DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

ANALYSIS OF MIMO COMMUNICATION SYSTEM UNDER SKEWED ALPHA-STABLE NOISE

by
İbrahim DEMİR

July, 2020
İZMİR

ANALYSIS OF MIMO COMMUNICATION SYSTEM UNDER SKEWED ALPHA-STABLE NOISE

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"ANALYSIS OF MIMO COMMUNICATION SYSTEM UNDER SKEWED ALPHA-STABLE NOISE"** completed by **İBRAHİM DEMİR** under supervision of **ASST. PROF. DR. MEHMET EMRE ÇEK** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

....

Asst. Prof. Dr. Mehmet Emre ÇEK

Supervisor

....

Assoc.Prof. Dr. Olcay AKAY

(Jury Member)

....

Asst. Prof. Dr. İlhan BAŞTÜRK

(Jury Member)

Prof. Dr. Özgür ÖZÇELİK

Director

Graduate School of Natural and Applied Sciences

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ANALYSIS OF MIMO COMMUNICATION SYSTEM UNDER SKEWED ALPHA-STABLE NOISE

ABSTRACT

In this thesis, Multiple Input Multiple Output (MIMO) communication system is analyzed under non-Gaussian noise. The non-Gaussian noise model is given as $\alpha -$stable distribution. Differing from the previous studies in the literature, the thesis study is concentrated on effect of both symmetric and asymmetric impulsive noise on MIMO communication systems. The characterization of the receivers is investigated to yield energy efficient solution.

Briefly, there are two main contributions in the thesis. The first one relies on modifying the conventional maximum likelihood and minimum mean square error detectors to give stable outputs by adaptation and/or reformulation of these detectors including Fractional Lower Order Statistics (FLOS). It is shown that better bit error rate performances can be achieved if these detectors are described by considering fractional lower order moments. The second contribution is based on compensation of possible asymmetric behavior of the impulsive channel noise. It is shown that, increasing skewness of the channel noise having the same impulsiveness causes degradation on detector performance. This problem is overcome by injection of intended noise having an opposite skewness at the receiver input which results in improvement on bit error rate. This enhancement is expressed as stochastic resonance which is more apparent when the impulsiveness of the noise increases.

By reducing the transceiver scheme to SISO system, the effect of the impulsiveness and the skewness of the channel noise on capacity are also investigated. It is observed that capacity decreases with increasing impulsiveness of the channel noise whereas skewness has not an apparent effect, consistent with results in the literature.

Keywords: MIMO communication system, $\alpha -$stable distribution, stochastic resonance, bit error rate, capacity

MIMO HABERLEŞME SİSTEMİNİN EĞİK ALFA-KARARLI GÜRÜLTÜ ALTINDA ANALİZİ

ÖZ

Bu tezde, çoklu giriş-çoklu çıkış (MIMO) haberleşme sistemi Gauss dışı gürültü altında analiz edilmiştir. Gauss olmayan gürültü modeli alfa kararlı dağılım ile verilmiştir. Literatürdeki önceki çalışmalardan farklı olarak, tez çalışması hem simetrik hem de asimetrik dürtüsel gürültünün MIMO haberleşme sistemleri üzerindeki etkisine odaklanmıştır. Alıcıların karakterizasyonu verimli enerji elde edebilmek için araştırılmaktadır.

Özet olarak bu tezin iki ana katkısı vardır. Birincisi, kesirli düşük üs istatistikleri (FLOS) dahil olmak bu dedektörlerin uyarlanması ve / veya yeniden düzenlenmesi yoluyla kararlı çıktılar vermesi için geleneksel maksimum olasılık ve minimum ortalama kare hata dedektörlerini değiştirmeye dayanır. Bu dedektörler kısmi düşük dereceli momentler dikkate alınarak tanımlanırsa daha iyi hata oranı performanslarının elde edilebileceği gösterilmiştir. İkinci katkı dürtüsel kanal gürültüsünün olası asimetrik davranışının telafisine dayanmaktadır. Aynı dürtüselliğe sahip kanal gürültüsünün artan asimetrikliğinin dedektör performansında bozulmaya neden olduğu gösterilmiştir. Bu sorun, alıcı girişinde ters bir asimetriye sahip olan maksatlı bir gürültü enjeksiyonu ile aşılır ve bu da bit hata oranında iyileşmeye neden olur. Bu geliştirme, gürültünün dürtüselliği arttığında daha belirgin olan skotastik rezonans olarak ifade edilmiştir.

Ayrıca, alıcı verici şeması SISO sisteme indirilerek, kanal gürültüsünün dürtüselliğinin ve asimetrikliğinin kapasite üzerindeki etkisi araştırılmıştır. Kanal gürültüsünün dürtüselliği arttıkça kapasitenin azaldığı gözlemlenirken, asimetrikliğin literatürdeki sonuçlarla tutarlı olarak belirgin bir etkisi olmadığı görülmektedir.

Anahtar kelimeler: MIMO haberleşme sistemi, α −kararlı dağılım, stokastik rezonans, bit hata oranı, kapasite

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CHAPTER ONE
INTRODUCTION

In digital communication, channel capacity and/or probability of error characterize the performance of the proposed baseband or band-pass communication system. According to this motivation, there exist numerous papers to improve communication system performance. The usage of multiple antennas at the receiver and transmitter in wireless communication is one of the challenging topics being widely studied. Namely, Multiple-Input Multiple-Output (MIMO) technique is utilized to increase the channel capacity and that also has a certain effect on the bit error rate in communication system.

A basic MIMO system can be expressed with respect to the number of antennas at the transmitter and the receiver, as described below (Biglieri et al., 2007).

1. Single Input Single Output (SISO) System: This can be considered as conventional communication scheme including one transmitter and one receiver antenna. SISO system design is not complex but its capacity is lower than other MIMO type systems.
2. Single Input Multiple Output (SIMO) System: In this communication system there is one transmitter and more than one receiver antennas. The antennas are combined to minimize errors and optimize data speed. SIMO is one of the several forms of smart antenna technologies. It is preferable in the case of interference and fading from ionosphere.
3. Multiple Input Single Output (MISO) System: This technique uses more than one transmitter and one receiver antenna. In MISO systems, the exact same bit stream is transmitted for each transmitter antenna. The receiver is capable of receiving the optimum signal, thus optimum signal is used for extracting the transmitted bit stream. The main advantage is expressed in terms of cost and battery life.

4. Multiple Input Multiple Output (MIMO) System: This technique uses multiple antennas at transmitter and receiver. System design is more complex than the other techniques but MIMO provides a certain capacity improvement.

MIMO communication systems have been used in many areas. For future communication systems MIMO can be used on 5G technologies as Millimeter-Wave Massive MIMO to evolve a cellular network that remarkably pushes forward the limits of legacy mobile systems across all dimensions of performance metrics as introduced by (Busari et al., 2018). In order to support 5G multimedia communication systems and reducing the cost, massive MIMO systems are used for their high spectral efficiency in (Xiaohu et al., 2016). Secure transmission is studied by using massive MIMO to adopt low-resolution digital-to-analog converters (Xu et al., 2019).

Also, spectrum sharing with MIMO is proposed in (Li et al., 2016) for radar systems in order to minimize interference between signals, decreasing communication bandwidth and power. In radar systems, joint system design is studied in (Qian et al., 2018) and beamforming techniques are proposed for joint MIMO radars where a single device behave as base station and communicate synchronize with downlink users and detecting radar targets by (Liu et al., 2018).

Increasing demand for higher bandwidth has lead MIMO communication systems to increase number of antennas at transmitter and receiver, and for the higher speed, higher frequency bands are being explored. Ultra-massive MIMO communication system is introduced (Akyildiz & Jornet, 2016).

MIMO systems promise improved performance compared to conventional systems by using multiple antennas at receiver (Jethva & Porwal, 2014). The MIMO technique using multiple antennas in receiver increases the channel capacity for today’s wireless communication and is reported to provide a conventional improvement on the communication system (Goldsmith, 2005). Basically, there are three detector types for MIMO communication systems. These detectors are given below (Proakis, Salehi & Bauch, 2013).

1. Maximum Likelihood Detector (MLD)
2. Minimum Mean Square-Error Detector (MMSE)
3. Inverse Channel Detector (ICD)

Even though MIMO technique is reported to enhance the channel capacity, channel noise is also important. In the literature, although the distribution of channel noise is modeled as Gaussian, it is stated that channel noise in wireless communication systems exhibits impulsive characteristics modeled by α −stable distribution.

This section represents a brief survey of MIMO communication systems, α-stable distribution, detectors which are used in MIMO systems. Also, in this thesis, experimentally determined bit error rate of MIMO communication system is analyzed under skewed α −stable distributed noise. It is observed that skewness parameter increases the error probability within a certain signal to noise ratio (SNR), the error probability decreases as the noise becomes symmetrized. Since the effect of asymmetry disappears, output signal can be manipulated like symmetrical signal. Therefore, stochastic resonance is observed by adding intentional noise having same impulsiveness with opposite skewness and the noise probability is decreased. The resonance phenomenon is analyzed with respect to skewness and intentional noise intensity parameters.

Furthermore, bit error rate and the channel capacity are calculated using different detectors and illustrated by using MATLAB, and the simulation results showed that the error probability of the channel increases as the channel noise becomes more asymmetrical and the effect of the asymmetry parameter decreases as the impulsiveness decreases. Also, as a result of work fulfilled, even though channel has asymmetrical noise, it is seen that improves the bit error rate performance and the channel capacity with symmetrization process, which is proposed in this study.

Since asymmetrical signal is used in this thesis, modulation types which contains complex signals are cannot be used. The bit error rate changes depending on the change of antenna number in the receiver and / or transmitter of the MIMO communication system which was examined in (Zhu & Murch, 2002) using the highest likelihood detection in the Gauss distribution channel under the BPSK modulation. Rather than using Gaussian distribution, channel noise is modeled by Gaussian mixture in the study of (Le et al., 2016).

In the literature, although the distribution of channel noise is generally modeled as Gaussian, it is stated that channel noise in wireless communication systems exhibits impulsive characteristic modeled by $\alpha-$stable distribution (Mahmood, Chitre & Armand, 2014). In signal processing perspective, maximum likelihood detection is adopted to perform under infinite variance introducing l_p norm minimization (Zeng, So & Zoubir, 2013) where p is the fractional lower order moment satisfying the criteria $p < \alpha$.

Since $\alpha-$stable noise is described in terms of its four parameters in which especially characteristic exponent α responsible for degree of impulsiveness and the skewness β responsible for the amount of symmetry take an important place. Accordingly, the capacity change under symmetric $\alpha-$stable noise is also examined with a theoretical study (Fahs & Abou-Faycal, 2012). Although the Gaussian mixed model is used as an alternative noise model for the analysis of channel capacity in fading channels (Freitas et al., 2017), the studies on the extraction of channel capacity under symmetrical $\alpha-$stable noise still continue (Freitas et al., 2017). In addition, there is a recent study on the modeling of stable distributions of heterogeneous networks (Egan et al., 2017). Almost all the studies in the literature assume that $\alpha-$stable noise has a symmetrical distribution.

Although numerous studies exist related to MIMO system characterization under various channel behavior, there are only a few studies investigating non-Gaussian noise in MIMO communication system design. The first of the two main contributions of the thesis study can be emphasized that, the conventional detectors used in MIMO communication system can be enhanced including fractional lower order moments. The second novelty arises from the analysis of performance of MIMO system when the channel noise has skewed (asymmetrical) stable distribution. To compensate for the performance degradation of the detectors, intended noise is injected at the receiver to achieve resultant noise having symmetrical distribution. The bit error rate performance enhancement due to adding noise corresponds to a well-known physical phenomenon named stochastic resonance.

Discarding the multiple antenna case in the communication system, it is investigated whether a similar behavior exists on capacity analysis with respect to varying impulsiveness and skewness of the stable noise. It is observed that skewness parameter β has almost no effect on capacity in the range of $1 < \alpha < 2$ whereas the decreasing parameter α, i.e. increasing impulsiveness has a certain degradation on error performance of the digital communication system.

The thesis is organized as follows: In Chapter 2, the fundamentals of MIMO system is introduced containing conventional detector type used in Gaussian noise in the case of channel matrix assumed to remain constant. In Chapter 3, the $\alpha-$stable noise and some of its properties used in the operation while designing the detectors are introduced. In Chapter 4, the MIMO communication system under symmetrical and skewed $\alpha-$stable noise is analyzed and the results are presented. In conclusion chapter the findings are discussed.

CHAPTER TWO
MIMO COMMUNICATION SYSTEM

2.1 Introduction

In this chapter, multiple antenna uses are identified in the receiver and / or transmitter, types of multiple antenna usage and their channel models are given. Also, it is mentioned that differences between different cases, performance and bit error rate performance.

A communication system employing N_T transmit antennas and N_R receive antennas is generally called a multiple input-multiple output (MIMO) system and resulting spatial channel in such a system is called MIMO channel. There are 4 special cases regarding to multiple antenna usage. First one is known as conventional communication system named single input-single output (SISO) communication system which is using single antenna on transmitter and receiver. The case of using one antenna in the transmitter and more than one antenna in the receiver is called single input-multiple output system (SIMO), and the corresponding channel is called a SIMO channel. The other case of using one antenna in the receiver and more than one in the transmitter is called multiple input-single output (MISO), and the corresponding channel is called MISO channel. As the last case and the case that is used in this thesis is using more than one antenna on both transmitter and receiver is called multiple input-multiple output (MIMO), and the corresponding channel is a MIMO channel. In communication systems y is denoted by the channel transmission matrix **H**, **s** represents the input vector and **n** is the noise vector.

Single input-single output communication systems consist of a transmitter and a receiver antenna. It is known as conventional communication systems. This system employs no diversity technique. Both the transmitter and receiver have one RF chain. SISO is relatively simple and cheap to implement and it has been used age long since the birth of radio technology. It is currently used in radio and TV broadcast and our personal wireless technologies.

In SISO systems **H** has single dimension, so it is used as h since it is a scalar value. There is a channel model illustrated on the following figure for SISO communication system, Tx and Rx represent transmitter and receiver antennas, respectively.

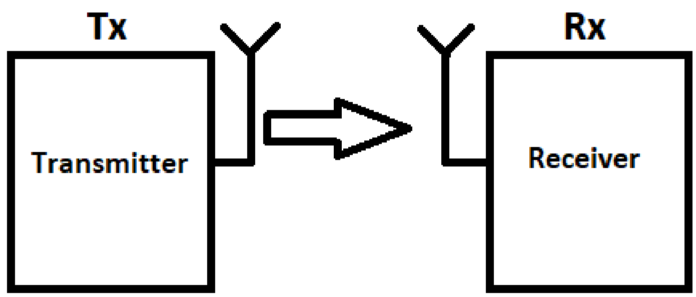

Figure 2.1 Basic channel model of SISO communication system

SISO wireless channel output signal formula is given in Equation (2.1), it is described as (Proakis et al., 2013)

$$\boldsymbol{y} = \boldsymbol{h} \times \boldsymbol{s} + \boldsymbol{w} \tag{2.1}$$

Single input-multiple output systems consist of a transmitter and more than one receiver antennas. To improve performance, multiple antenna technique has been developed. The receiver can choose the best antenna to receive a stronger signal or combine signals from all antennas in such a way that maximizes the signal to noise ratio (SNR). The first technique is known as switched diversity or selection diversity.

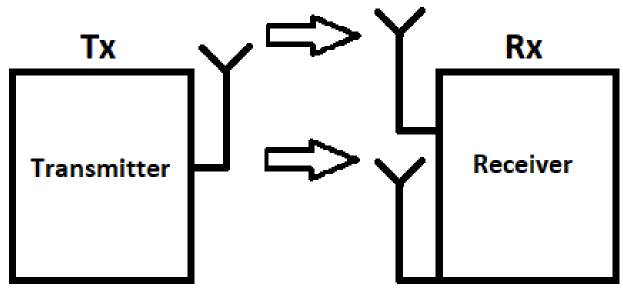

Figure 2.2 Basic channel model of SIMO communication system

Multiple input-single output communication systems use multiple antennas at the transmitter and a single antenna at receiver. A technique known as Alamouti STC (Space Time Coding) is employed at the transmitter with two antennas. STC allows the transmitter to transmit signals (information) both in time and space, meaning the information is transmitted by two antennas at two different times consecutively.

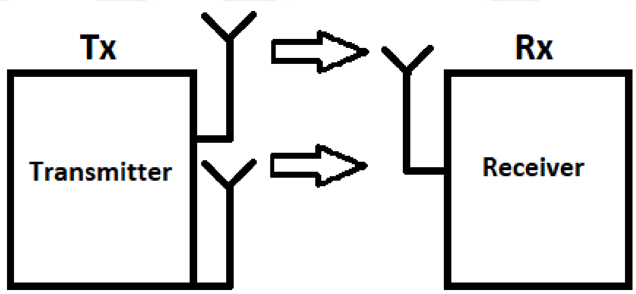

Figure 2.3 Basic channel model of MISO communication system

To multiply throughput of a radio link, multiple antennas (and multiple RF chains accordingly) are put at both the transmitter and the receiver. This system is referred to as Multiple input-multiple output (MIMO). A MIMO system with similar count of antennas at both the transmitter and the receiver in a point-to-point link is able to multiply the system throughput linearly with every additional antenna. For example, a 2×2 MIMO will double the throughput.

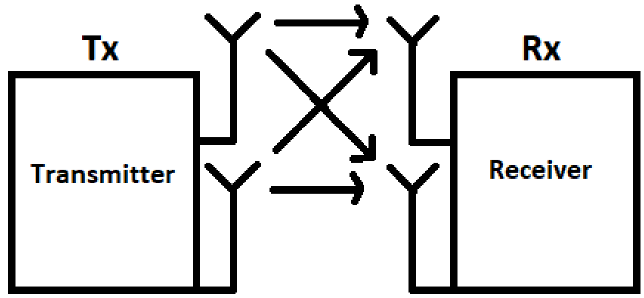

Figure 2.4 Basic channel model of MIMO communication system

2.2 Channel Model MIMO System

MIMO system increases capacity significantly when employing multiple antennas both at transmitter and receiver. But in a conventional communication system can receive only one signal at the receiver, MIMO system configured of several antennas at both ends. MIMO channel path formed between all transmit and receive antenna that considered an $N_R \times N_T$ antenna, beside that MIMO channel express a linear time-variant as presented by $N_R \times N_T$ channel matrix model and in Figure 2.5 MIMO system model is illustrated.

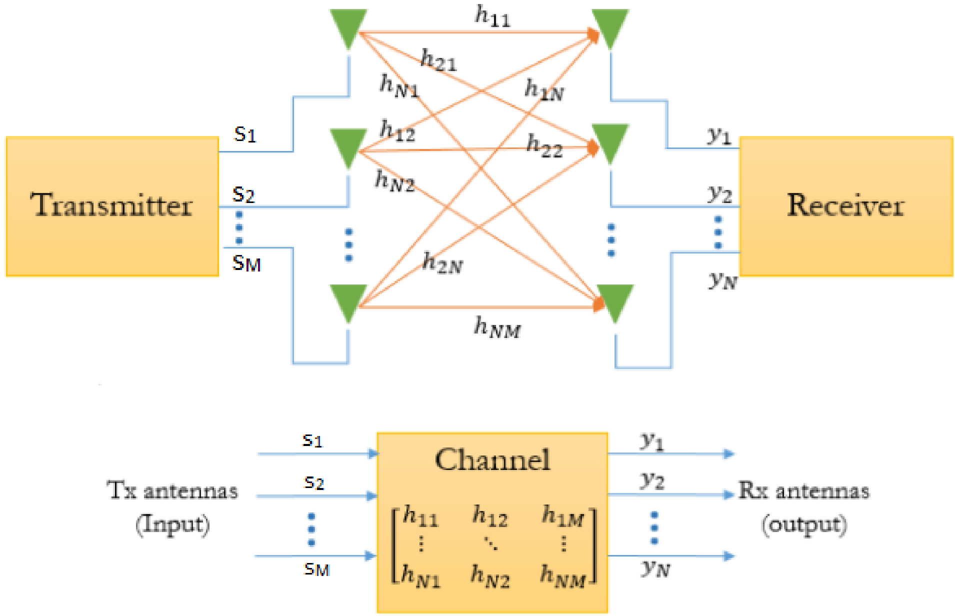

Figure 2.5 Multiple Input Multiple Output (MIMO) System *(*Mathuranathan, 2019*)*

2.3 Gaussian Channel Capacity

In this section, it is considered that the evaluation of the channel capacity of AWGN MIMO channel characterized by the channel matrix **H**. The MIMO channel refers to the matrix model that consists of more than one vector channel that can transmit and receive various signals at the same time and frequency band. All signals not reach to the receiver together, some signals scattered, reflected or weaken. Using multiple antennas increases the capacity of the channel, the transmitter sends data by N_T transmitters and for encoding input data stream uses vector encoder, also at receiver by N_R number of antennas received the data. That is the basic processing of the channel MIMO system, the power of the input signal, noise and channel properties have a great effect on the channel (Proakis et al., 2013).

$$C = log_2 \det(\boldsymbol{I}_{N_R} + \frac{1}{N_0}\boldsymbol{H}\boldsymbol{R}_{ss}\boldsymbol{H}^H) \qquad \text{bps/Hz} \qquad (2.2)$$

where $\mathbf{R}_{ss}$ denotes the trace of the signal covariance $\mathbf{R}_{ss}$. This is the maximum rate per Hz that can be transmit reliably over the MIMO channel for any given realization of the channel matrix $\mathbf{H}$. Eq. (2.2) can be extended to the following equation by (Proakis et al., 2013),

$$C = \sum_{i=1}^{r} log_2(1 + \frac{E_S}{N_T N_0} \lambda_i) \text{ bps/Hz} \tag{2.3}$$

where r refers the rank of the channel matrix, $\frac{\boldsymbol{E_S}}{\boldsymbol{N_0}}$ is the signal to noise ratio (SNR) and N_T is the number of transmit antennas. In a SISO channel, it is expected to see that $\lambda_1 = |\mathbf{h}_{11}|^2$, so Eq. (2.3) is simplified in (Proakis et al., 2013) to

$$C_{SISO} = log_2(1 + \frac{E_S}{N_0} |\boldsymbol{h}_{11}|^2) \text{ bps/Hz} \tag{2.4}$$

The capacity under Gaussian noise for single user SISO is illustrated in Figure 2.6. with respect to SNR.

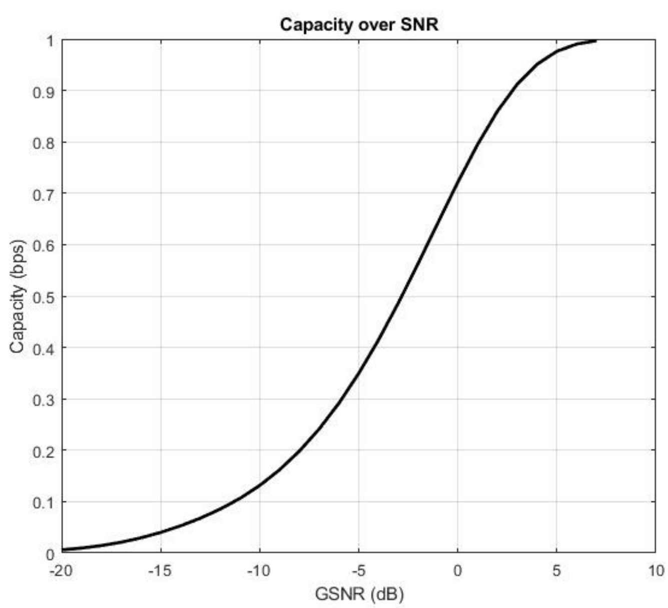

Figure 2.6 Channel capacity under AWGN noise

A SIMO channel ($N_T = 1,\ \ N_R > 1$) is characterized by the vector $\boldsymbol{h} = \left[\mathbf{h}_{11}\ \mathbf{h}_{21} \dots \mathbf{h}_{N_R 1}\right]^T$. In this case, the rank of the channel matrix is unity and the eigenvalue λ_1 assumed is given by (Proakis et al., 2013).

$$\lambda_1 = \sum_{i=1}^{N_R} |\boldsymbol{h}_{i1}|^2 \tag{2.5}$$

Therefore, it is assumed that N_R elements of the channel are deterministic and known to the receiver capacity is given as (Proakis et al., 2013).

$$C_{SIMO} = log_2(1 + \frac{E_S}{N_0} \sum_{i=1}^{N_R} |\boldsymbol{h}_{i1}|^2) \text{ bps/Hz} \tag{2.6}$$

In a MISO channel ($N_T > 1$, $N_R = 1$) assumed can be characterized by the vector $\boldsymbol{h} = \left[\boldsymbol{h}_{11}\ \boldsymbol{h}_{12} \dots \boldsymbol{h}_{1N_T}\right]^T$. In this case, the rank of the channel matrix is also unity and eigenvalue λ_1 assumed is given as (Proakis et al., 2013)

$$\lambda_1 = \sum_{j=1}^{N_T} \left|\boldsymbol{h}_{1j}\right|^2 \tag{2.7}$$

It is also assumed that if $\boldsymbol{N}_T$ elements of the channel matrix are deterministic and known to the receiver, capacity is given as (Proakis et al., 2013).

$$C_{MISO} = log_2(1 + \frac{E_S}{N_T N_0} \sum_{j=1}^{\boldsymbol{N}_T} \left|\boldsymbol{h}_{1j}\right|^2) \text{ bps/Hz} \tag{2.8}$$

If the channel is assummed to be memoryless and using the BPSK modulation with $\{-A, A\}$ symbols, mutual information is maximized where $\mathrm{P}(X = A) = \mathrm{P}(X = -A) = \frac{1}{2}$. Thus, Equation (2.9) is obtained by (Proakis, 2001).

$$C_{MIMO} = \frac{1}{2} \int_{-\infty}^{\infty} p(y|A)\, log_2 \frac{p(y|A)}{p(y)} dy + \frac{1}{2} \int_{-\infty}^{\infty} p(y|-A)\, log_2 \frac{p(y|-A)}{p(y)} dy \text{ bps/Hz} \tag{2.9}$$

As an illustration, it is reported by the capacity of SISO and MIMO systems in terms of SNR in Figure 2.7. In SISO case ($N_T = 1$, $N_R = 1$) capacity ranges is lower compared to MIMO case ($N_T = 4$, $N_R = 4$). The capacity improvement for increased antenna number is quite apparent.

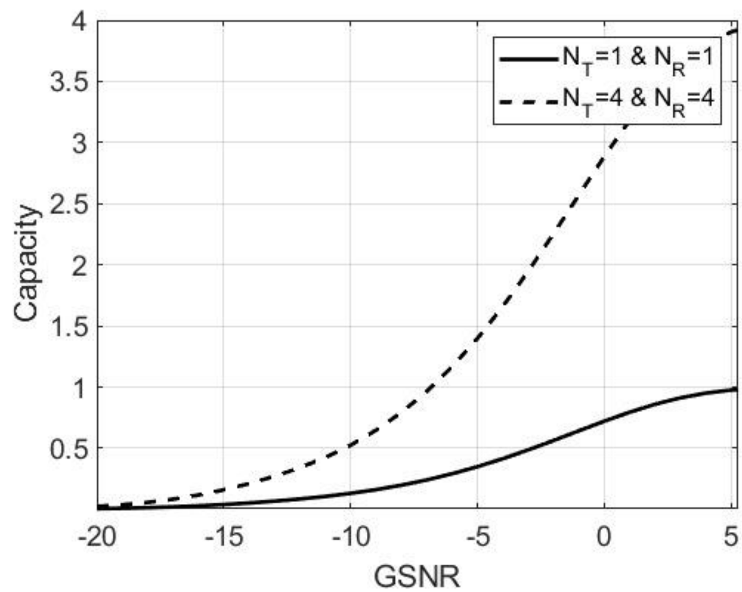

Figure 2.7 Capacity comparison between SISO and MIMO

To illustrate the increase of channel capacity with respect to the number of antennas for all communication systems leads to Figure 2.8.

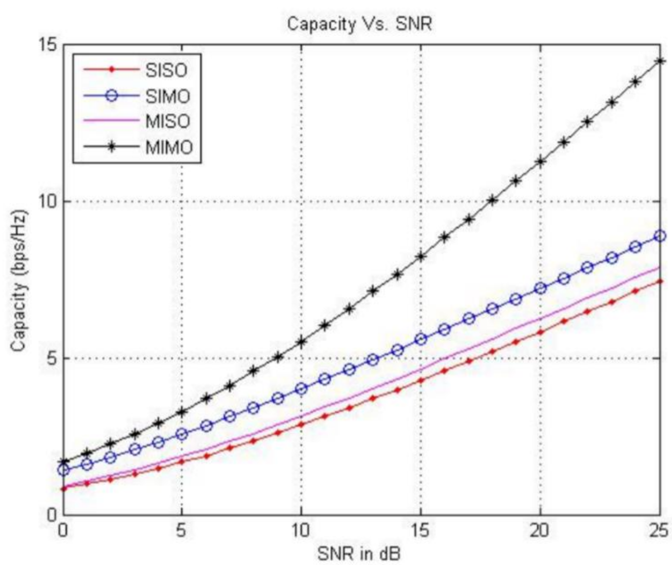

Figure 2.8 Capacity comparison for MIMO, SIMO, MISO and SISO systems (Sarangi & Datta, 2018)

It is seen from Figure 2.8 that MIMO channel capacity increases linearly with respect to the number of antennas and it can be said that MIMO capacity is $\boldsymbol{N_T}$ times larger than SISO communication systems. Also, it is seen that SIMO system is more efficient than MISO and SISO because SIMO used $\boldsymbol{N_R}$ antennas at receiver and multiple antenna usage in receiver increases the capacity in accordance with the Equation (2.6) and Equation (2.8).

2.4 Error Performance

Since the major aim of the thesis is to analyze the effect of the non-Gaussian noise to the MIMO systems, the modulation type is fixed and selected to be binary phase shift keying (BPSK). Therefore, the error performance is considered only for BPSK communication. For the binary symmetric memoryless channel, the probability of error for BPSK modulation under additive white Gaussian noise (AWGN) channel which is given in Equation (2.10) is analytically described by (Proakis, 2001).

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{2.10}$$

The baseband correspondence of BPSK modulated communication is the antipodal symbol transmission where the symbols have the amplitude A or $-A$. The Monte-Carlo simulations relying on the ratio of erroneous bits to the total transmitted bits are used to illustrate bit error rate.

In the literature, the bit error rate alters regarding to the modulation, channel noise, detectors and the number of multiple antenna usage. The change of bit error rate with respect to different number of antenna selection under Rayleigh fading is analyzed using BPSK and QPSK modulation types by (Bactor & Kaur, 2015).

2.5 Detector Types

Based on the frequency nonselective MIMO channel model described in Section 2.2 where the channel matrix is considered to be constant, three conventional detectors are introduced in the literature for recovering the transmitted data symbols and evaluate their performance under additive white Gaussian noise. Without loss of generality, demodulator output can be found in the MIMO communication system with N_t transmitter antennas and N_r receiver antennas corresponding to the m_{th} receiver antenna with $n = 1, \dots, N_t$ and $m = 1, \dots, N_r$.

$$\boldsymbol{y}_m = \sum_{n=1}^{N_t} \boldsymbol{h}_{mn} \boldsymbol{s}_n + \boldsymbol{w}_m \qquad (2.11)$$

In the above equation, the size of receiver $\mathbf{y}$ and the transmitter $\mathbf{s}$ are $N_r \times 1$ and $N_t \times 1$, respectively. $\mathbf{h}_{mn}$ is the element of channel matrix $\mathbf{H}$ and the multi-message information sent by the vector $N_t \times 1$ is obtained by the vector $\mathbf{y}$ whose size is $N_r \times 1$ in the receiver as in Equation (2.12).

$$\mathbf{y} = \mathbf{Hs} + \mathbf{w} \qquad (2.12)$$

It is assumed that all three detectors have the exact information on channel matrix $\mathbf{H}$ including the transmitter and receiver together with the assumption of channel matrix to be fixed. In the sequel these detectors are explained briefly.

2.5.1 Maximum Likelihood Detector (MLD)

Maximum likelihood estimation is a method of estimating the parameters of a probability distribution by maximizing a likelihood function. The object of the receiver is to obtain an estimate of the message, $\hat{\mathbf{s}}$, from the given information in $\mathbf{y}$ and $\mathbf{H}$. Maximum likelihood detector (MLD) provides to achieve minimum probability of error, so it is also known as the optimum detector. Due to Gaussian channel consideration, maximum likelihood estimation corresponds to determining a symbol that minimizes the

Euclidean distance between received symbol and possible transmitted symbols sent through the channel matrix given in (2.13).

$$\hat{s} = \ argmin \sum_{m=1}^{N_R} \left| \boldsymbol{y}_m - \sum_{n=1}^{N_T} \boldsymbol{h}_{mn} \boldsymbol{s}_n \right|^2 \tag{2.13}$$

2.5.2 Minimum Mean Square-Error (MMSE) Detector

The Minimum mean square-error detector (MMSE) combines the received signals linearly to form an estimate of the transmitted symbols and represent in matrix form as

$$\hat{\mathbf{s}} = \ \mathbf{W^H y} \tag{2.14}$$

where $\mathbf{W}$ is given as $\boldsymbol{N}_R \times \boldsymbol{N}_T$ weighting matrix and it is used to minimize the mean square error and solution is obtained for the optimum weight vectors $\boldsymbol{\omega}_1$, $\boldsymbol{\omega}_2$, …, $\boldsymbol{\omega}_{N_T}$ as

$$\boldsymbol{\omega}_n = \ \mathbf{R}_{yy}^{-1} \boldsymbol{r}_{s_n y} \qquad \text{n} = 1, 2, 3, \ldots, N_T \tag{2.15}$$

where $\boldsymbol{\omega}_n$ is the nth column of $\mathbf{W}$, and $\mathbf{r}_{s_n^* y} = E[s_n^* \mathbf{y}]$.

2.5.3 Inverse Channel Detector (ICD)

Inverse channel detector is also combining the received signals linearly to form an estimate of transmitted signals. But in this detector, it is set $N_T = N_R$ unlike the case of MMSE, so it eliminates the interchannel interference definitely. $\mathbf{W}^H = \mathbf{H}^{-1}$ and, so the following equation is generated.

$$\hat{\mathbf{s}} = \ \mathbf{H^{-1} y} \tag{2.16}$$

In the case of $N_R > N_T$, the weighting matrix is selected as the pseudoinverse of the channel matrix;

$$\boldsymbol{W}^H = (\boldsymbol{H}^H \boldsymbol{H}) \boldsymbol{H}^H \tag{2.17}$$

In the following figures (Figure 3.1 and Figure 3.2) these three detectors are compared for a range of SNR values. According to Section 2.5 maximum likelihood detector shows the best error performance theoretically. Also, it is seen that minimum mean square-error detector (MMSE) shows better performance than inverse channel detector (ICD). In Figure 3.1 number of receiver antennas is used as ($N_R = 2$). Error performance of single input single output (SISO) case is shown to compare multi antenna usage.

CHAPTER THREE
ALPHA STABLE NOISE

Among the wide variety of distributions as candidates to model the channel noise, it is reported by (Win et. al., 2009) that random interference in wireless systems exhibits α −stable distribution. The reason is the existence of impulsive noise components in the channel and α −stable distribution to be a proper selection of the noise having heavy tail. In the sequel, stable distribution is explained in terms its parameters.

3.1 Alpha-Stable Distribution

As the definition, α −stable distribution in one dimension is described by its characteristic function as follows (Samorodnitsky, 1994).

$$\varphi(\omega) = \begin{cases} exp\left\{-\sigma^{\alpha}|\omega|^{\alpha}\left(1 - j\beta sgn(\omega) tan\left(\frac{\pi\alpha}{2}\right) + j\mu\omega\right)\right\} & \alpha \neq 1 \\ exp\left\{-\sigma|\omega|\left(1 + j\beta\frac{2}{\pi} sgn(\omega) ln|\omega|\right) + j\mu\omega\right\} & \alpha = 1 \end{cases} \tag{3.1}$$

where $sgn(\omega)$ in (3.1) is given as;

$$sgn(\omega) = \begin{cases} 1, & \omega > 0 \\ 0, & \omega = 0 \\ -1, & \omega < 0 \end{cases} \tag{3.2}$$

Noise parameters; characteristic exponent α, $\alpha \in (0,2]$, the skewness parameter β, $-1 \leq \beta \leq 1$, scale parameter σ, $\sigma > 0$, and position parameter μ, $\infty < \mu < \infty$ denote the amount of impulsiveness, asymmetry, intensity and the location of the noise, respectively. The intensity of noise is also defined in the literature by the dispersion parameter as $\gamma = \sigma^{\alpha}$ (Swami & Sadler, 2002). If $\beta = 0$ and $\mu = 0$, the distribution is said to be symmetric. Since the effect of the location of the distribution is not examined in this thesis, it is assumed $\mu = 0$ as the most frequent cases in the literature. The probability density function can be found from the characteristic function as given in (3.3) (Samorodnitsky & Taqqu, 1994).

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(\omega)e^{-j\omega x}d\omega \tag{3.3}$$

Instead of an anonymous representation $f(x)$, it is more convenient to use an analogous expression to the probability density functions as in parameterized form $S(x; \alpha, \beta, \sigma, \mu)$. Unfortunately, it cannot be determined analytically except for special cases given below (Janicki & Weron, 1994).

Gaussian distribution Gauss ($\alpha = 2$),

$$S(x; 2{,}0, \sigma, \mu) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3.4}$$

Cauchy distribution ($\alpha = 1$) having density

$$S(x; 1{,}0, \sigma, \mu) = \frac{2\sigma}{\pi(x-\mu)^2 + 4\sigma^2} \tag{3.5}$$

Levy distribution ($\alpha = {}^{1}/_{2}, \beta = 1$).

$$S\left(x; \frac{1}{2}, 1, \sigma, \mu\right) = \left(\frac{\sigma}{2\pi}\right)^{1/2} (x-\mu)^{-3/2} e^{-\frac{\sigma}{2(x-\mu)}} \tag{3.6}$$

In Figure 3.1, the effect of the symmetry parameter is illustrated when the characteristic exponent α is constant. Figure 3.2 illustrates the density function with respect to various characteristic exponent values. If the tails are being concentrated, it is seen that as the impulsiveness of the distribution increases (α decreases), the area covered by the distribution increases. Therefore, α −stable distributions are also called heavy-tailed distributions.

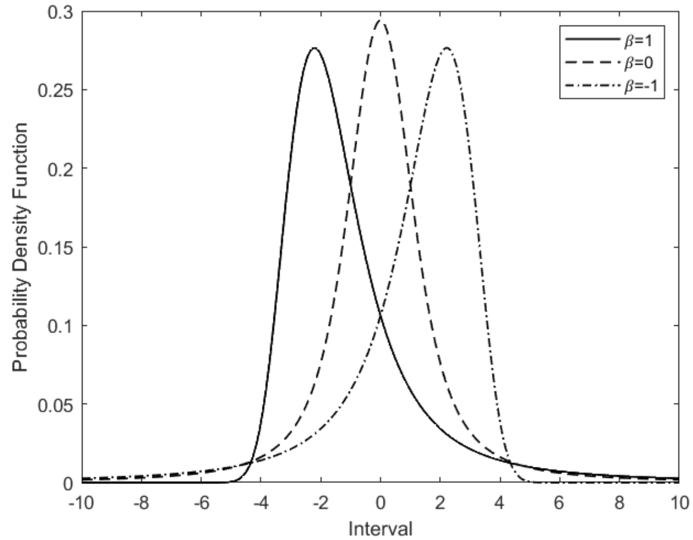

Figure 3.1 Variation of probability density function with respect to β ($\alpha = 1.3$)

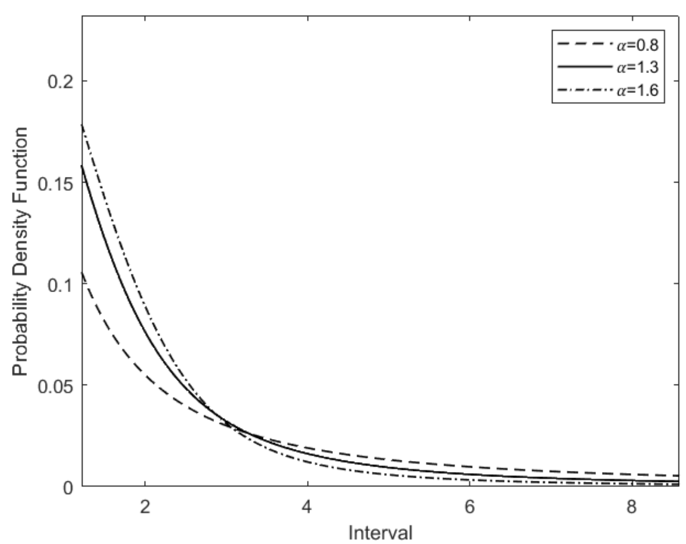

Figure 3.2 Tail distributions for various α, ($\beta = 0$)

3.2 Covariance under Alpha-Stable Noise

It should be noted that one of the distinctive properties of α −stable distributions is infinite variance nature. Correspondingly, an important feature of α −stable distribution is that only moments less than α are finite. This leads to formulate the moments of stable random process by assigning lower order than 2 to obtain finite values. This feature is described for a stable random variable X as in Equation (3.4) below, where E is the expectation operator (Samorodnitsky & Taqqu, 1994).

$$E|X|^p < \infty \quad p < \alpha \qquad (3.7)$$
$$E|X|^p = \infty \quad p \geq \alpha$$

According to this feature, Fractional Lower Order Moment (FLOM) can be used to statistically characterize the signal having stable distribution such as autocorrelation, auto or cross covariance which is alternatively called as Fractional Lower Order Statistics (FLOS). Although the term *covariation* is sometimes used in the literature to differentiate the conventional covariance from Gaussian distribution and the covariance correspondence in infinite variance distribution, the term fractional lower order correlation/covariance is a satisfactory expression to describe the statistical relation of stable processes. From the set of discrete time observation $x[n]$ having N samples, the sample fractional lower-order auto-covariance (FLOC) is described as (Cek, 2015).

$$R[k] = \frac{\sum_{n=N_1+1}^{N_2} |x[n]|^a |x[n+k]|^b sign(x[n]x[n+k])}{N_2 - N_1} \qquad (3.8)$$

where $N_1 = \max(0, -k)$, $N_2 = \min(N-k, N)$, $a = b = \frac{\alpha}{4}$. This correlation function provides the evaluation of analytical operations of detectors in MIMO minimum mean square error (MMSE) detection in which covariance information is required.

3.3 Generation of Alpha-Stable Distribution

Any random variable can be generated by applying appropriate transformation of random variables having uniform and exponential distributions. As the simpler case, a random variable X having symmetric $\alpha-$stable $(S\alpha S)$ with unit intensity represented by $X \sim S(\cdot\,; \alpha, 0,1,0)$ is obtained according to the direct method (Janicki & Weron, 1994)

- Generate a random variable $V \sim \mathcal{U}(-\pi/2, \pi/2)$ where $\mathcal{U}(\cdot)$ is the uniform distribution and exponential random variable W having mean 1;
- Determine X by applying the following formulation

$$X = \frac{\sin(\alpha V)}{(\cos V)^{1/\alpha}} \cdot \left[\frac{\cos(V - \alpha V)}{W}\right]^{(1-\alpha)/\alpha} \tag{3.9}$$

When the problem is more generalized to cover the generation of skewed α −stable random variable $Y \sim S(\cdot\,; \alpha, \beta, 1{,}0)$, the following transformation is applied (Janicki & Weron, 1994)

$$Y = D_{\alpha,\beta} \cdot \frac{\sin\left(\alpha\left(V + C_{\alpha,\beta}\right)\right)}{(\cos V)^{1/\alpha}} \cdot \left[\frac{\cos\left(V - \alpha\left(V + C_{\alpha,\beta}\right)\right)}{W}\right]^{(1-\alpha)/\alpha} \tag{3.10}$$

where

$$C_{\alpha,\beta} = \frac{\arctan(\beta \tan(\pi\, \alpha/2))}{1 - |1 - \alpha|}$$

$D_{\alpha,\beta} = [\cos(\arctan(\beta \tan(\pi\, \alpha/2)))]^{-1/\alpha}$.

Note that $\alpha \in (0,1) \cup (1,2]$ and $\beta \in [-1,1]$.

CHAPTER FOUR

CAPACITY and MIMO SYSTEM ANALYSIS UNDER SKEWED ALPHA-STABLE DISTRIBUTION

Although there are various publications to analyze SISO communication system under symmetric α −stable ($S\alpha S$) noise, there are a few recent papers discussing capacity limits under impulsive non-Gaussian distributions in which the impulsive noise is modeled with $S\alpha S$. To the best of our knowledge, only the publication which discusses the effect of skewness on channel capacity is given by (Wang & Kuruoğlu & Zhou, 2011). One of the contributions of the thesis is to analyze the capacity variation under both symmetrical and skewed α −stable noise. Since MIMO obviously has an enhancement on the capacity, the investigation is performed considering SISO model with respect to varying stable noise parameters, discussed in section 4.1.

During the analysis of both capacity in SISO and detectors in MIMO communication systems, the baseband BPSK modulation is utilized to reflect the effect of noise more apparently rather than discussing different modulation types. Equivalently, the baseband BPSK symbol is modeled with carrying binary information with amplitude $\{-A, A\}$. Due to infinite variance property of the channel noise indicated by Equation (3.7), the signal to noise ratio is not conventionally expressed in terms of variance. Alternatively, the generalized signal to noise ratio (GSNR), given by (Sureka & Kiasaleh, 2013), can be used to represent signal power to noise power ratio given by (4.1)

$$GSNR = 10 \log \frac{A^2}{\sigma^\alpha} \tag{4.1}$$

The amplitude of the signal is set to $A = \sqrt{10^{\frac{GSNR}{10}}}$ for desired $GSNR$ value, since it is accepted as $\sigma = 1$ for sake of simplicity in the literature.

Basically, the maximum likelihood detector (MLD) in MIMO communication relies on the determination of the symbol which yields minimum error norm. The norm expression denotes the square of the error term due to channel noise. In order to overcome unstable results at the receiver outputs for each received symbol having N_T and N_R number of transmitter and receiver antennas, the MLD detector is reformulated considering the fractional lower order moment (FLOM) approach where the condition $p < a$ is sufficient to satisfy the error to lie within a certain bound. Considering a similar approach given by (Zeng et al., 2013) p^{th} fractional lower order norm is replaced differing from the conventional representation of MLD detector in MIMO system given by (Proakis & Salehi & Bauch, 2013).

$$\hat{\boldsymbol{s}} = argmin \sum_{m=1}^{N_R} \left| \boldsymbol{y}_m - \sum_{n=1}^{N_T} \boldsymbol{h}_{mn} \boldsymbol{s}_n \right|^p \tag{4.2}$$

The variation of probability of error depends on the p parameter as shown in Figure 4.1, and it is seen that selecting $p < \alpha$ provides a significant improvement in the probability of error. Figure 4.1 is illustrated by using MATLAB, the number of antennas is assumed to be $N_T = N_R = 4$, $\alpha = 1.2$ and in Monte Carlo simulations 2^{14} bits are randomly generated and the average of 10 realizations is taken.

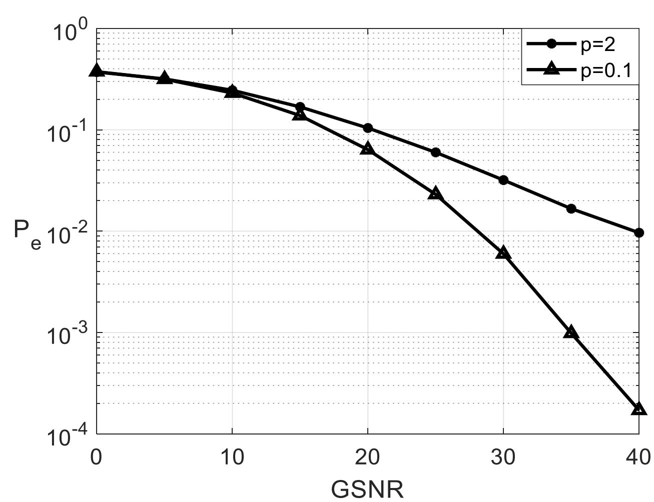

Figure 4.1 Variation of error probability due to p value under α −stable noise

In order to observe the effect of both α and β parameters together at a certain $GSNR$ value, the bit error rate results are illustrated in Figure 4.2 and Figure 4.3 for $\alpha = 1.1$ and $\alpha = 1.2$, respectively. The common result for both plots is that the bit error rate performance increases when the noise becomes symmetric and there exist a valley along with the line $\beta = 0$, any positive or negative skewness results in degradation in BER performance. This leads to an idea that any manipulation at the receiver which symmetrizes the noise can be used to enhance the BER performance. The second consideration is on the characteristic exponent α. Increasing impulsiveness, i.e., decreasing α, causes BER performance to become poorer. The sensitivity to asymmetry of the channel is more apparent when the impulsiveness increases. This means that when the noise gets closer to Gaussian distribution, the effect of the skewness in weakened.

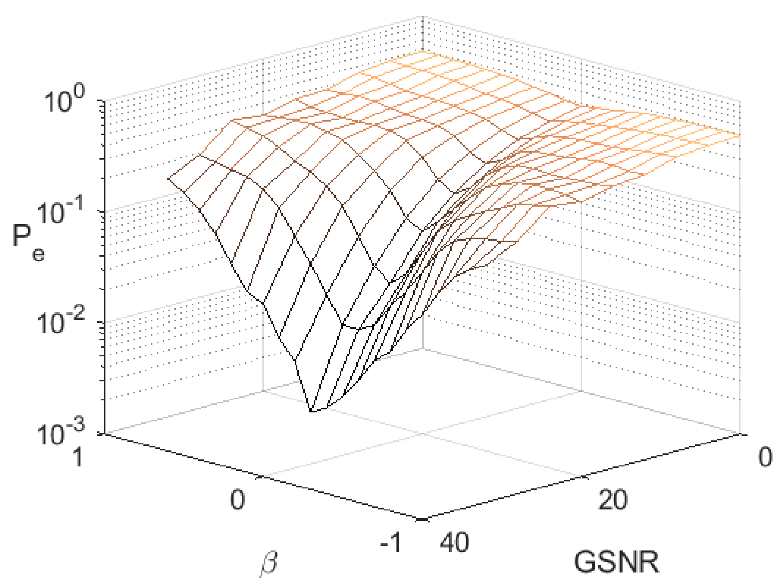

Figure 4.2 The probability of error due to GSNR and β under skewed α −stable noise ($\alpha = 1.1$)

It is also seen in Figure 4.2 and 4.3 that even if a small variation occurs in characteristic exponent α, the impulsiveness of the α −stable distribution is strongly affects the error results. In order to illustrate the error performance, Monte Carlo simulations need more random data bit and realization which requires more computation time. On the other hand, even though error performance of MIMO communication system changes due to parameters of α −stable noise, it is also changes regarding to usage of number of antennas at transmitter and receiver. As in the case of having the same transmitter and receiver antenna ($N_T = N_R$), MIMO system may also include unequal number of antennas at the transmitter and the receiver ($N_T \neq N_R$).

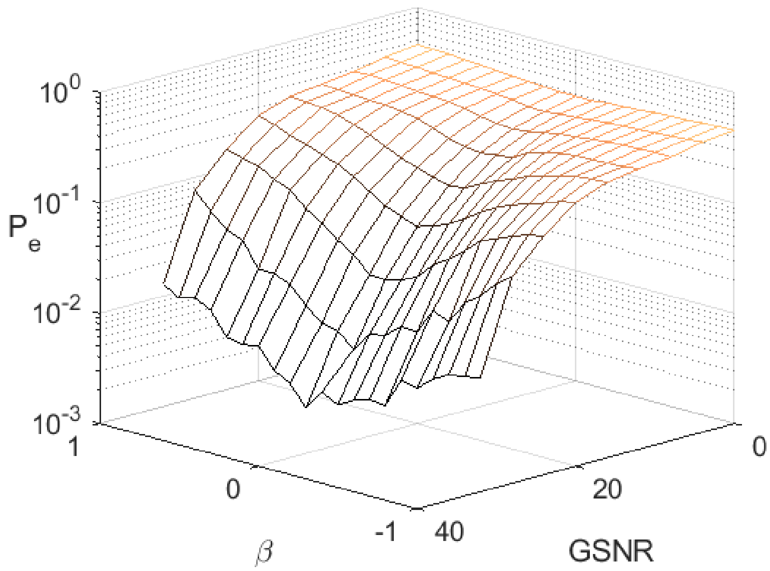

Figure 4.3 The probability of error due to GSNR and β under skewed α −stable noise ($\alpha = 1.2$)

Error performance is changed when channel matrix is rectangular for $N_T > N_R$ or $N_T < N_R$ cases. Figure 4.4 illustrates the variation of error performance comparison is illustrated with respect to different number transmitter and receiver usage. In order to perform a fair comparison, noise parameters are taken fixed as characteristic exponential $\alpha = 1.2$, symmetry parameter $\beta = 0$, scale parameter $\sigma = 1$ and $\mu = 0$, respectively. The total number of 2^{14} bits are transmitted in Monte Carlo simulations.

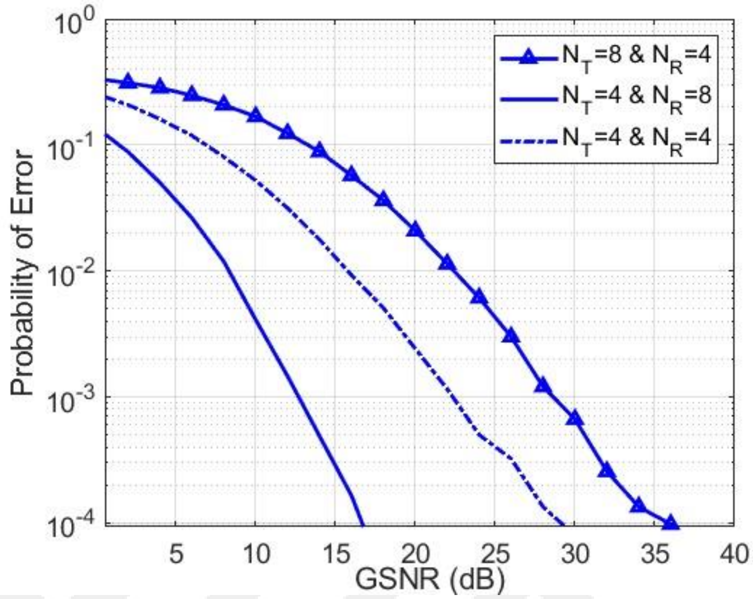

Figure 4.4 The probability of error having different number of antennas at the transmitter and receiver.

It is seen that $\mathrm{N_T} < \mathrm{N_R}$ case has the best error performance than other cases. The reason is that as the number of receiver antennas increases, the data rate increases also and antennas are combined to minimize the error.

4.1 Capacity

The conventional channel capacity of the MIMO communication systems under Gaussian noise is examined comprehensively in the literature. To define the channel capacity of MIMO communication channel, mutual information is calculated between transmitted and received signals, denoted as $I(s; y)$. In Equation 4.3 mutual information computation is given (Proakis et al., 2013).

$$C = \ max\ I(\boldsymbol{s}; \boldsymbol{y}) \tag{4.3}$$

C is the channel capacity of MIMO channel in bits per second per frequency (bps/Hz) for the channel matrix **H** and is given in Equation 4.4 (Proakis et al., 2013).

$$C = log_2 det(\boldsymbol{I}_{N_R} + \frac{1}{N_0}\mathbf{H}\mathbf{R}_{ss}\mathbf{H}^H) \quad (4.4)$$

Since there is lack of publication discussing the effect of impulsive noise parameters in the literature, rather than discussing channel capacity directly in terms of MIMO system parameters, SISO system under both symmetric and skewed α −stable distribution is investigated in the thesis. Only the study by (Wang et. al., 2011) investigates the effect of noise parameters, it is reported to be observed that channel capacity decreases as the parameter α decreases, due to increase of the impulsivity of noise shown in Figure 4.5 when the noise is kept to be symmetric.

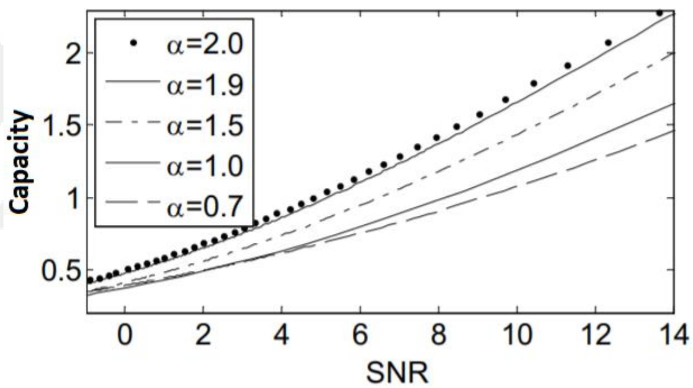

Figure 4.5 Channel capacity with varies α (Wang et al., 2011)

It is reported by (Wang et. al., 2011) that the skewness of the noise for fixed characteristic exponent ($\alpha = 1.3$) affects more slightly shown in Figure 4.6 compared with the effect of characteristic exponent α given in Figure 4.5. It is reported that when $\beta = 0.5$ and $\beta = -0.5$ channel capacity have the same effect. It means asymmetry parameters effects the channel capacity only with its absolute value, any positive or negative divergence from the symmetry causes the same difference on capacity change. It is also declared by (Wang et. al., 2011) that the channel capacity does not change with scaling and location parameters of the α −stable noise.

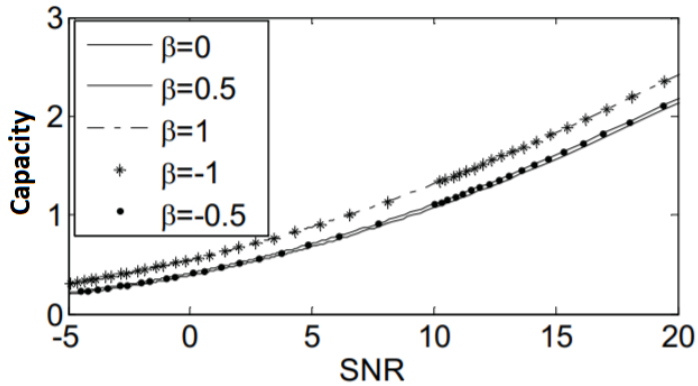

Figure 4.6 Channel capacity with varies β (Wang et al., 2011)

Considering the transmission of band-limited signal in additive non-Gaussian noise channel, the input output relation can be formulized as (Proakis, 2001)

$$y_i = x_i + w_i \tag{4.5}$$

where w_i has $\alpha-$stable distribution and considered to be uncorrelated. Using the information that the capacity of the channel expressed in terms of bits per channel corresponds to the maximum value of the average mutual information between discrete input $X = \{x_1, \cdots, x_j\}$ and the output $Y = \{-\infty, \infty\}$. Then the capacity expression becomes

$$C = \max_{P(x_i)} \sum_{i=1}^{j} \int_{-\infty}^{\infty} p(y|x_i)\, P(x_i) \log_2 \frac{p(y|x_i)}{p(y)} dy \tag{4.6}$$

where

$$p(y) = \sum_{i=1}^{j} p(y|x_i) P(x_i) \tag{4.7}$$

The conditional density $p(y|x_i)$ is obtained from $\alpha-$stable pdf $S(x; \alpha, \beta, \sigma, \mu)$ as

$$p(y|x_i) = S(y - x_i; \alpha, \beta, \sigma, 0) \tag{4.8}$$

If the problem is reduced to obtain capacity for binary-input α −stable distributed memoryless channel, then the capacity is formulated as in the case given by (Proakis, 2001)

$$C = \frac{1}{2}\int_{-\infty}^{\infty} p(y|A)\, log_2 \frac{p(y|A)}{p(y)}\, dy + \frac{1}{2}\int_{-\infty}^{\infty} p(y|-A)\, log_2 \frac{p(y|-A)}{p(y)}\, dy \quad (4.9)$$

Since the analytical closed from expression of α −stable distribution does not exist for arbitrary parameter selection, integration in equation (4.9) is performed numerically using trapezoidal rule to obtain capacity C in figures Figure 4.7, 4.8 and Figure 4.9, respectively. The variation of capacity with respect to $GSNR$ given by equation (4.1) for antipodal binary input is analyzed with respect to the following impulsive noise parameters such as characteristic exponent and skewness.

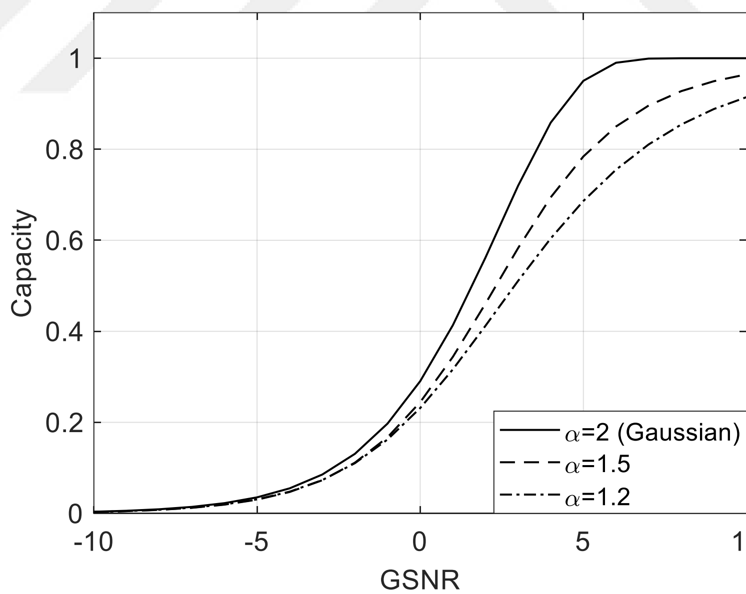

Figure 4.7 Channel capacity under α −stable noise with respect to $\alpha, (\beta = 0)$.

The effect of the characteristic exponent on the channel capacity is shown in Figure 4.7 It is seen that when the impulsiveness of the α −stable noise increases, i.e. α decreases, the capacity of the channel for fixed $GSNR$ decreases. Similarly, the channel capacity is also analyzed with respect to variation of skewness parameter shown in Figure 4.8. It is observed that the channel capacity slightly increases when the channel noise becomes skewed. One can clearly see that the binary memoryless channel capacity variation results with respect to stable noise parameters are consistent with the study (Wang et. al., 2011) in the literature shown in Figure 4.5 and Figure 4.6.

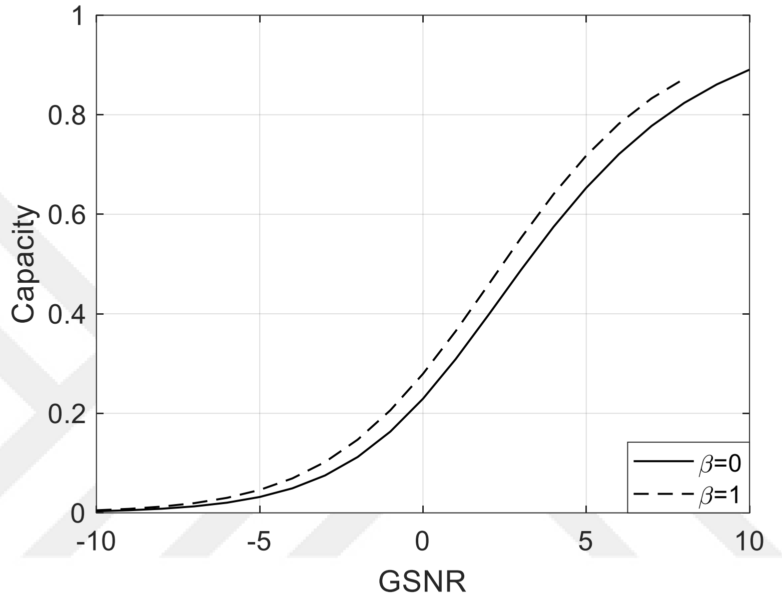

Figure 4.8 Channel capacity variations with respect to skewness. ($\alpha = 1.1$)

Although the channel capacity dependence on noise parameters are illustrated separately in Figure 4.7 and Figure 4.8, the divergence from the symmetry within a certain GSNR range in three dimensions is shown in Figure 4.9. Since the increased impulsiveness of the noise reflects effect of asymmetry more clearly, the characteristic exponent of the channel noise in Figure 4.9 is taken as $\alpha = 0.8$. It is seen that there is a small valley when the noise becomes symmetric which corresponds the capacity to be worse than skewed value at a certain $GSNR$ value. However this effect disappears when the impulsiveness of the noise weakens, i.e., α gets closer to 2, Gaussian distribution.

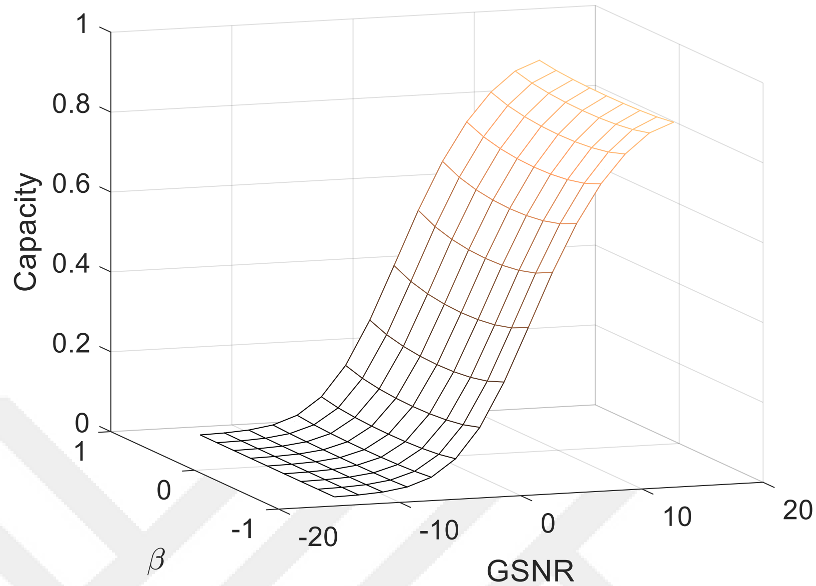

Figure 4.9 Channel capacity variation for both β and $GSNR$. ($\alpha = 0.8$)

4.2 Receiver Types

In this section maximum likelihood detector (MLD) and inverse channel detector (ICD) are introduced for MIMO communication systems and comparison of bit error rate performance is illustrated under skewed α −stable distribution.

In Figure 4.10 characteristic exponential is taken as $\alpha = 0.8$, symmetry parameter is taken as $\beta = 0$, scale parameter is taken as $\sigma = 1$. 2^{14} bits are used as input signal and number of antennas are taken as ($N_T = N_R = 4$).

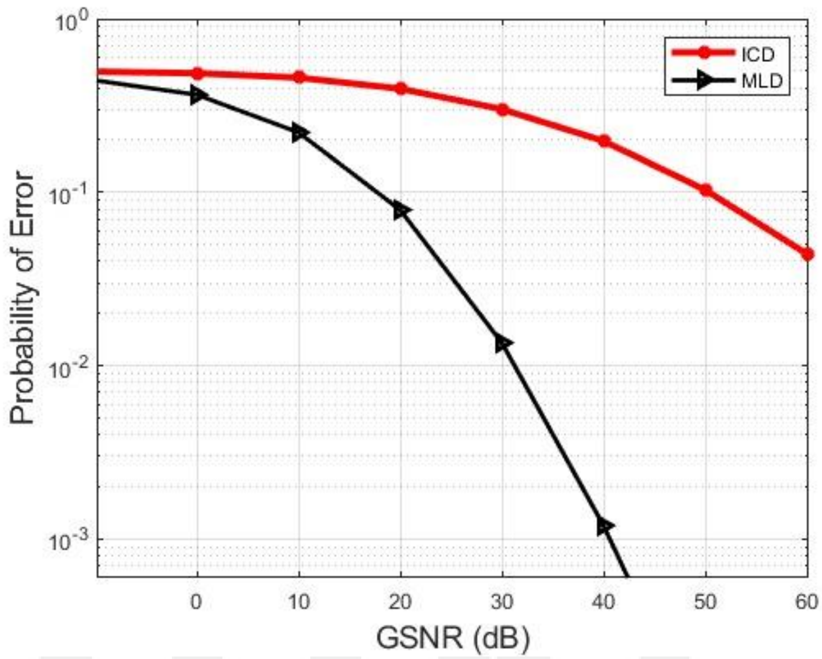

Figure 4.10 Bit error rate comparison of MLD and ICD detectors

It is approved that maximum likelihood detector has better performance than inverse channel detector by using the same parameters.

4.3 Stochastic Resonance

Due to the finding that the probability of error decreases in case the stable noise is symmetrical, it is suggested in the thesis that symmetrization process can be achieved at the receiver by adding intentional noise having inverse symmetry in addition to the noise in the channel. This noise injection results in stochastic resonance and causes a decrease bit error rate.

Intentional noise is expressed by $\mathbf{w}_{int_m}[\cdot] \sim S(\alpha, \beta_{int}, \sigma_{int}, 0)$ distribution parameter where $\beta_{int} = -\beta$ and the receiver design is revised by Equation (4.5).

$$\mathbf{y} = \mathbf{Hs} + \mathbf{w} + \mathbf{w}_{int} \tag{4.5}$$

In Equation (4.5), the distribution of the total noise arises from the properties of the α −Stable noise and shown as $\mathbf{w}[\cdot]\sim S(\alpha, \beta_{res}, \sigma_{res}, 0)$ where the resultant noise parameters are as in Equation (4.6) (Samorodnitsky & Taqqu, 1994).

$$\sigma_{res} = (\sigma^{\alpha} + \sigma_{int}^{\alpha})^{1/\alpha} \qquad \beta_{res} = \frac{\beta\sigma^{\alpha} + \beta_{int}\sigma_{int}^{\alpha}}{\sigma^{\alpha} + \sigma_{int}^{\alpha}} \tag{4.6}$$

The receiver output generates a total symmetrical behavior by adjusting the intensity and / or asymmetry of the noise in the channel. Bit error rate is illustrated in Figure 4.11 and Figure 4.12. Using Monte Carlo simulations 2^{14} bits are utilized as baseband BPSK modulated input signal and number of antennas are taken as $N_T = N_R = 4$. In Figure 4.11 characteristic exponential parameter is taken as $\alpha = 1.1$, intensity parameter is taken as $\sigma = 1$ and intensity parameter of intentional noise is taken as 1 ($\sigma_{int} = 1$). Number of multiple antenna usage is managed as ($N_T = N_R = 4$).

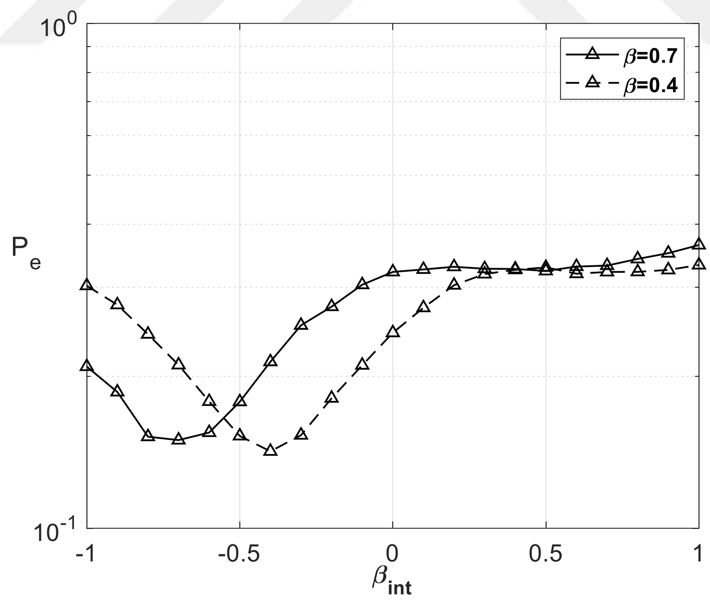

Figure 4.11 Error probability due to β, under skewed α −stable noise

Figure 4.11 shows the probability of error of the channel due to intentional noise, respectively, while $\sigma = 1$ and $\sigma_{int} = 1$, while the asymmetry parameter of the channel noise is $\beta = 0.7$ and $\beta = 0.4$, the probability of error varies according to the value of skewness parameter. It is seen that the error probability is minimized at $\beta = -0.7$ and $\beta = -0.4$ which makes the total noise symmetrical.

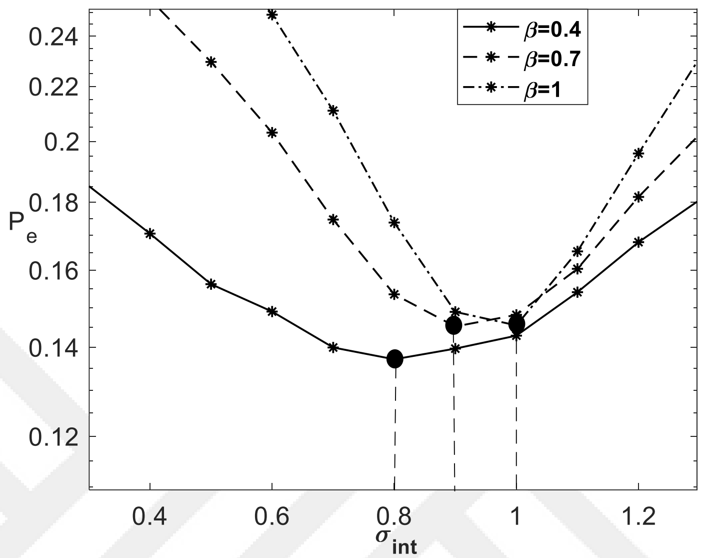

Figure 4.12 Error probability due to σ_{int}, under skewed α −stable noise

Figure 4.12 shows the bit error rate variation with respect to the variation of intentional noise intensity when the channel noise is $\sigma = 1$, the skewness parameters $\beta = -0.4$, $\beta = -0.7$ and $\beta = 1$, respectively. When the asymmetry parameter of the intentional noise has a value of $-\beta$, the probability of error changes due to the intentional noise intensity σ_{int}. As the channel noise get closer symmetrical (decreasing β as absolute value), it is seen that there is a deviation from $\sigma_{int} = 1$ which is theoretically expected to give the best value. When the findings obtained in Figure 4.8 and Figure 4.12 are generalized, the error probability reduction corresponding to the resonance state is obtained by selecting the pair of β_{int} and σ_{int} which provides the Equation (4.7), even if the direct asymmetry value or intensity of the noise in the channel is not known in advance.

$$\beta_{int}\sigma_{int}^{\alpha} = -\beta\sigma^{\alpha} \tag{4.7}$$

Only the strong requirement is that the characteristic exponent of the channel should be exactly known at the receiver.

CHAPTER FIVE
CONCLUSION

In this thesis, the MIMO communication systems are analyzed under $\alpha-$stable noise exhibiting impulsive behavior. Differing from the previous studies in the literature, the thesis study analyses the MIMO communication system in the presence of asymmetric noise. $\alpha-$stable distributed noise is considered as non-Gaussian noise which also has the ability to exhibit asymmetric behavior. The skewness parameter of the $\alpha-$stable noise is also taken into account to observe the effect of asymmetry on detector performance. It is shown that, the error performance becomes poorer when the noise becomes positive or negative skewed. The best error performance can be achieved when the noise is symmetric for fixed characteristic exponent, α. Additionally, when the characteristic exponent decreases, i.e, the channel noise becomes more impulsive, bit error rate increases monotonically, which is consistent with the literature. In order to reduce the error rate, $\alpha-$stable noise having opposite skewness is proposed, called intentional noise, at the receiver. This results in stochastic resonance phenomenon and the improvement on error performance is illustrated in terms of error parameters. Since the increment on characteristic exponent leads to more symmetrical behavior of the noise even the skewness parameter remains the same, the stochastic resonance is hardly apparent when noise probability approaches to Gaussian.

Secondly, the detectors are modified by redefining by fractional lower order power in decrease the error rate due to the infinite variance property of the stable noise. This constitutes the second contribution of the thesis study.

Asymmetric impulsive noise effect is also considered to observe channel capacity under skewed $\alpha-$stable noise environment. Since the conventional capacity measurement is defined for Gaussian distribution, the capacity variation with respect to noise parameters are evaluated directly by finding mutual information. It is observed that, there is no an apparent effect of skewness of the noise even there is a slight improvement on error performance under skewed α −stable noise.

Since the main contribution of the thesis is concentrated on error performance improvement of MIMO systems under non-Gaussian noise environment including also asymmetric distribution, the modulation type is considered to be BPSK during all simulations.

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