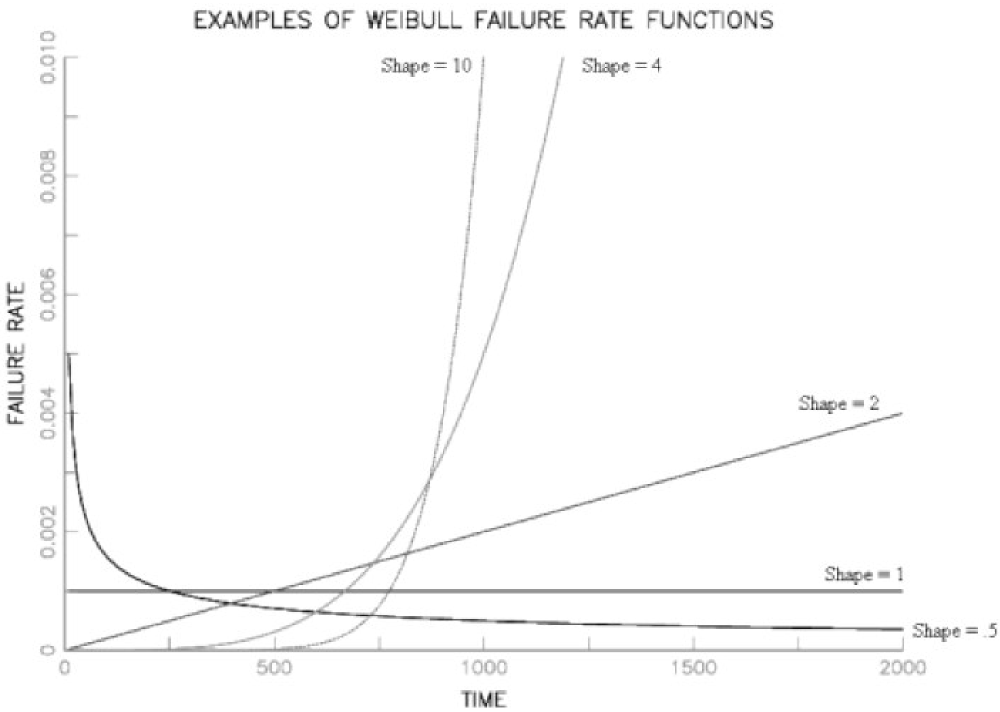

Figure 3.3 Failure rate function versus different values of β (NIST, 2003)

It is obviously depicted in Figure 3.2 that shape parameter plays the major role in determining how the failure rate looks like. All the three regions of the bathtub curve can be seen from the graph. This flexibility can also be encountered when shape parameter is equal to 1 or 2. In the former case Weibull reduces to exponential distribution whereas in the latter Rayleigh distribution.

The unreliability function or CDF is given by

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta} \right)^{\beta} \right] \tag{3.3}$$

Since R(t)=1-F(t), reliability or survival function is obtained by

$$R(t) = \exp\left[-\left(\frac{t}{\eta} \right)^{\beta} \right] \tag{3.4}$$

According to Equation 2.5 the failure rate of Weibull distribution is derived by

$$h(t) = \frac{\frac{\beta}{\eta}\left(\frac{t}{\eta} \right)^{\beta-1} \exp\left[-\left(\frac{t}{\eta} \right)^{\beta} \right]}{\exp\left[-\left(\frac{t}{\eta} \right)^{\beta} \right]} = \frac{\beta}{\eta}\left(\frac{t}{\eta} \right)^{\beta-1} \tag{3.5}$$

Average failure rate is obtained by

$$AFR(t_1, t_2) = \frac{[(t_2/\eta)^{\beta} - (t_1/\eta)^{\beta}]}{t_2 - t_1} \tag{3.6}$$

The Weibull CDF has four quantities which can be calculated if any of the three of them are known. These quantities are cumulative fraction failed F(t), the time t, shape parameter β, and scale parameter η.

$$\beta = \frac{\ln[-\ln(1-F)]}{\ln(t/\eta)} \tag{3.7}$$

$$t = \eta[-\ln(1-F)]^{1/\beta} \tag{3.8}$$

$$\eta = \frac{t}{[-\ln(1-F)]^{1/\beta}} \tag{3.9}$$

To find out the expected or mean value of the Weibull distribution, it is required to introduce a mathematical function called gamma function which is symbolized by Γ. This function is given by

$$\Gamma(v) = \int_0^\infty x^{v-1} e^{-x} dx \tag{3.10}$$

When v is an integer, the formula reduces to

$$\Gamma(v) = (v-1)! \tag{3.11}$$

When v=0.5, gamma function has value of

$$\Gamma(0.5) = \sqrt{\pi}$$

The mean or from reliability standpoint MTTF of the Weibull distribution is given in terms of gamma function below

$$\mu = \eta\Gamma(1+1/\beta) \tag{3.12}$$

3.2.2 Exponential Distribution

Exponential distribution is one of the most common and handy distribution in statistics. The distribution exhibits a constant failure rate so the failures of the useful life period of the bathtub curve can be modeled with exponential distribution. It is known that exponential distribution can be derived from Weibull distribution in case the Weibull's shape parameter equals to 1. The formulae of exponential distributions can thus easily be obtained.

When β takes a value of 1 in Equation 3.1 PDF of the exponential distribution will look like

$$f(t) = \frac{1}{\eta} e^{-\left(\frac{t}{\eta}\right)} \tag{3.13}$$

If λ is placed over 1/η, PDF will be

$$f(t) = \lambda e^{-\lambda t} \tag{3.14}$$

Where

$f(t) \geq 0$

λ= constant failure rate

The unreliability function or CDF is obtained again in the same way by converting Equation 3.3. If β=1 and λ is placed over 1/η, CDF will be

$$F(t) = 1 - e^{-\lambda t} \tag{3.15}$$

And the reliability function will be

$$R(t) = e^{-\lambda t} \tag{3.16}$$

The reason why exponential distribution has a constant failure rate is provided by calculating the failure rate function which is f(t) over R(t).

$$h(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda \tag{3.17}$$

Average failure rate is also constant

$$AFR(t_1, t_2) = \lambda \tag{3.18}$$

Distribution overview for exponential distribution with a parameter of λ=1 is represented in Figure 3.4.

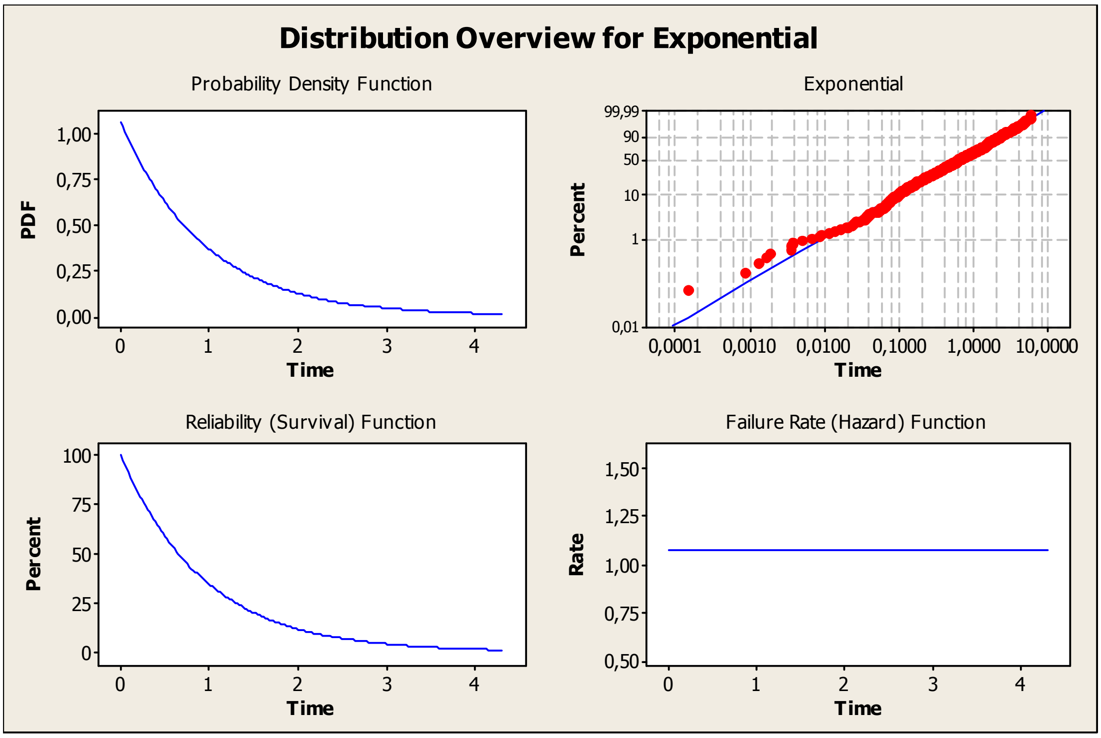

Figure 3.4 Overview of Exponential distribution graphs with λ=1

For the exponential distribution h(t)=λ shows that failure rate function reduces to the value of λ for all times as Figure 3.4 shows how the failure rate looks like along time. This is a characteristic property of the exponential distribution. As mentioned in chapter 2, failure rate is expressed as units of failures per unit time. Thus, if time is in hours, then λ is in failures per hours.

It can also be stated that exponential distribution does not have any shape parameter as it has only one shape and a constant failure rate. For different values of the λ, PDF looks like as it is shown in Figure 3.5.

The distribution starts at time zero and f(t) has a value of λ. Later as time elapses, PDF decrease exponentially.

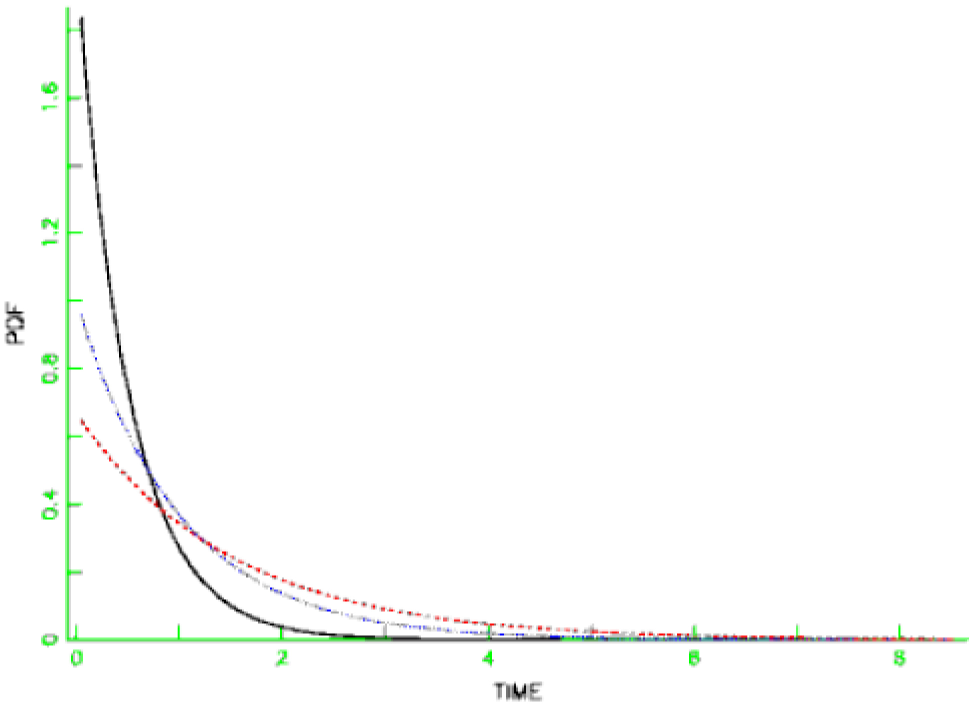

Figure 3.5 PDF versus different values of λ (NIST, 2003)

The mean or MTTF of the exponential distribution is calculated by the following

$$\mu = \int_0^\infty t\lambda e^{-\lambda t}\,dt \qquad (3.19)$$

This expression can be integrated and the following will be attained

$$MTTF = \mu = \frac{1}{\lambda} \qquad (3.20)$$

MTTF for a population with a constant failure rate of λ is the reciprocal of that failure rate which is 1/λ. It should be paid attention that for both Weibull and exponential distributions the mean of the distribution is not the same as the time when half of the population fails. To calculate the latter, one should investigate the median time T_{50}; that is $F(T_{50})=0.50$

To investigate what portion of the population has been failed when MTTF value is reached, MTTF is placed in unreliability function in Equation 3.15.

$$F(MTTF) = F\left(\frac{1}{\lambda}\right) = 1 - e^{-\lambda\left(\frac{1}{\lambda}\right)} = 1 - e^{-1} = 0.632$$

This means that 63.2% of an exponential population with a failure rate of λ has failed by the time the MTTF or 1/ λ is reached.

This can also be adapted to Weibull distribution and the question that why scale parameter η indicates the point in CDF where 63.2% of the population fails by the characteristic life point which is η. As referred to Equation 3.3 let us find out the value of CDF when time is the characteristic life.

$$F(\eta) = 1 - \exp\left[-\left(\frac{\eta}{\eta}\right)^{\beta}\right] = 1 - e^{-1} = 0.632$$

As it is seen above no matter what value the shape parameter takes, the characteristic life point will always indicate the same value of 63.2% in CDF.

3.2.2.1 Lack of Memory Property

Except for the constant rate property, exponential distribution has another property called lack of memory. An item following exponential distribution does not remember how long it has been operating. The probability that the item will fail in the next hour of operation is the same if it were new, one month old, or several years old (Tobias & Trindade, 1995). In terms of statistics this is explained through conditional probability.

P(fail in next h| survive to t)=P(new item fails in h)

$$\frac{F(t+h) - F(t)}{R(t)} = F(h) \tag{3.21}$$

The most significant founding signaled by this property is that products do not age or wear out or degrade with time or use. Failure is a chance happening, always at the same constant rate. That is why exponential lifetime distribution model is used for modeling the useful life failures of the bathtub curve. Exponential model is thus said to be a good choice of modeling failures when products have no significant wear-out mechanism at least for their intended application life and early failures of infant mortality are not expected and not obligated to weed out.

3.2.3 The Other Distributions

Besides Weibull and exponential distributions, normal, lognormal and gamma distributions are summarized in terms of their PDFs.

3.2.3.1 Normal Distribution

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t-\mu)^2/2\sigma^2} \tag{3.22}$$

Where

μ= location parameter

σ= scale parameter

The case where μ=0 and σ=1 is called standard normal distribution which is given by

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \tag{3.23}$$

PDF of normal distribution is given as an example in Figure 3.6.

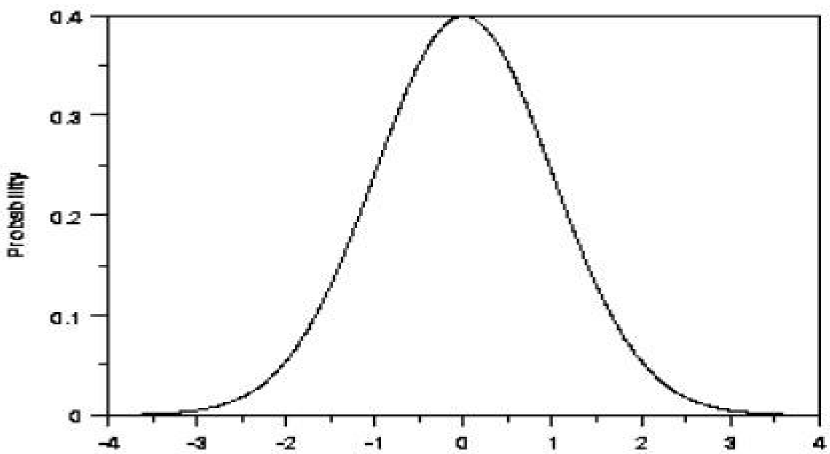

Figure 3.6 PDF of normal distribution (NIST, 2003)

3.2.3.2 Lognormal Distribution

$$f(t) = \frac{1}{\sigma t\sqrt{2\pi}} \exp[-(1/2\sigma^2)(\ln t - \ln T_{50})^2] \tag{3.24}$$

Where

σ= shape parameter (note that not the standard deviation of lifetimes of the population)

T_{50}= median parameter (median time to fail of the population)

PDF of lognormal distribution is given as an example in Figure 3.7.

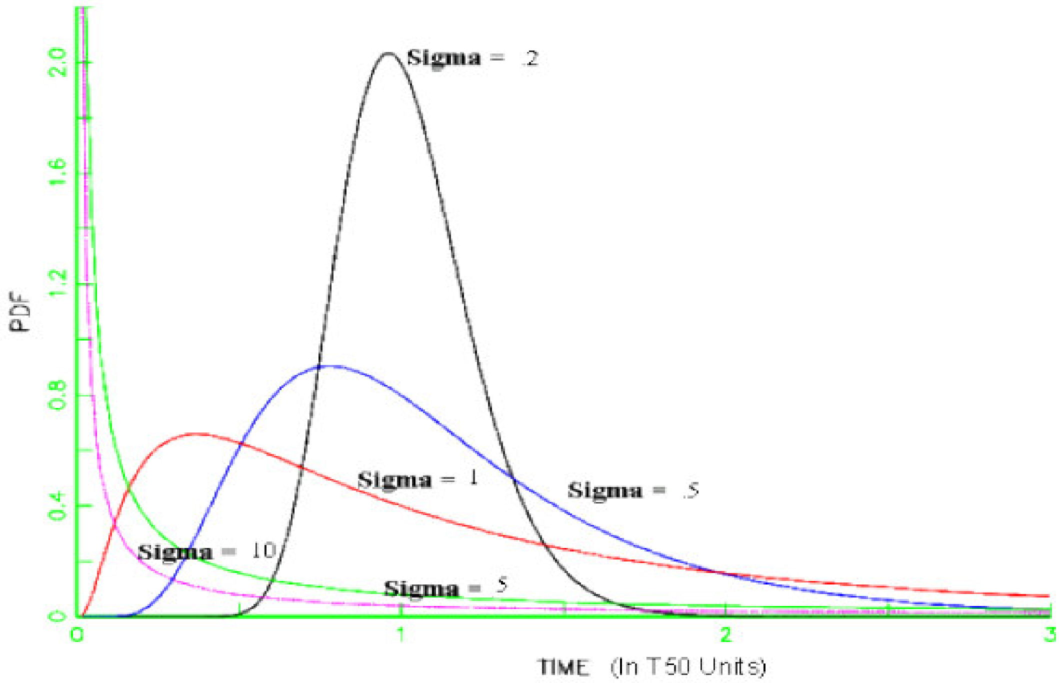

Figure 3.7 PDF of lognormal distribution (NIST, 2003)

3.2.3.3 Gamma Distribution

$$f(t) = \frac{\left(\frac{t-\mu}{\beta}\right)^{\gamma-1} \exp\left(-\frac{t-\mu}{\beta}\right)}{\beta\Gamma(\gamma)} \tag{3.25}$$

Where

γ= shape parameter

μ= location parameter

β= scale parameter

The case where μ=0 and β=1 is called the standard gamma distribution whose equation is the following

$$f(t) = \frac{t^{\gamma-1} e^{t}}{\Gamma(\gamma)} \tag{3.26}$$

PDF of gamma distribution and effect of the gamma function is given as an example in Figure 3.8.

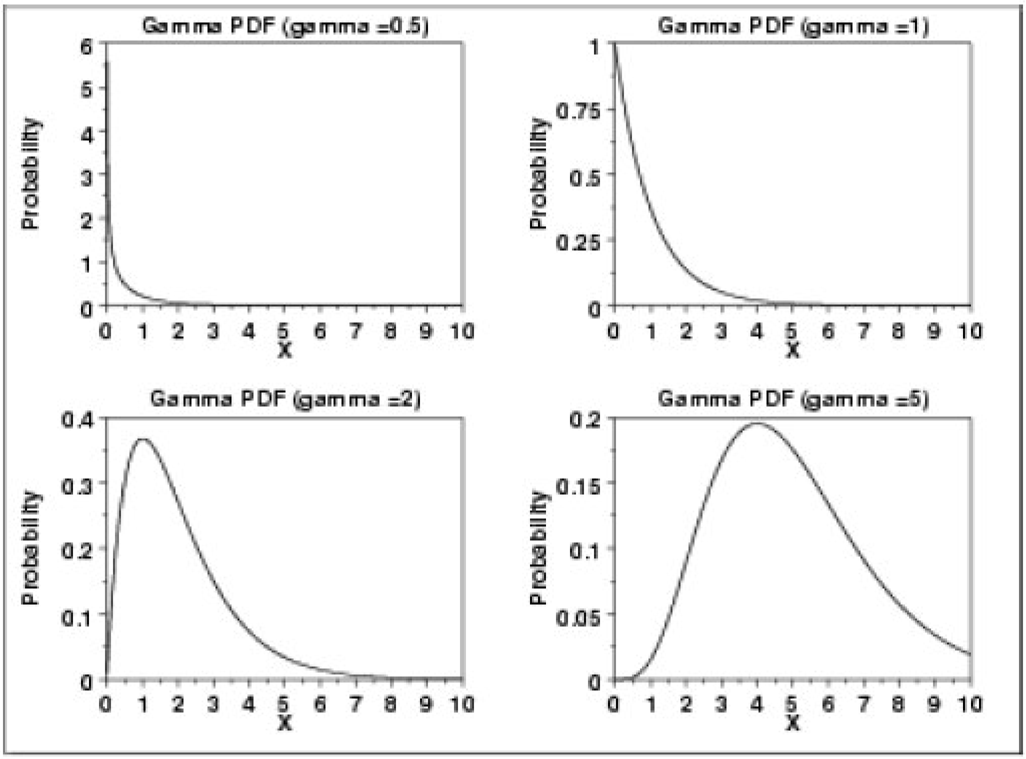

Figure 3.8 PDF of gamma distribution (NIST, 2003)

3.3 Estimating Parameters of Lifetime Distributions

To accomplish a reliability model one should firstly fit the reliability data to the best possible distribution and then estimate the parameters of that distribution. Hence various properties from the fitting and distribution such as the failure probabilities and quantiles can be estimated.

There are mainly two kinds of ways to estimate the distribution parameters: Analytical and graphical methods. Analytical methods consist of maximum likelihood and linear regression (rectification or least square) methods. Graphical method is probability plotting on a special paper. These methods are valid for both Weibull and exponential distributions so the following parts deal with Weibull parameter estimation.

3.3.1 Maximum Likelihood Estimation Method

Maximum likelihood estimation (MLE) is one of the best and well known techniques of estimation in statistics. The method starts with writing the mathematical expression of likelihood function which contains the unknown parameters that will be estimated. The estimates of the parameters are the values of parameters that maximize the likelihood function.

The method is a desired choice when estimating the parameters especially in the case where the data has censorship. But on the other hand sometimes it becomes tough to solve the likelihood equation and computer aid must be involved. Recently there are several options of statistical software packages that help using MLE since they provide the algorithms for MLE and commonly used lifetime distributions.

Besides MLE has some own advantages and they can be stated as (Montgomery & Runger, 2003):

- Lack of bias: The expected value of the estimate equals to the true parameter.

  $E(\hat{\alpha}) \cong \alpha$

  Where $\hat{\alpha}$ is maximum likelihood estimator for parameter α.

- Minimum variance: The variance of the estimator is as small as the variance that could be obtained with any other estimators.
- Estimators have an approximate normal distribution.

On the other hand except for its calculation challenge MLE has another drawback that with small sample size it becomes heavily biased. In practice it may be difficult to estimate without both a lack of bias and having minimum variance. Therefore the benefits enumerated above are applicable when only sufficient sample size of failures is obtained. Another technique of estimation should be used instead when there is not large amount of reliability data.

To estimate the parameters of distributions, the likelihood function in Equation 3.27 should be written.

$$L = c\left(\prod_{i=1}^{r} f(t_i)\right)\left[1 - F(T)\right]^{n-r} \tag{3.27}$$

Where

L= likelihood function

c= likelihood constant

t_i= failure time of item i

r= number of failures

T= test duration

In likelihood function if the data is censored type 1, then the test duration is fixed and is denoted by T. Otherwise if the data is type 2 censored, which the number of failures are fixed, T stands for the time that last item fails. The likelihood constant c will not be of importance during the calculations.

For instance to estimate the two parameters of Weibull distribution, the Equation 3.27 will look like

$$L = c\prod_{i=1}^{r}\left\{\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1}\exp\left[-\left(\frac{t_i}{\eta}\right)^{\beta}\right]\right\}\exp\left[-\left(\frac{T}{\eta}\right)^{n-r}\right] \tag{3.28}$$

Since it is difficult to solve the Equation 3.28 without a software help, the easier example of MLE method is given in estimating the parameters of exponential distribution which is also derived from Weibull. As it is known from distributions the exponential distribution has only one parameter, λ. The following steps will help estimating λ.

When the likelihood function is rearranged due to exponential distribution, Equation 3.29 is obtained.

$$L = c\lambda^r \left[\exp(-\lambda \sum_{i=1}^{r} t_i) \right] (e^{-\lambda T})^{n-r} \tag{3.29}$$

If logarithm of Equation 3.29 is taken, the function takes this form

$$\ln L = \ln c + r \ln \lambda - \lambda \sum_{i=1}^{r} t_i - (\lambda T)(n-r) \tag{3.30}$$

As stated in the beginning the estimate of the parameter is the value of parameter that maximizes the likelihood function. To find the value of λ which maximizes the function above, the partial derivative with respect to λ is taken and it is set equal to 0.

$$\frac{\partial \ln L}{\partial \lambda} = \frac{r}{\lambda} - \sum_{i=1}^{r} t_i - (n-r)T = 0 \tag{3.31}$$

If the Equation 3.31 is solved for λ, the estimator for parameter λ is given by

$$\hat{\lambda} = \frac{r}{\sum_{i=1}^{r} t_i + (n-r)T} \tag{3.32}$$

So once the failure rate λ is computed, reliability function, R(t), unreliability function, F(t), and quantiles can be estimated.

3.3.2 Linear Regression Method

The other analytical parameter estimation method linear regression is based on a procedure called rectification in which CDF is converted into a form that through proper substitutions the CDF becomes a linear equation. Therefore this method is sometimes called linear rectification. After the rectification process, the linear equation is used to estimate the parameters of distribution just like the regression technique in statistics. As the linear regression uses least square error to estimate the coefficients of regression line, another name for this parameter estimation is also least square error method.

The least square error fit is defined as drawing a line though data points on scatter diagram such that the sum of the squares of the deviations of the predicted values from the observed values is a minimum. In statistics this line is said to be regression line as illustrated in Figure 3.9. The circles represent the data points and the dotted lines coming perpendicular to the line plainly depict the deviation from the predicted values.

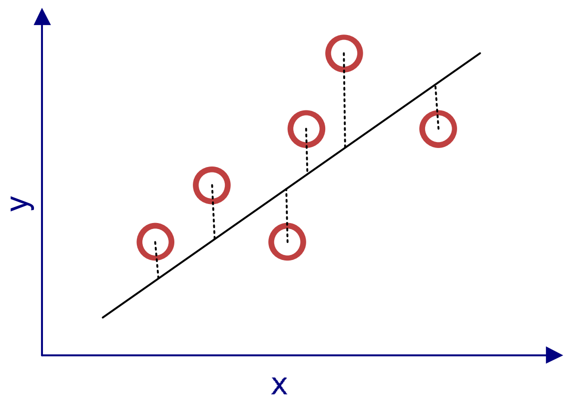

Figure 3.9 Linear regression line

To illustrate how the parameter estimation is done through linear regression method, Weibull distribution is taken up and both parameters β and η will be estimated in the following equations.

Recall from Equation 3.4, $1\text{-F(t)} = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$ and the logarithm of this expression is given by

$$\ln[1 - F(t)] = -\left(\frac{t}{\eta}\right)^{\beta} \tag{3.33}$$

If another logarithm of Equation 3.33 is taken, then it is converted into

$$\ln\{-\ln[1 - F(t)]\} = \beta \ln\left(\frac{t}{\eta}\right) = \beta \ln t - \beta \ln \eta \tag{3.34}$$

So the rectification is provided that CDF of Weibull distribution is converted into a linear expression. The Equation 3.34 designates a regression line if the parameters of a regression line y=mx+b is defined as follows

$y = \ln\{-\ln[1-F(t)]\}$ (3.35)
$m = \beta$ (3.36)
$b = -\beta\ln\eta$ (3.37)
$x = \ln t$ (3.38)

To estimate y, F(t) should be estimated. Wu, Zhou and Li (2005) has defined four ways to estimate F(t). The following four expressions give the probability estimator of F(t).

- $y_i = \dfrac{i - 0.5}{n}$
- $y_i = \dfrac{i}{n + 1}$
- $y_i = \dfrac{i - 0.3}{n + 0.4}$
- $y_i = \dfrac{i - 3/8}{n + 1/4}$

Where, y_i= probability of failure for the ith item (probability estimator)
n= sample size

The first equation yields the least biased estimators whereas the second the largest biased estimators. The second equation is called mean rank equation. The most common estimation method of F(t) is the third equation called median rank equation having slightly more accurate results.

After estimating F(t) and applying linear regression rules, through equations 3.36 to 3.38 the estimators of both shape and scale parameters are simplified to

$\hat{\beta} = \beta$ and,

$$\hat{\eta} = e^{\frac{b}{\beta}}$$

As stated in previous parts MLE method leads to the highest estimation precision of parameters. However, the most widely used may be the linear regression method due to its simplicity. Moreover, MLE method results more often in an overestimation of the shape parameter of Weibull distribution, and hence results in a lower safety than linear regression method in reliability prediction. From an engineering point of view, linear regression method is therefore to be preferred (Wu et al.).

Lastly it must not be overlooked that the linear regression method is not applicable for readout data.

3.3.3 Kaplan-Meier Approach

The analytical methods of MLE and linear regression are parametric ways of estimating parameters. Another approach of estimating expect for the ways above is Kaplan-Meier (KM) technique. It is a nonparametric estimation method; that is there should not be any distribution to fit the data. KM technique is a simple method to estimate reliability values of data. Reliability is estimated until the last failure occurs. In this respect KM approach is easy to apply without any complex statistical methods.

Steps for computing KM estimates are the following (NIST, 2003):

- Actual failures times from t_1 to t_r, where there are r numbers of failures, are ordered.
- Corresponding to each t_i, the number n_i is associated with n_i= the number of operating units just before the ith failure occurred at time t_i.
- $R(t_1)$ is estimated from $(n_1-1)/n_1$
- Likewise $R(t_i)$ is estimated from $R(t_{i-1})(n_i-1)/n_i$
- CDF is estimated from $1-R(t_i)$

3.3.4 Graphical Method

In this method the failure data are plotted on a special Weibull graph paper which is specifically designed on a logarithmic scale. The paper consists of time and cumulative probability axis. A line through these data points is drawn to estimate the shape and scale parameters.

These papers provide crude estimates of the parameters but on the other hand they are easy to use and make a visual estimation. Moreover, the calculations are simple and no special software is needed.

A sample Weibull probability paper is shown in Figure 3.10. It is clear that these papers are on logarithmic scale since both the time and unreliability axis grows logarithmically. These papers have their own kind of properties. Graphical method's comfort comes from its straightforward estimation of parameters. That is why both shape and scale parameters are estimated on this specially scaled paper. As it is depicted in the sample paper, the plotting is done on time (t) versus unreliability [F(t)]. On this paper there is a special point which is %63.2 indicating the scale parameter η. As it would be remembered earlier, 63.2% of the population fails by the characteristic life point η, independent of the value of the shape parameter so this shows the 63.2 percentile. Therefore this percentile number is marked on the paper. On the upper side possible values of shape parameter β are pointed out. The dashed lines on that side shows possible Weibull slopes.

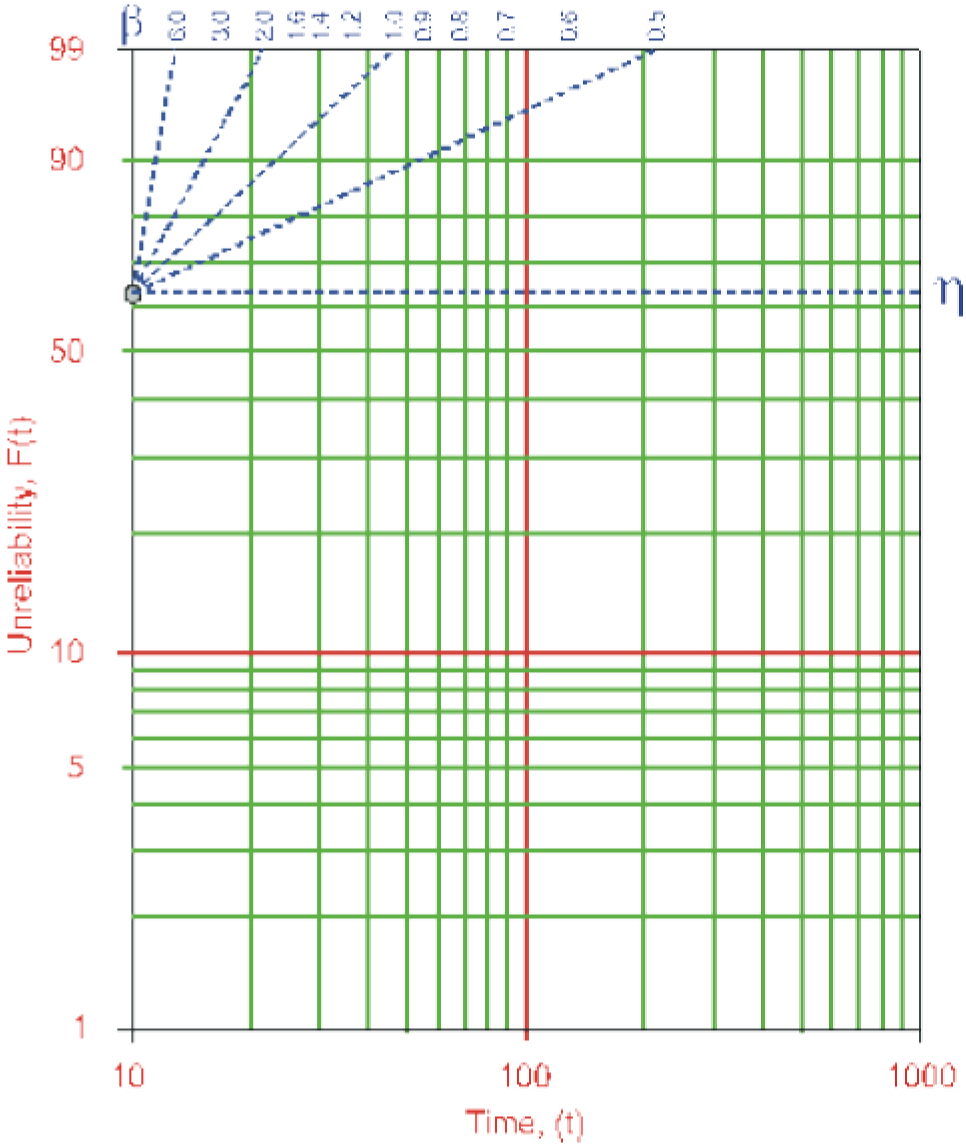

Figure. 3.10 Sample Weibull probability plot

Retrieved from http://www.weibull.com/LifeDataWeb/estimation_of_the_weibull_parameter.htm

To illustrate how plotting is done on these papers, a small example is given below in Figure 3.11. Suppose that there is a failure data consisting of six failure times which are 16, 34, 53, 75, 93, and 120 respectively. Firstly these data points are plotted on the paper. To find out the probabilities of these six failure times F(t) should be estimated. As mentioned before F(t) can be generally estimated by median rank method. These values in Figure 3.11 are obtained through median rank estimation. Then the best possible line passing through these points is drawn. If the data points do not fall on the line then the data are bad and rest of the data falling apart from the line are called outliers.

The slope of this line would give the shape parameter of the Weibull distribution. To obtain the value of the shape parameter a parallel line is drawn through the point of %63.2 on the left. When this parallel line is extended to the β scale the point that the line crosses would represent the estimation of shape parameter. In this example the line extends to the value of 1.4. To estimate the scale parameter η, from the point of %63.2 horizontal line is drawn until it intersects the fitted line. From the intersection point, a vertical line is drawn to time axis. The point where this vertical line crosses would give the value of scale parameter. In this case this value is 76.

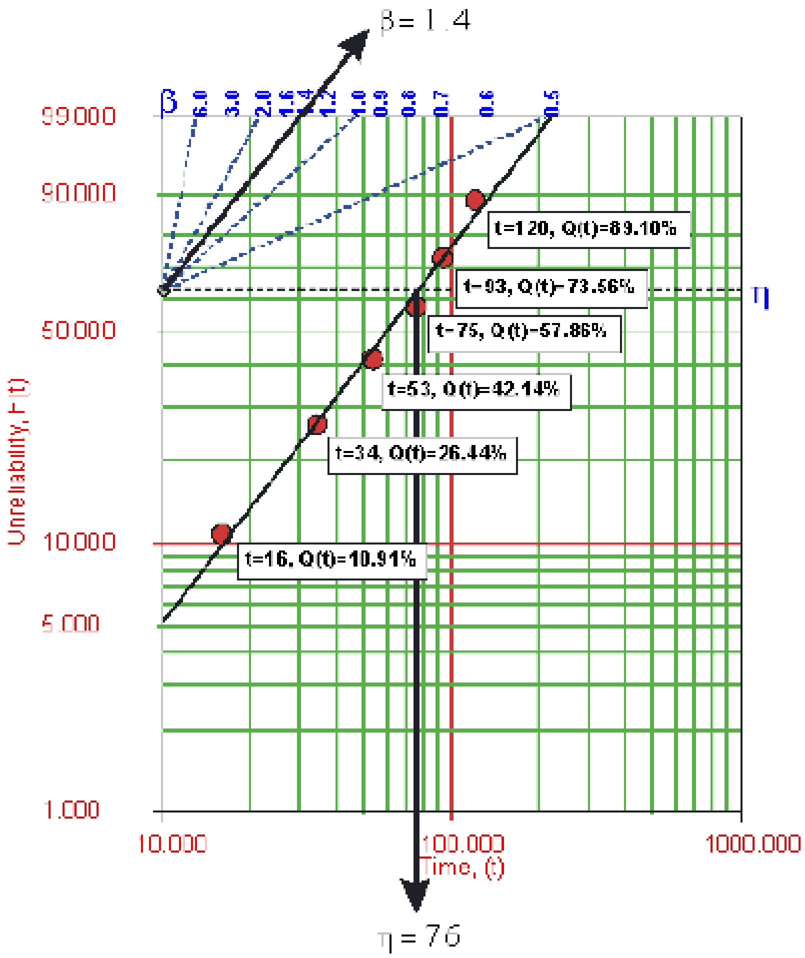

Figure 3.11 Probability plotting of sample data

Retrieved from http://www.weibull.com/LifeDataWeb/estimation_of_the_weibull_parameter.htm

CHAPTER FOUR
RELIABILITY APPLICATION IN ELECTRONICS COMPANY

In this chapter to illustrate the significance of reliability, the following case was provided as of a practical application. A brief note about the company was introduced in the following section. Detailed information follows next.

VESTEL has been the leader in Turkey's consumer electronics sector since its foundation in 1984. In the following years, the group expanded its operations into television components, personal computers, PC monitors. Today VESTEL is comprised of seventeen companies operating in manufacturing, technology development, marketing, and distribution fields in the consumer electronics, digital technologies. The Group's products include; flat TVs, plasma TVs, TFT-LCD (Liquid Crystal Display) TVs, conventional TVs, TV-DVD Combos, IDTVs, digital and analogue receivers, PC monitors, personal computers. The product groups are differentiated in order to provide consumers a wide selection range.

Television plant, in which this study was carried out, is the oldest and most professional company of VESTEL. This plant is the biggest exporter company of Turkey. Almost 10 million products are being produced in a year which is 80 percent exported.

The goal of applying the study in VESTEL is to decrease warranty claims and benefit from using the reliability concepts. The company tends to manufacture of better quality and more reliable products in order to incur less warranty cost. They want to evaluate product families according to these reliability results. Under these circumstances both ongoing forecasting for current TVs and future estimates at production planning stage are determined. If the existing family meet the expectations in terms of reliability and warranty, the manufacturing will be kept going with minor modifications. Contrarily, if the family shows poor results, then the existing product will be in question. A new design for the future product will be put on the agenda.

4.1 Methodology

As the CRT TVs are placed in declining phase of their life, LCD Tvs (abbreviated by LCDs here) become more popular. VESTEL manufactures four main types of LCDs as own label products with 32”, 37”, 42” and 46” displays. From the reports of sales department, LCDs with 32” displays have been the leader product among the bigger ones in the plant. They have a positioning advantage over the other types. Consequently it has been decided to investigate the reliability character of 32” LCDs. When making decisions about these 32”s in the lighting of reliability estimates, it is probable to get more gains as compared to other LCDs. As a matter of fact 32” LCD is the most sensible product in terms of consumer satisfaction.

As it is looked into the domestic market, LCDs of VESTEL are positioned in mid-range with its basic models. There are also sophisticated models manufactured in the same plant, that are designed for upscale, but these are excluded in the study. Only the basic models were assessed. VESTEL LCDs have a warranty length of two years. Anyway the EU (European Union) has passed a legislation that requires all products sold in the EU to have a two year warranty (Murthy 2006). Unfortunately LCDs are in no way defect-free products. Extended warranty, which means you can extend the length of warranty by purchasing, seems a tempting offer for consumers. As insurance it protects consumers against after-warranty costs. It is offered by either a third-party company or the manufacturer. Repairing an LCD is more costly than a CRT as it’s the price of owning cutting-edge technology. Hence some consumers are eager to purchase, although there is currently no defect occurring in LCD. So some companies apply this sort of strategy to catch the attention of the market. In the domestic market some brands offer more than two years as warranty length. However most of the brands’ LCDs have two years warranty length. VESTEL therefore does not want to lose its competitive advantage when the market shifts to a longer period.

The data that was used in reliability analysis were warranty claims data. Lifetimes as warranty claims data under warranty were obtained from the service department. To narrow the timeline of product family, data were censored. These lifetime values show the time at which the product fails so sometimes called time to failure data. Furthermore the censored data has some assumptions and therefore the knowledge of failures was hampered in the following ways:

- It is assumed that the claim date is equal to the failure data. There may be lag before the consumer notifies the claim.
- It is assumed that a claim is actually a failure. In practice all the claims do not have to concern a failure.
- Failures, which happen outside warranty, are not known. Only the claims under warranty are inputs of the study. Claims over warranty are hard to be determined. Information provided by dealers must be reliable. Besides, consumers may tend to return the products to unofficial service place. In this case claims are inaccessible.

Reliability analysis was conducted using months to failure data. The life variable, which is months in service in this study, was fairly straightforward because the sales date of products sold was precisely known. Data provided by claims were used instead of laboratory data since it displays more general behavior of products.

A parametric model was developed by using these lifetime data of 32" LCDs. Weibull model was used to estimate the chance of failure of randomly selected units and thus reliability of 32"LCD family.

The reasons why a Weibull model was used as the best choice to practice are stated in three ways:

1. Weibull analysis is the leading method for fitting and analyzing life data. The Weibull family includes broad range of distribution shapes.
2. Weibull analysis forecasts failures with extremely small samples for engineering analysis although higher samples are needed for statistical relevance. Small samples allow the study to be in a cost effective manner with samples from fifteen months.
3. Weibull analysis provides a simple and useful graphical plot of the failure data.

Estimation of parameters was implemented through linear regression method. F(t) values were estimated by median ranks. Henceforth the data were tested for goodness of fit whether they are consistent with the Weibull model.

4.2 Reliability Analysis

For the analysis data concerning the sales quantities between January 2006 and March 2007 are collected from service and sales departments. Then they were tabulated as seen in Table 4.1. On the table the failure quantities are plotted along months. By the help of the table, it is known when and how many products were sold. Furthermore it is shown how many products of a certain month's sales were failed along time. On Table 4.2 the percentage values are aligned according to the months since sales. In this case these values are said to be cumulative failure percentages. Rightmost column designates the weighted average percentage of failures which means failures over cumulative time. For instance on the second row of the column 0.32% implies that in two months after the sales of the whole products (grand total) 0.32% of the products were failed. Sales quantity of every month was considered as a sample and was analyzed individually. Later a reliability analysis covering for the first six months sales were separately conducted. Below these two tables, the scatter of failures of Jan 06, Feb 06, and Mar 06 along time are plotted. See these histograms in Figure 4.1, 4.2, and 4.3.

Table 4.1 Failures along months since sales

	Jan-06	Feb-06	Mar-06	Apr-06	May-06	Jun-06	Jul-06	Aug-06	Sep-06	Oct-06	Nov-06	Dec-06	Jan-07	Feb-07	Mar-07	Grand Total
Sales Qty.	10243	3524	33086	14537	7869	5098	6909	10989	23089	39039	53926	6859	12212	16206	1.023	244609
Jan-06	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Feb-06	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	10
Mar-06	33	6	42	0	0	0	0	0	0	0	0	0	0	0	0	81
Apr-06	47	16	97	2	0	0	0	0	0	0	0	0	0	0	0	162
May-06	43	8	177	12	1	0	0	0	0	0	0	0	0	0	0	241
Jun-06	31	15	172	44	23	0	0	0	0	0	0	0	0	0	0	285
Jul-06	47	7	171	39	25	9	0	0	0	0	0	0	0	0	0	298
Aug-06	29	8	166	55	15	12	7	0	0	0	0	0	0	0	0	292
Sep-06	46	4	188	46	22	13	22	17	3	0	0	0	0	0	0	361
Oct-06	47	11	188	45	21	21	42	40	55	7	0	0	0	0	0	477
Nov-06	64	11	229	68	33	33	59	54	116	119	45	0	0	0	0	831
Dec-06	37	7	155	60	32	11	29	24	109	146	215	2	0	0	0	827
Jan-07	58	21	191	87	35	21	28	47	172	401	513	40	0	0	0	1614
Feb-07	58	19	211	78	34	39	28	44	149	270	538	50	23	4	0	1545
Mar-07	42	18	168	78	46	30	36	75	155	310	438	62	62	37	14	1571
Total Fail.	591	152	2155	614	287	189	251	301	759	1253	1749	154	85	41	14	8595
Fail %	5,77%	4,31%	6,51%	4,22%	3,65%	3,71%	3,63%	2,74%	3,29%	3,21%	3,24%	2,25%	0,70%	0,25%	1,37%	3,51%

Table 4.2 Cumulative failure percentages along months since sales

% of Sales	4,19%	1,44%	13,53%	5,94%	3,22%	2,08%	2,82%	4,49%	9,44%	15,96%	22,05%	2,80%	4,99%	6,63%	0,42%	100%
Sales Quantity	10243	3524	33086	14537	7869	5098	6909	10989	23089	39039	53926	6859	12212	16206	1023	244.609
Months since Sales Date	Jan-06	Feb-06	Mar-06	Apr-06	May-06	Jun-06	Jul-06	Aug-06	Sep-06	Oct-06	Nov-06	Dec-06	Jan-07	Feb-07	Mar-07	Weighted Average
1	0,00%	0,03%	0,13%	0,01%	0,01%	0,00%	0,00%	0,00%	0,01%	0,02%	0,08%	0,03%	0,00%	0,02%	1,37%	0,05%
2	0,09%	0,20%	0,42%	0,10%	0,30%	0,18%	0,10%	0,15%	0,25%	0,32%	0,48%	0,61%	0,19%	0,25%		0,32%
3	0,41%	0,65%	0,96%	0,40%	0,62%	0,41%	0,42%	0,52%	0,75%	0,70%	1,43%	1,34%	0,70%			0,81%
4	0,87%	0,88%	1,47%	0,67%	0,81%	0,67%	1,03%	1,01%	1,23%	1,72%	2,43%	2,25%				1,39%
5	1,29%	1,31%	1,99%	1,05%	1,09%	1,08%	1,88%	1,23%	1,97%	2,42%	3,24%					1,86%
6	1,59%	1,50%	2,49%	1,36%	1,36%	1,73%	2,30%	1,66%	2,62%	3,21%						1,48%
7	2,05%	1,73%	3,06%	1,67%	1,78%	1,94%	2,71%	2,06%	3,29%							1,20%
8	2,33%	1,84%	3,63%	2,14%	2,19%	2,35%	3,11%	2,74%								1,07%
9	2,78%	2,16%	4,32%	2,55%	2,63%	3,12%	3,63%									1,14%
10	3,24%	2,47%	4,79%	3,15%	3,06%	3,71%										
11	3,87%	2,67%	5,37%	3,69%	3,65%											
12	4,23%	3,26%	6,01%	4,22%												
13	4,79%	3,80%	6,51%													
14	5,36%	4,31%														
15	5,77%															

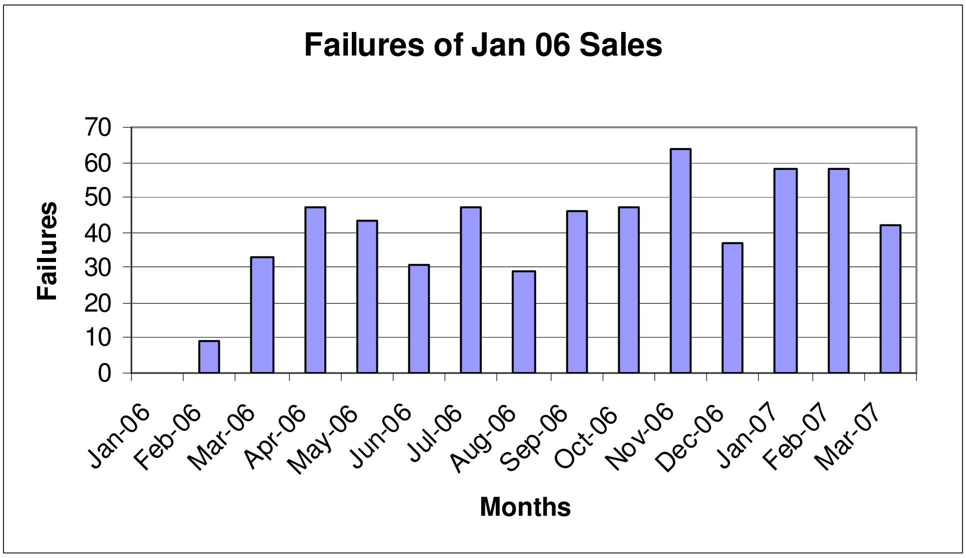

Figure 4.1 Failures of January 2006 sales

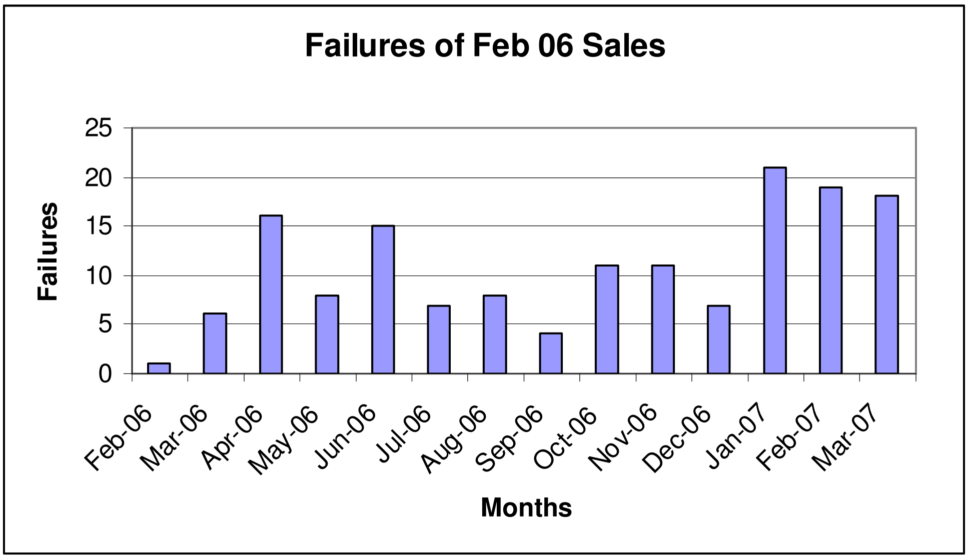

Figure 4.2 Failures of February 2006 sales

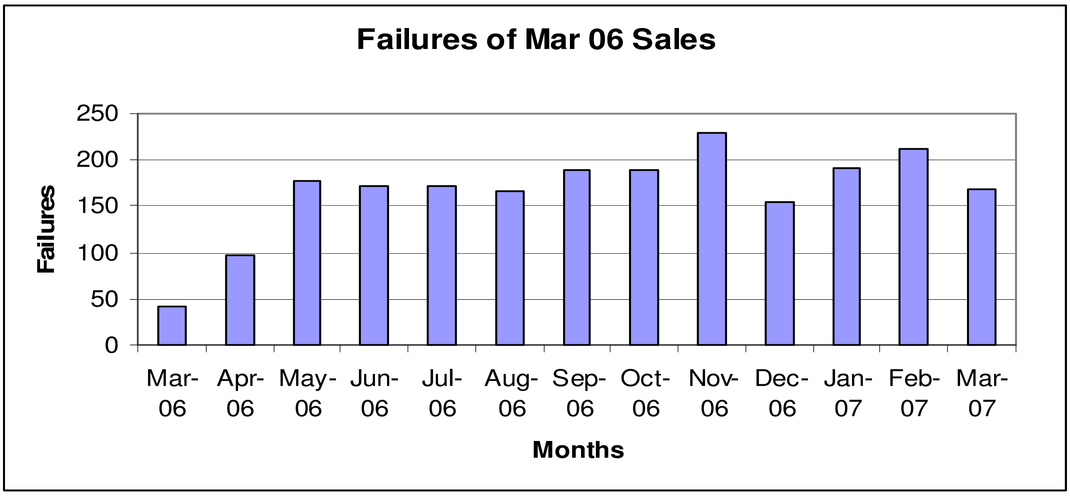

Figure 4.3 Failures of March 2006 sales

After the preliminary analysis, the data were used in Minitab 14 to produce the reliability results of the LCDs. Data were both left and right censored and had a readout characteristic. Firstly the sales of January 06, February 06 and March 06 were individually analyzed. January's failure data were traced until March 07. Beyond this 15 month period, the failures were right censored. Likewise, failure data of February 06 were traced until March 07 but this time the test period was 14 months. Using 2 parameter Weibull distribution parametric reliability analysis was conducted. Parameters for each month were estimated and related graphs were derived. Moreover, an overall reliability for the first six months sales was also put into an analysis. Semiannual product sales were considered as whole and were chased for a 9 month of test period. Table 4.3 summarizes the findings of the reliability analysis.

Table 4.3 Estimated parameters (in months) of Weibull

	Shape (β)	Scale (η)	Mean	Std dev.	Q1[1]	Q3[2]
Jan 06	1,95133	58,3829	51,7678	27,6673	30,8315	69,0212
Feb 06	1,79773	69,4788	67,7903	35,5622	34,7431	83,3221
Mar 06	1,53447	70,3781	63,3666	42,1337	31,2475	87,0730
Jan-Jun06	1,82282	52,2979	46,4806	26,4175	26,4024	62,5614

[1] The time 25 % of products fail (in months)
[2] The time 75 % of products fail (in months)

For the first three months of the year 2006, reliability of the products tends to slightly increase in terms of average time to failure. It can be detected in the decrement of the estimated shape parameter along months. This may lead a consequence that the products which were earlier manufactured have more tendencies to be less reliable. But the six months sales reliability analysis made it fallacious deduction since the characteristic life is about 53 months which is less than life of Jan 06, Feb 06 and Mar 06 as well as the quartiles are behind those months.

The reason why an annual study was not driven is to derive more reliable results from the analysis, because the more months included in the test (i.e. increasing the sample size), the more censored data we have and the less test period. There exists only 15 months of data spanning from January 06 to March 07. If the reliability was questioned on an annual basis, then after 4 months since sales the remaining survived products would be right censored. Hence the test period would be only 4 months which is not appropriate to derive reliable results. That is in numbers approximately %97 of the sales. But when it was applied, the test yielded Weibull distribution with parameters β=3.00933 and η=12.0328 that are much far away from the results of monthly and semiannual reliabilities. This shortage in determining failure time of products made annual analysis uncovered in the study.

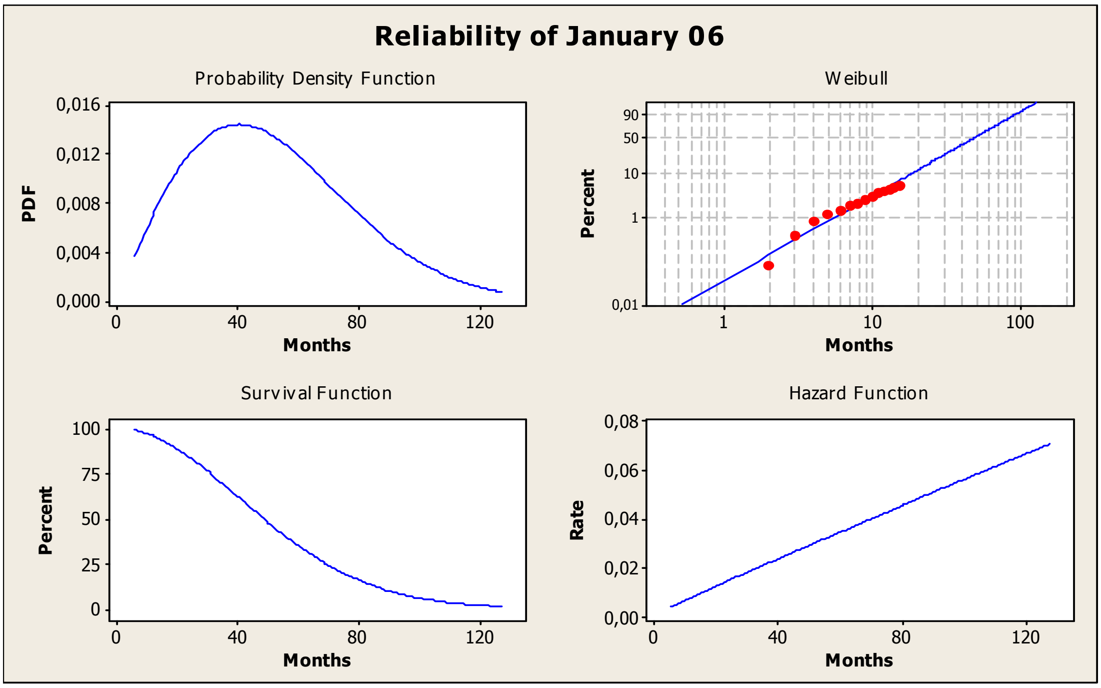

Figure 4.4 Reliability of January 06

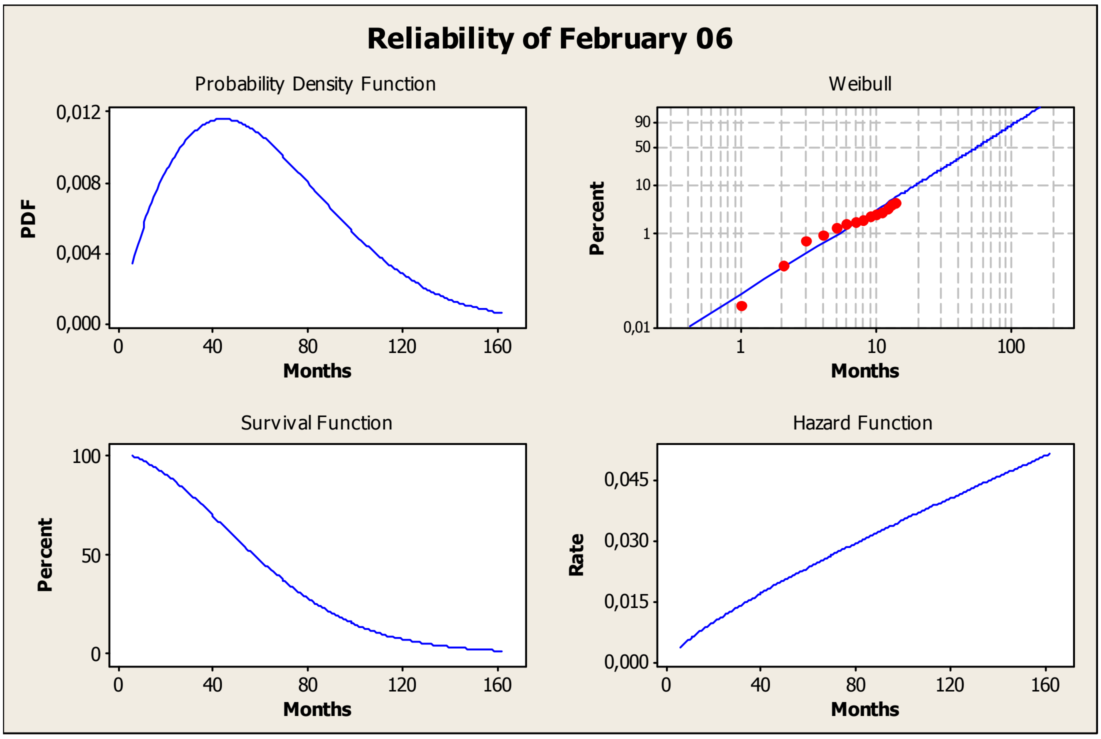

Figure 4.5 Reliability of February 06

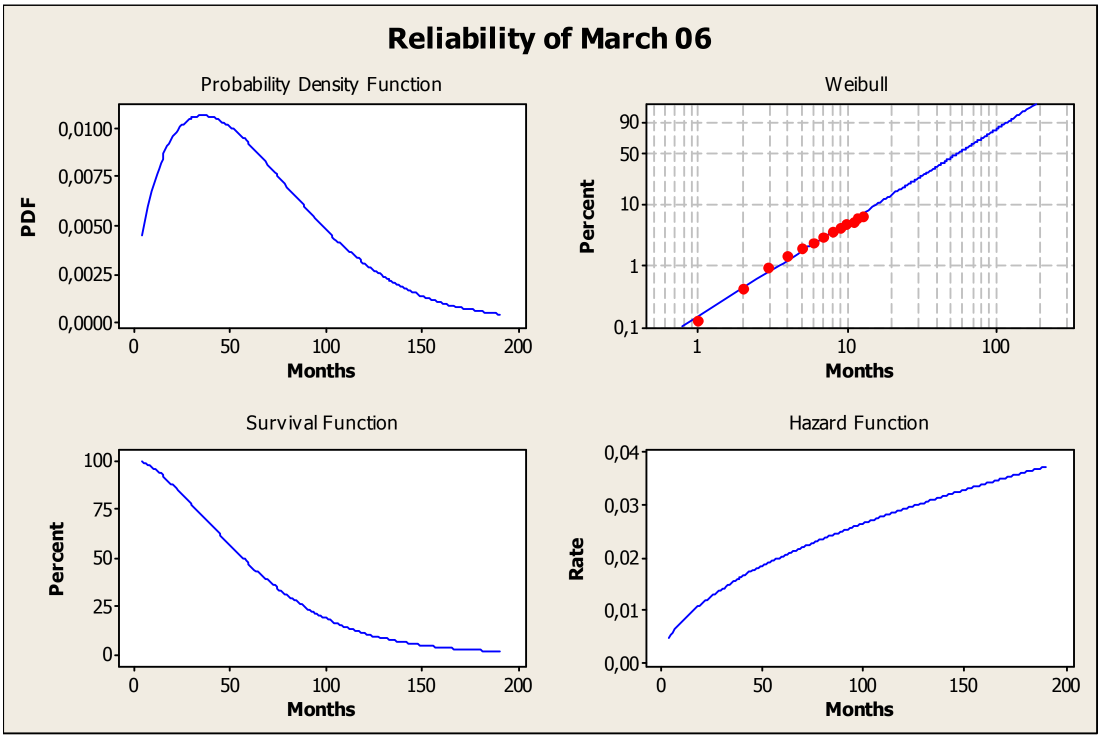

Figure 4.6 Reliability of March 06

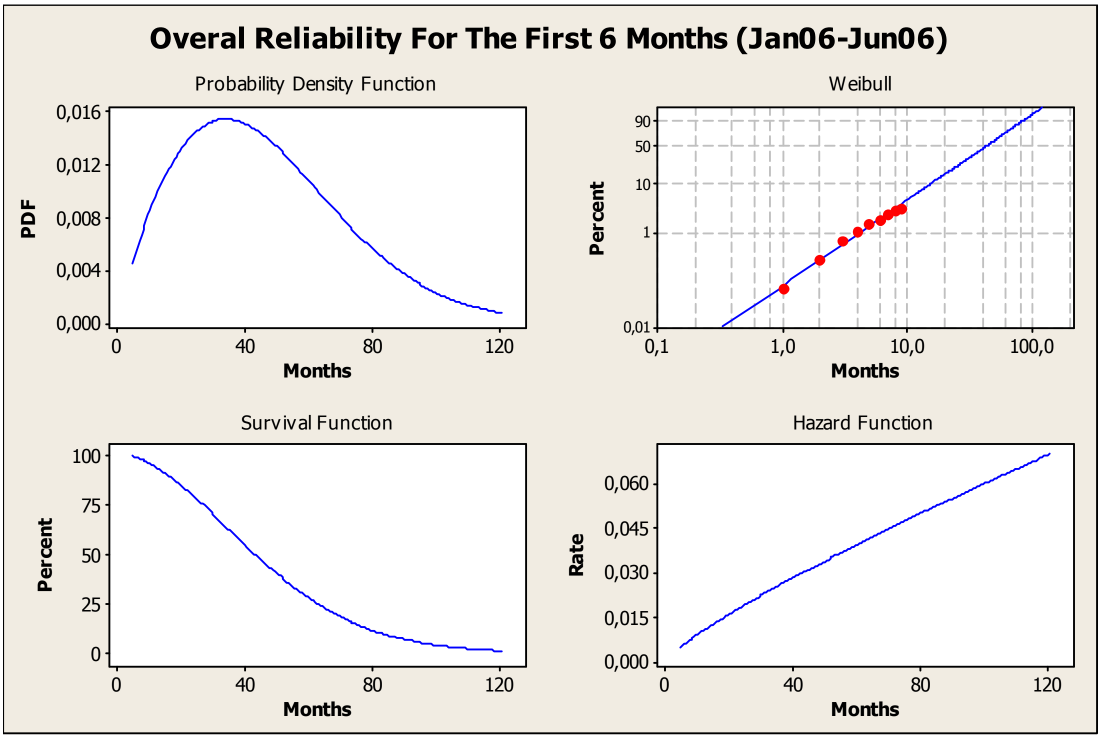

Figure 4.7 Overall reliability of 6 months sales

4.3 Impact of the Reliability Study on Warranty

The results of the analysis build up the basis for evaluating the performance of LCDs in means of service. By the help of it, upcoming failures were forecasted and defined when and how many of them could occur in say six months, a year or two years. The time at which a particular percentage of the production will have failed can be determined. It gives the impulse to take corrective actions to reduce the risk of losses because the costs associated with warranty and service can approximately be estimated. In this regard the spots servicing the warranty claims could be in confidence of ordering spare parts by reviewing the anticipated failure times of products. Consequently cash would not be stuck with the overstock. An optimal maintenance program can be scheduled with enough parts and labor supported by the reliability analysis. As the failure rates of both products which are in and out of the warranty length were known, this length could be regulated in terms of associated costs. If it is found to be rational, this period could be extended along with the marketing strategy. Besides those effects, reliability analysis provides manufacturer control over the production processes where process results tend to yield deviation from the expected.

What was obtained from this analysis supports to drive the warranty issue. Figuring out how and when a product family survives during its functional period is a question of matter for the manufacturers, because it is directly related to warranty scope and circumstances. Warranty can be perceived differently by both manufacturer and consumer sides. As the warranty is one of the fundamental parts of strategic plans for marketing, it may reflect reliability of products. Consumers may interpret products in means of their warranty as a reflection of their reliability. But as mentioned earlier, warranty is not a technical issue so the market may have a tricky feature. Although some companies have of lower quality product, they tend to have better warranty like the others which have attained satisfactory quality levels. But for another market this can be imaginary. The less quality companies can not afford to pretend having better quality products in case they can not cope with the cost of serving warranty claims because of the higher failure rate. On the other hand, consumer may have the comfort to maintain the product less carefully if the warranty agreement is on the extensive scope. This misleading point takes apart the accuracy of the reliability studies on those products. Therefore despite the fact that warranty as a marketing tool does not concern technically, it has broad consequences on the reliability. Reliability studies must be considered interrelated and noted that the nature of the market has an aspect.

4.4 Conclusion

This study introduces reliability in manufacturing and effects on warranty decision. It combines reliability as a technical issue with warranty as an extension of marketing issue. Today reliability plays an important role on how products are settled in market, because it defines whether a product is adequate to survive along time excluding the aging fact. Therefore evaluating the reliability of products is a must for companies which claim to improve their product range from development to sales.

As an application, a case study was conducted in an electronics company to combine the reliability aspects with products which are LCDs. The reliability was analyzed with a parametric Weibull model which constitutes best fit for different distribution shapes and useful for small sample sizes.

The analysis results predict how long the LCDs can function without any failure. This designates a prediction over time and gives the opportunity to compare between the existing and expected product characteristic. From this standpoint, the failure behavior of the products can be followed and controlled whether if they achieve the lifetime which was inferred in the analysis. If there is a substantial deviation, then this may be named as problems which may caused by manufacturing so this will result taking some steps to improve the manufacturing and having the required precautions.

For the first three months of the year 2006, reliability of the products tends to slightly increase in terms of average time to failure. It can be detected in the decrement of the estimated shape parameter along months. This may lead a consequence that the products which were earlier manufactured have more tendencies to be less reliable. But the six months sales reliability analysis made it fallacious deduction since the characteristic life is about 53 months which is less than life of Jan 06, Feb 06 and Mar 06 as well as the quartiles are behind those months.

Consequently, the effect of the study is to make the company to be more proactive and benefit from the data which are gathered from service points by analyzing the failure feature of products. In this regard reliability analysis is significant to be assessed in order to estimate the failure mechanism of a product family and turn this knowledge into warranty scope and length.

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