DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

CHAOTIC COMMUNICATION IN ALPHA-STABLE NOISE CHANNEL USING ROBUST ESTIMATORS

by
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İZMİR

CHAOTIC COMMUNICATION IN ALPHA-STABLE NOISE CHANNEL USING ROBUST ESTIMATORS

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M.Sc THESIS EXAMINATION RESULT FORM

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.

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ABSTRACT

The thesis study investigates the behavior of non-coherent chaotic communication system under non-Gaussian noise. Due to broadband nature of chaotic signals, conventional band pass filters cannot be applied for noise reduction. The robust estimators are utilized to remove impulsive non-Gaussian noise effect on the chaotic signal. These estimators are called as Median, Myriad and Meridian estimators. The non-Gaussian noise is modeled to have $\alpha -$stable distribution. The performances of these estimators are given in terms of error probability for varying characteristic exponent which tunes impulsiveness of the channel noise.

The estimators are used for time varying signal by assigning a sliding window function having certain length. It is shown that even if the selection of window length directly affects the amount of noise removal from the signal, all of the robust estimators provide a certain bit error rate performance improvement. It should be noted that the linearity parameter of Myriad and medianity parameter of Meridian filter should be assigned properly to achieve satisfactory bit error rate improvement.

Keywords: Skewed alpha-stable distribution, robust estimators, digital communication

GÜRBÜZ KESTİRİCİLER KULLANARAK ALFA-KARARLI GÜRÜLTÜLÜ KANALDA KAOTİK HABERLEŞME

ÖZ

Tez çalışması, Gauss olmayan gürültü altında, uyumlu olmayan kaotik haberleşme sistemini incelemektedir. Kaotik işaretlerin geniş bantlı doğasından dolayı geleneksel bant geçiren süzgeçler gürültü azaltımı için uygulanamaz. Gürbüz kestiricilerden, kaotik işaret üzerindeki Gauss olmayan dürtüsel gürültünün süzgeçlenmesinde faydalanılmaktadır. Bu kestiriciler, Median, Myriad ve Meridian kestiricilerdir. Gauss olmayan gürültü, $\alpha -$kararlı dağılıma sahip olarak modellenmektedir. Bu kestiricilerin başarımları, kanal gürültüsünün dürtüselliğini belirleyen değişen karakteristik üstel için hata olasılığı cinsinden verilmektedir.

Zamanla değişen işaret için kestiriciler belli bir uzunlukta kayan bir pencere fonksiyonu tanımlayarak kullanılmaktadır. Pencere uzunluğu seçimi işaretten gürültünün kaldırılma miktarına doğrudan etki etse bile, tüm gürbüz kestiricilerin kesin bir bit hata oranı başarımı iyileştirmesi sağladığı gösterilmektedir. Myriad süzgecinin doğrusallık parametresi ve Meridian süzgecinin medyanlık parametresinin, tatmin edici bit hata oranı başarımı iyileştirilmesine ulaşmak için uygun bir şekilde tanımlanması gerektiği not edilmelidir.

Anahtar Kelimeler: Alfa-kararlı dağılım, gürbüz kestiriciler, sayısal haberleşme

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CHAPTER ONE
INTRODUCTION

Security in digital communication systems has been a significant demand in recent years. Beginning with the synchronization of chaotic systems introduced by Pecora and Caroll in 1990, chaotic communication offers a proper alternative way of communication hiding the narrow band message, differing from the conventional spectrum spreading techniques (Pecora & Caroll, 1991). In the sequel, properties of signals generated by chaotic dynamic systems are briefly given which explains how the chaotic signals can be considered as candidate in secure communication.

1.1 Potential of Chaos in Communications

Aspects of the chaotic dynamical systems are well understood and the main idea behind chaotic communication is to use and exploit such aspects in communication applications. Such a system will have more simplified requirement then a standard, conventional communication system, due to features of chaotic signals. Basic aspects of such systems will be discussed in the following sections.

1.1.1 Broadband Aspect

Chaotic signals exhibit aperiodic behaviour, which leads the spectrum to be wideband. As an illustration, frequency spectrum obtained from one of the states of Rössler system expressed by Eq. (1.1) is shown in Figure 1.1

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{aligned} \tag{1.1}$$

where $a = 0.2, b = 0.2, c = 8$ (Liao & Yu, 2006). The broadband nature of chaotic signals provides a potential to camouflage the narrow band message signal by embedding it into the wideband chaotic signal. This property can be considered as an alternative approach to *spread spectrum* communication system which has a certain correspondence of covertness in communication (Abel & Schwarz, 2002).

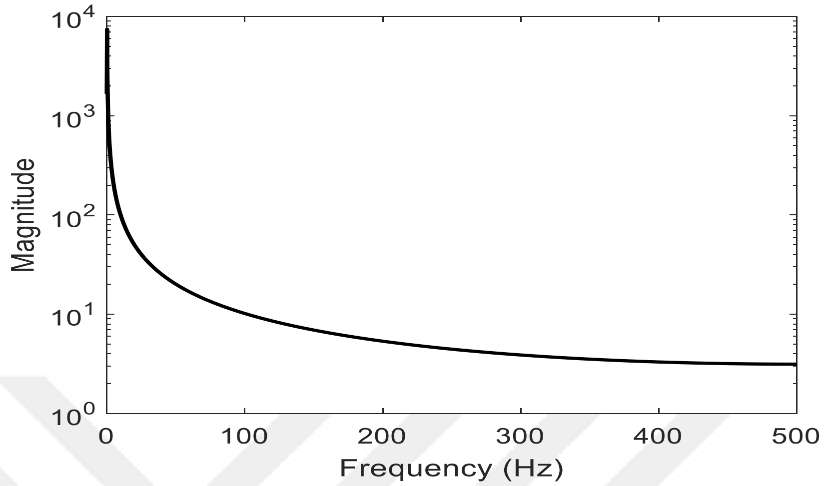

Figure 1.1 Frequency spectrum of Rössler signal $x(t)$ sampled at 1kHz

1.1.2 Complexity Aspect

One of the most distinctive properties of chaotic dynamic systems is sensitively dependence on initial conditions. It is always generated different signals with respect to the chosen initial condition. Variation of the signals as a function of time observed from Rössler system is illustrated in Figure 1.2, when the initial conditions are perturbed. It is obviously seen that one can generate completely different trajectory even a slight perturbation is applied to the initial conditions. It is not possible to extract information and estimate the signals over long time intervals. Such a complex and unpredictable behavior makes the chaotic signals a proper candidate in another area of chaos; *cryptography (*Abel & Schwarz, 2002).

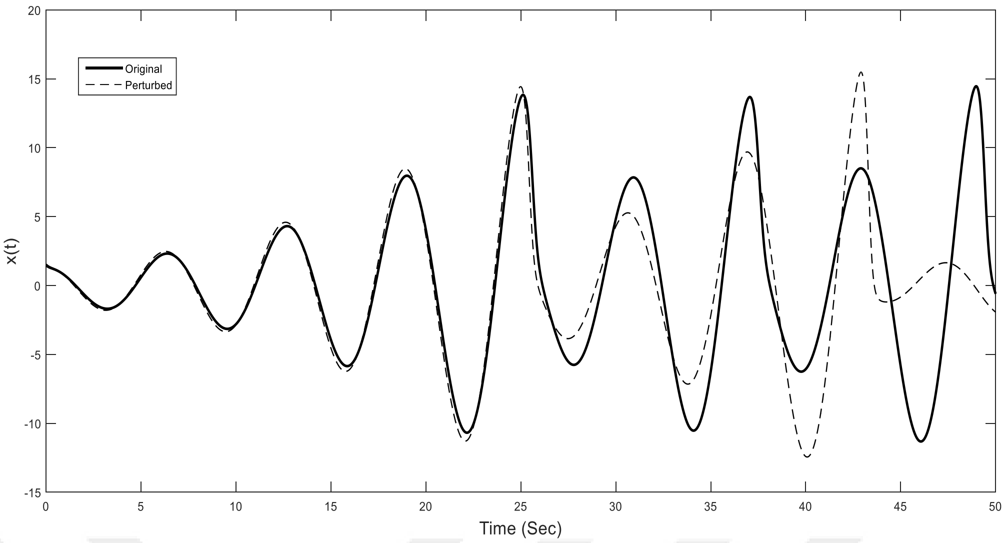

Figure 1.2 Two Rössler signals where the initial conditions are perturbed by 0.1

1.1.3 Orthogonality Aspect

Each physical channel has a limited message transmission capacity and the vital point in a communication system design is to adjust the share of limited sources efficiently. Orthogonal signals are used to share the resources among the large number of users without any overlap in time or frequency. Orthogonality principle given in Eq. (1.2) ensures the separability of used signals

$$\int x_1(t)x_2^*(t)\,\mathrm{dt} = 0 \tag{1.2}$$

where * denotes complex conjugate. Parseval's theorem given by Eq. (1.3) (Abel & Schwarz, 2002)

$$\int_{-\infty}^{\infty} x_1(t)x_2^*(t)\mathrm{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega)X_2^*(\omega)\mathrm{d}\omega \tag{1.3}$$

where $X_1(\omega)$ and $X_2(\omega)$ are Fourier transforms of $x_1(t)$ and $x_2(t)$ respectively, implies that orthogonality in the time domain indicates orthogonality in frequency domain. It can be achieved in different ways given as follows.

1.1.3.1 Signals Disjoint in Time

Eq. (1.2) holds trivially if either $x_1(t)$ or $x_2(t)$ becomes zero at any time, which corresponds to time division multiple access (TDMA).

1.1.3.2 Signals Disjoint in Frequency

Eq. (1.3) becomes zero if at any frequency either $X_1(\omega)$ or $X_2(\omega)$ vanishes which corresponds to frequency division multiple access (FDMA).

1.1.3.3 Uncorrelated Signals

Eq. (1.2) can hold without satisfying conditions given above even if the signals are disjoint neither in time nor in frequency. This is provided by applying an appropriate code to yield orthogonality and corresponds to code division multiple access (CDMA).

Basically, these features arise from the autocorrelation structure of chaotic signals. Due to the noise like behavior of chaotic signal, the autocorrelation function decays to zero within a small time lag. As an illustration, ensemble averaging of 50 realizations of autocorrelation function of the state $x(t)$ taken from Rössler system is shown in Figure 1.3. One can clearly observe that, the envelope of the autocorrelation decays to zero with respect to time lag. This property provides chaotic systems to be utilized for *multiuser applications* for any communication system (Abel & Schwarz, 2002).

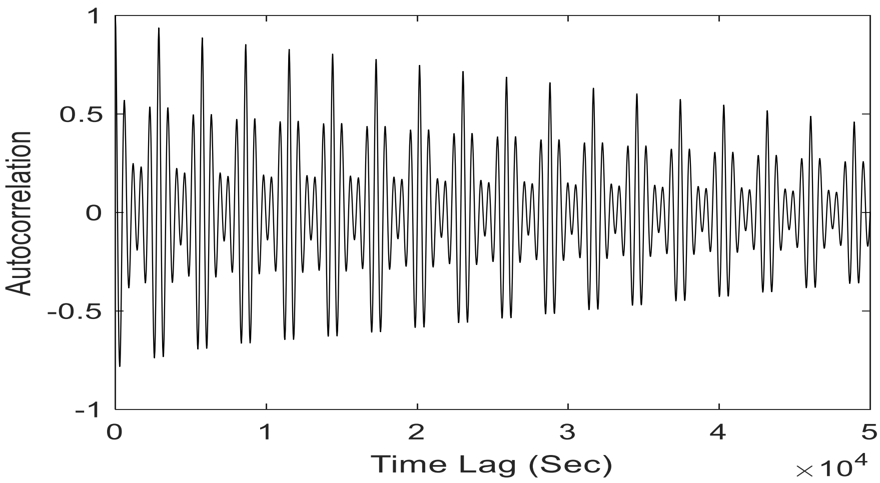

Figure 1.3 Autocorrelation function of Rössler System

1.2 Chaotic Synchronization

In order to be able to apply chaotic systems in communication applications, one of the requirements is to provide coherent connection between the transmitter and the receiver having different initial conditions in practice. (Pecora & Caroll, 1990) introduced synchronization between chaotic systems, the transmitter and receiver structures can be defined to establish entire communication system. Synchronization of two chaotic systems are defined in terms of two subsystem known as drive subsystem for transmitter and response subsystem for receiver. The structure of this system is comprehensively explained by (Pecora & Caroll, 1991). This drive and response systems are coupled where response subsystem at the receiver is driven by one of the state variables of the transmitter while the drive system is not influenced by response system. Mathematical expression of such a system is given in Eq. (1.4)

$$\begin{aligned} \dot{v} &= \mathrm{f}(v,u),\, v \in R^m \\ \dot{u} &= \mathrm{g}(v,u),\, u \in \mathrm{R}^k \\ &\text{drive} \\ \dot{w} &= \mathrm{h}(v,w),\, w \in \mathrm{R}^l \quad \text{response} \end{aligned} \tag{1.4}$$

where $n = m + k + l$ is the total dimension of the this composite system. The objective here is to adjust a stable subsystem, and then the synchronization error between drive and response systems disappears. To analyze the stability of $w(t)$, a second trajectory, $w'(t)$ with different initial condition is considered and the difference equation $\Delta w(t) = |w'(t) - w(t)| \rightarrow 0$ is analyzed in terms of vector fields expressed in Eq. (1.5);

$$\begin{aligned} \dot{\Delta} w &= h(v,w') - h(v,w) \\ &= D_w h(v,w) \Delta w + o(v,w) \end{aligned} \tag{1.5}$$

where $D_w h$ is the Jacobian of the vector field with respect to response variables and $o(w)$ represents the higher order terms. For the limit case, $\Delta w(t)$ becomes small and Eq. (1.6) is obtained

$$\dot{\Delta}\mathrm{w} = \mathrm{D}_w h(\mathrm{v,w}) w \tag{1.6}$$

Instead of Eq. (1.6), a more general formulation that does not depend on the initial choice of a displacement is given in Eq. (1.7) where matrix Z is used to determine the stability

$$\dot{\Delta}\mathrm{Z} = \mathrm{D}_w h(\mathrm{v,w})\mathrm{Z}\,, \qquad \mathrm{Z}(0) = \mathrm{I} \tag{1.7}$$

As $t \rightarrow \infty$, for a particular drive trajectory $v(t)$ and $u(t)$, the matrix $Z(t)$ can be used to find the Lyapunov exponents for the system w subsystem. The Lyapunov exponents related with this subsystem are called as conditional Lyapunov exponents since it is driven by drive signal $v(t)$, (Pecora & Caroll, 1991). It can be said that $w(t)$ is asymptotically stable trajectory if the conditional Lyapunov exponents are negative. As it is obvious, condition of negativity of the conditional Lyapunov exponents for the w system is a necessary condition for the stability of the response, (Pecora & Caroll, 1991; Oppenhein et. al., 1993).

Eq. (1.1) can be evaluated as drive system since its dynamics are independent of response system, for the Rössler system; and a dynamical response system driven by $y(t)$ presented in Eq. (1.8). (Pecora & Caroll, 2015).

$$\dot{x}_2 = -y - z$$
$$\dot{z}_2 = b + z(x_2 - c) \tag{1.8}$$

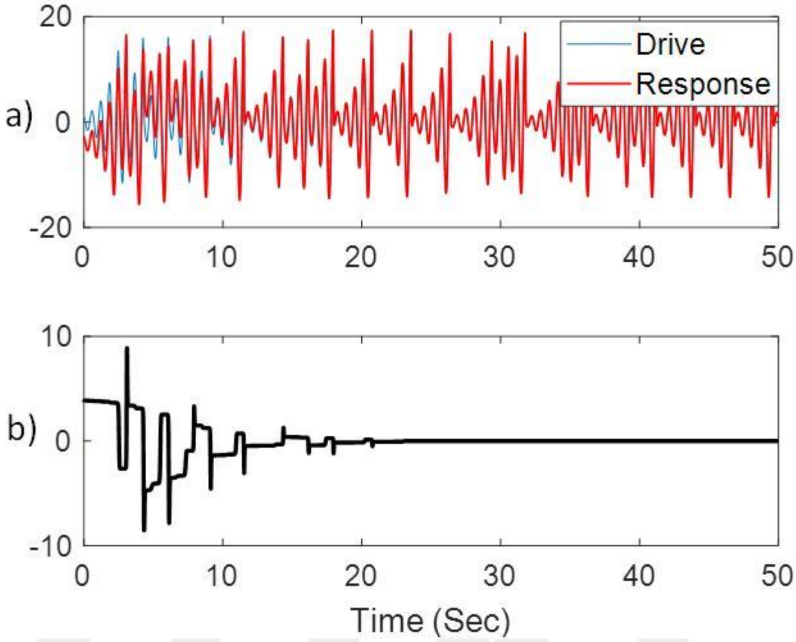

Figure 1.4 Illustration of synchronization for Rössler signal $x(t)$ in time domain

Figure 1.4 depicts the convergence of the response system $x_2(t)$ to the drive system $x(t)$ which is illustrated for y driven (x,z) Rössler subsystem. Synchronization error decays to zero after a short time interval as it can be clearly seen from Figure 1.4. Here it becomes obvious that the chaotic synchronization has significant importance in building a secure communication scheme based on chaotic dynamics and it has triggered the development of chaotic secure communication systems. Recently, the coherent communication based on chaos synchronization is proposed by (Wang et. al., 2018) introducing dual channel structure in order to increase the information security. Recently, the coherent communication based on chaos synchronization is proposed by (Wang et. al., 2018) introducing dual channel structure in order to increase the information security.

CHAPTER TWO
METHODS OF CHAOTIC COMMUNICATION

A bunch of communication systems using chaotic principles has been proposed in past decades. In this section, basic chaotic communication systems are introduced under the topics of two main categories; chaotic analog modulation, chaotic digital modulation.

Like conventional digital communication systems, chaos based communication is performed using coherent and noncoherent detection techniques. In coherent detection based chaotic communication, the transmitter and receiver should satisfy synchronization, which becomes a disadvantage when the signal to noise ratio decreases and the probability of error increases dramatically. In order to reconstruct data from the signals received at receiver side, chaotic carriers must be copied as replicas. Correlator type receiver can be considered as detector of coherent chaos shift keying system. To achieve a satisfactory detection performance at coherent receivers, several studies investigated coherent communication by applying robust synchronization techniques (Itoh & Murakami, 1995), (Kolumbán Géza, Kennedy & Chua, 1998), (Kennedy & Kolumban, 1999), (Hasler & Schimming, 2000), (Francis, Lau & Chi, 2002). Due to the practical problems related with chaos synchronization, it is also proposed to apply chaotic communication without requiring synchronzation (Kis, Jako, Kennedy & Kolumban, 1998). The binary Chaos Shift Keying (CSK) communication system given the Figure 2.1 is an example for correlator type coherent CSK communication systems.

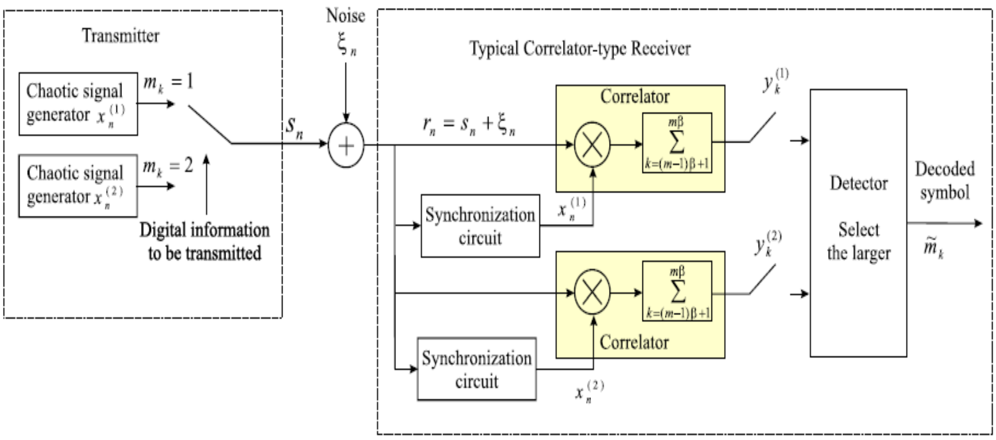

Figure 2.1 Binary CSK digital communication system (Francis, Lau & Chi, 2001)

On the transmitter side, two chaotic sequences are generated which are expressed as $\left\{x_n^{(1)}\right\}$, $\left\{x_n^{(2)}\right\}$ and spreading factor that is expressed as β defines the number of chaotic symbols to transmit a single binary symbol. Through the kth symbol, $n = (k-1)\beta + 1, (k-1)\beta + 2, \cdots, k\beta$ transmitted signal, s_n, is expressed as follows (Francis, Lau & Chi, 2001)

$$s_n = x_n^{(1)} if \;\; m_k = 1 \tag{2.1}$$

$$s_n = x_n^{(2)} if \;\; m_k = 2 \tag{2.2}$$

Alternatively, noncoherent chaos based communication systems do not require synchronous chaotic signal generator at the receiver and extract the binary information from received signal itself according to the prescribed scheme. Differential chaos shift keying, generalized chaos shift keying and correlation delay shift keying can be given as examples of such communication systems (Kolumbán et. al., 1997), (Kolumban et. al., 1998), (Kolumbán & Kennedy, 2000).

2.1 Chaotic Analog Modulation

Among the several approaches on chaotic modulation by analog modulating signal such as amplitude and frequency modulation of chaotic signals, most common methods are known to be chaotic masking and chaotic system forced by information signal. Chaotic masking is simply obtained by adding the information signal to the noise-like chaotic signal (Yang, 2004). Detection of analog message is based on subtracting the chaotic signal generated by receiver from the received signal. Although the complexity is low, a robust synchronization circuit is required to reproduce the chaotic signal at the receiver. Therefore, this method yields satisfactory result only for high signal noise ratios (SNR). Alternatively, the information signal is used to force the chaotic system at the transmitter (Savacı et. al., 2003) where it is reported that the frequency of message signal should be sufficiently far away from the fundamental frequency of chaotic system or message signal should be at least 20 dB or 30 dB weaker than the chaotic masking signal to maintain the chaotic behavior.

However, similar to the chaotic masking, synchronization between transmitter and receiver determines the performance of the communication system and it is hard to recover the information if the chaotic signal is corrupted by noise through the channel.

Figure 2.2 depicts a chaos masking scheme which consists of two identical chaotic systems in both the transmitter and receiver parts. The chaotic masking signal, $c(t)$, is one of the state variables of the chaotic system in the transmitter and message signal, $m(t)$, is added to chaotic mask signal giving the transmitted signal, $s(t)$. It is proposed that an intruder is unable to recover the message signal, $m(t)$, from $s(t)$ without having exact information about $c(t)$. To be able to recover the message signal, a chaotic synchronization block is utilized at the receiver. Synchronization error vanishes at the receiver side within a certain time interval and the hidden analog message signal is obtained when the synchronization is completely achieved. However, this scheme cannot to be used in practice since the masking technique is very sensitive to channel noise and parameter mismatch between chaotic systems in the transmitter and the receiver, and such a scheme only has a very limited security level.

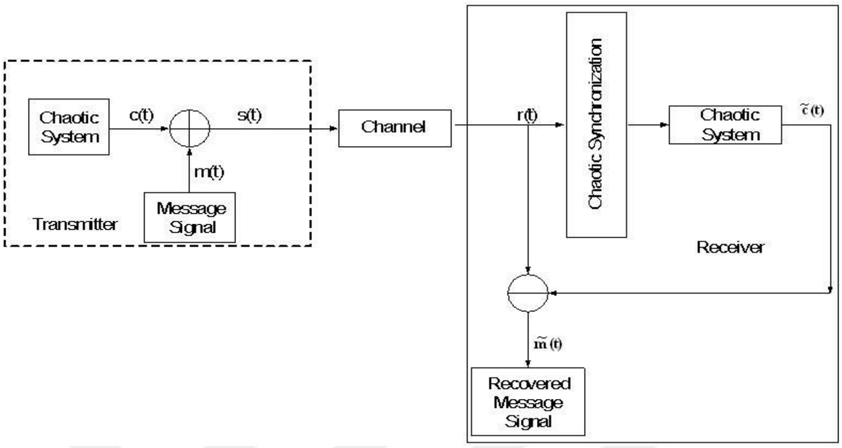

Figure 2.2 Illustration of basic chaotic masking scheme (Stavroulakis, 2006)

2.2 Chaotic Digital Modulation

Similar to the conventional digital communication systems, chaotic digital modulation is provided by mapping symbols to analog chaotic waveforms. There are several different techniques for mapping symbols to chaotic waveforms, such as Chaos Shift Keying (CSK) and Chaotic Phase Shift Keying (CPSK) based on coherent detection, differential chaos shift keying (DCSK), frequency-modulated differential chaos shift keying (FM-DCSK), correlation delay shift keying (CDSK) based on noncoherent detection. These techniques are explained briefly in the sequel.

2.2.1 Chaos Shift Keying (CSK)

The digital modulation scheme in which the chaotic carriers are generated by different or same chaotic dynamical system with different initial conditions acting as basis function is called as chaos shift keying (Kennedy et. al., 2000). As the mathematical description, let $s_m(t), m = 1,2,\cdots, M$ denote the elements of the signal set defined by Eq. (2.3)

$$s_m(t) = \sum_{j=1}^{N} s_{mj} g_j(t), \quad j = 1,2,\cdots,N \tag{2.3}$$

where the basis functions $g_j(t)$ are chaotic waveforms. CSK signal transmitter and receiver structures are shown in Figure 2.3a and Figure 2.3b, respectively.

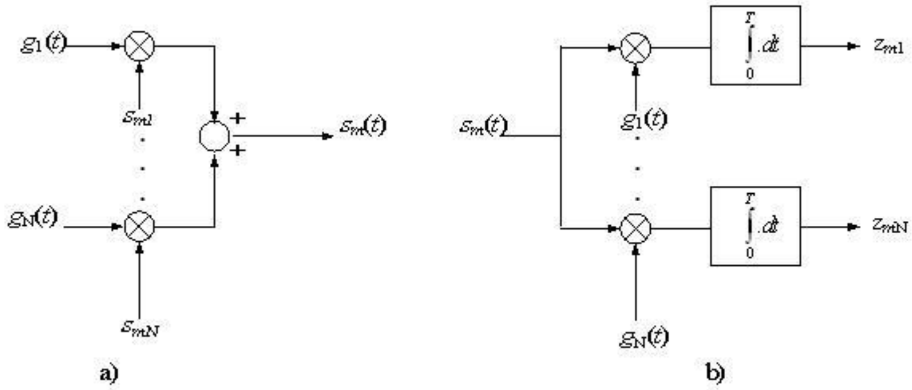

Figure 2.3 Basic CSK communication scheme a) Transmitter b) Receiver (Stavroulakis, 2006)

Chaotic basis functions are orthonormal in the mean that is,

$$E\left[\int_0^T g_i(t) g_j(t) dt\right] = \begin{cases} 1 & if \ \ i = j \\ 0 & otherwise \end{cases} \tag{2.4}$$

where $E[\cdot]$ denotes the expectation operator and T is the bit duration. The element z_{mj} of the observation vector at the output of the jth correlator, when signal $s_m(t)$ is transmitted, is given by Eq. (2.5) as

$$z_{mj} = \int_0^T s_m(t) g_j(t) \mathrm{dt} = \int_0^T \left[\sum_{j=1}^{N} s_{mj} g_j(t)\right] g_j(t) \mathrm{dt} = s_{mj} \int_0^T g_j(t) g_j(t) \mathrm{dt} = s_{mj} \tag{2.5}$$

Transmitted symbol based on single chaotic basis function such as $g_1(t)$, i.e., $s_m(t) = s_{m1} g_1(t)$ can be used to realize simplest case of CSK. Three main types of CSK can be generated with a single basis function.

2.2.1.1 Chaotic On-Off Keying (COOK)

In COOK scheme, the bit "1" is represented by $s_1(t) = \sqrt{2E_b}g_1(t)$ and the bit "0" is given by $s_2(t) = 0$. Similarly, $s_{11} = \sqrt{2E_b}$, $s_{21} = 0$ where E_b denotes the average energy per bit.

2.2.1.2 Unipodal CSK

Unipodal CSK scheme assigns bits "1" and "0" having bit energies E_{b1} and $E_{b2} = kE_{b1}$, with $0 < k < 1$. Bit "1" is represented by $s_1(t)$=$s_{11}g_1(t)$ and symbol "0" is given by $s_2(t)$=$s_{21}g_1(t)$, where $s_{11} = \sqrt{\frac{2E_b}{1+k}}$ and $s_{21} = \sqrt{\frac{2kE_b}{1+k}}$.

2.2.1.3 Antipodal CSK

Antipodal CSK scheme has the symbol "1" represented as $s_1(t)$=$s_{11}g_1(t)$ and symbol "0" as $s_2(t)$=$s_{21}g_1(t)$, where $s_{11} = \sqrt{E_b}$ and $s_{21} = -\sqrt{E_b}$. In addition to the coherent matched filters, demodulation can also be performed by coherent correlator receivers and noncoherent receivers as shown in Figure 2.4a and Figure 2.4b, respectively.

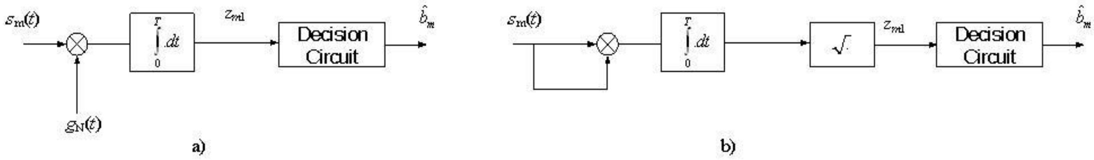

Figure 2.4 a) Coherent receiver for CSK signal with one basis function b) Noncoherent receiver for CSK signal with one basis function (Stavroulakis, 2006)

Estimated symbol by coherent receiver is formulized by Eq. (2.6) as

$$z_{m1} = \int_0^T s_m(t)g_1(t)\mathrm{dt} = s_{m1}\int_0^T g_1^2(t)\mathrm{dt} = s_{m1} \qquad (2.6)$$

The noncoherent demodulator determines the bit energy of the received signal and, differing from coherent detector, the received signal is correlated with itself, without utilizing basis function $g_1(t)$. The demodulator output is given as;

$$z_{m1} = \sqrt{\int_0^T s_m^2(t)\mathrm{dt}} = \sqrt{s_{m1}^2 \int_0^T g_1^2(t)\mathrm{dt}} = |s_{m1}| \tag{2.7}$$

Same receiver structure depicted here can be used to demodulate both COOK and Unipodal CSK, though because of absolute value, it cannot to be used to recover s_{m1}'s of opposite sign and thus, it is unsuitable for demodulating CSK. Though, coherent correlation receiver can be used for all of these schemes.

2.2.2 Differential Chaos Shift Keying (DCSK)

In binary DCSK, two elements of the signal set are given by Eq. (2.1) and Eq. (2.2),

$$s_{\mathrm{m}}(t) = s_{\mathrm{m1}} g_1(t) + s_{\mathrm{m2}} g_2(t) \tag{2.8}$$

where $(s_{11}\ s_{12}) = \left(\sqrt{E_b}\ \ 0\right)$ and $(s_{21}\ s_{22}) = \left(0\ \sqrt{E_b}\right)$. In the case of DCSK, the basis functions have the special form

$$\begin{aligned} g_1(t) &= \frac{+1}{\sqrt{E_b}} c(t), && 0 \leq t < T/2 \\ g_1(t) &= \frac{+1}{\sqrt{E_b}} c(t - T/2), && T/2 \leq t < T \\ g_2(t) &= \frac{+1}{\sqrt{E_b}} c(t), && 0 \leq t < T/2 \\ g_2(t) &= \frac{-1}{\sqrt{E_b}} c(t - T/2), && T/2 \leq t < T \end{aligned} \tag{2.9}$$

where $c(t)$ is a chaotic waveform and E_b is the energy of each bit. The first half of the basis function is named as reference chip, and the second half is called the information-bearing chip. In such a scheme, bit "1" is sent by transmitting $s_1(t) = \sqrt{E_b} g_1(t)$, bit "0"by transmitting $s_2(t) = \sqrt{E_b} g_2(t)$. The following figure shows block diagram of a DCSK modulator as explained above. Appropriate basis functions are generated according to b_m, and the modulation input by the modulation driver, delay circuit and switch (Kaddoum et al. 2015, 2012).

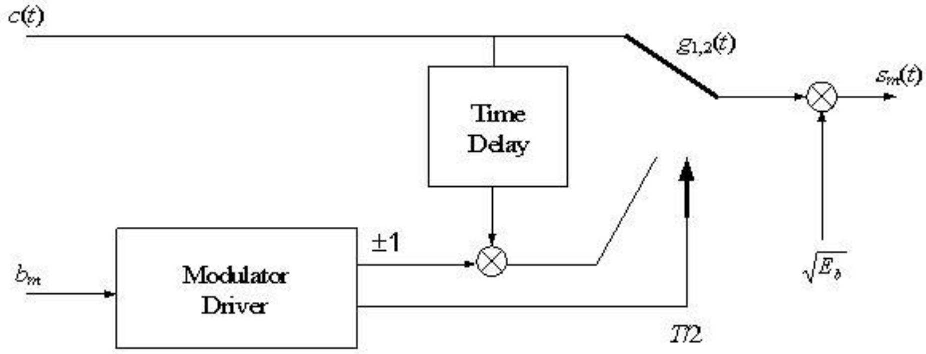

Figure 2.5 Block diagram of DCSK Modulator (Stavroulakis, 2006)

Actually the DCSK modulation scheme is a variant of CSK with two basis functions. Hence, it can be demodulated by the coherent receiver where the signal space diagram for CSK is also valid for DCSK which is the same as in conventional coherent FSK. Differentially coherent DCSK receiver can be used as an alternative approach for demodulation. A block diagram for this approach is shown in Figure 2.6. In this method, output of the demodulator is found in the form of Eq. (2.10) and by using the formula; $E\left[\int_{T/2}^{T} g_m^2(t)\mathrm{dt}\right] = 1/2$, where it is known $z_1 \approx +\mathrm{E}_b/2$ and $z_2 \approx -E_b/2$. The decision of which symbol was transmitted can be made by a simple level comparator with its threshold set to zero.

$$z_m = \int_{T/2}^{T} s_m(t)s_m(t-T/2)\mathrm{dt} = \int_{T/2}^{T} E_b g_m(t)g_m(t-T/2)\mathrm{dt} \tag{2.10}$$

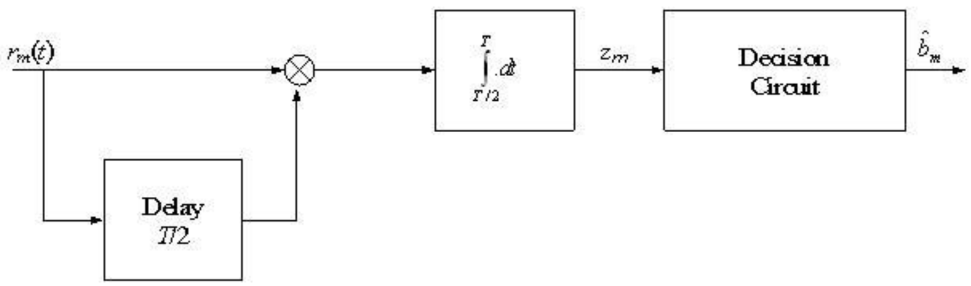

Figure 2.6 Block diagram of differentially coherent DCSK receiver (Stavroulakis, 2006)

2.2.3 Chaotic PSK

As mentioned in the first section, chaotic signals have wideband nature and thus they are more resistant to multipath propagation than any schemes with sinusoidal functions. In addition, another advantage of Chaotic Phase Shift Keying (CPSK) method is that one chaotic generator can be sufficient for the CPSK scheme, considering that the classical CSK scheme requires two chaotic generators, one in the transmitter and one in the receiver (Sandhu & Berber, 2005). The basic scheme of CPSK system is shown in Figure 2.7.

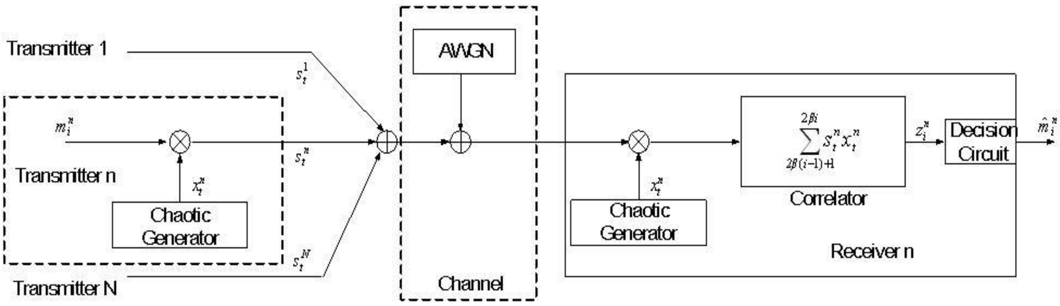

Figure 2.7 A multiuser CPSK communication system (Stavroulakis, 2006)

On the other hand, transmitter structure of the CPSK system can be mathematically expressed as follows. Consider a *communication* system where the zero-mean chaotic sequence generated by the chaos generator is denoted by x_t, m_i is the the i^{th} bit sequence with $m_i \in (-1, +1)$, 2β is the number of chaotic samples in each transmitted bit. During the i^{th} bit duration, i.e. for time $t = 2\beta(i - 1) + 1, 2\beta(i - 1) + 2 \ldots\ldots, 2\beta$, *i* the transmitter's output of the n^{th} user is given as

$$s_t^n = \begin{matrix} +x_t^n & \text{if } m_i = +1 \\ -x_t^n & \text{if } m_i = -1 \end{matrix} \tag{2.11}$$

Such an approach is called chaotic phase shift keying since only one generator is needed at the transmitter side, and a 180° phase difference is observed in case the transmitted bit is –1, in other words the signal is multiplied by –1 (Sandhu & Berber, 2005). Any noise in channel distorts the transmitted signal, and the input of the receiver *n* at time *t* is given as

$$r_t = \sum_{n=1}^{N} s_t^n + n_t \tag{2.12}$$

In which the first term is the output of the N transmitters at time t, and the second term, n_t, is the zero mean AWGN. By assuming exact synchronized samples are available at the receiver, the output of the correlator of the n^{th} user at the end of the i^{th}bit duration is obtained as

$$z_i^n = \sum_{t=2\beta(i-1)+1}^{2\beta i} r_t x_t^n \tag{2.13}$$

The demodulating process is concluded after comparing z_i^n to the threshold value of zero.

2.2.4 FM Differential Chaos Shift Keying

Since chaotic signals are different with respect to the initial conditions and has different bit energy for each generated symbol, there exists a problem of normalizing bit energy for each binary information. Otherwise, demodulator output of the communication system yields varying correlator outputs. In other words, it causes "estimation problems" (Kolumbán Géza et al. 1997; Kolumban et al. 1999; Xu et al. 2010; Kolumbán et al. 1998). To overcome this problem, an FM signal which has constant envelope is used (Volkovskii et. al 2005). In FM-DCSK, sinusoidal signal having chaotic instantaneous frequency provides binary information having equal energy. Output of the chaotic FM generator is bandlimited and its power spectral density is tuned by FM modulator (Stavroulakis, 2006). The FM-DCSK scheme is basically same with DCSK scheme. As the chaotic signal, output of the FM signal generator is given to the system shown in Figure 2.8 (Stavroulakis, 2006).

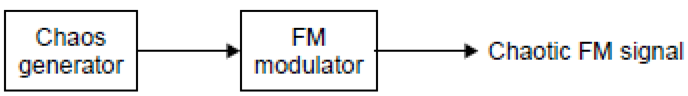

Figure 2.8 Chaotic frequency-modulated signal generator (Stavroulakis, 2006)

2.2.5 Correlation Delay Shift Keying

Correlation delay shift keying can be considered as it is derived from differential chaos shift keying. In correlation delay shift keying, chaotic signal generated by chaotic signal generator multiplied by information and it is added to the delayed chaos signal. Obtained signal is the one to be transmitted (Stavroulakis, 2006; Bassam & Jerew, 2015; Tam, 2006). Figure 2.9 depicts the CDSK modulator.

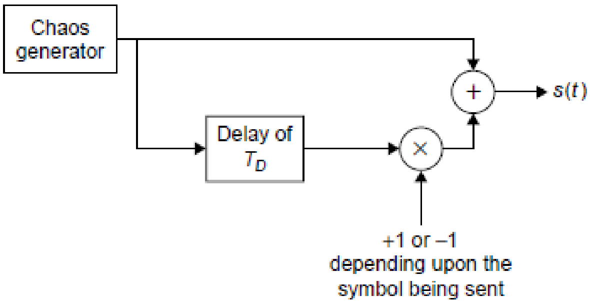

Figure 2.9 Correlation delay shift keying system modulator (Stavroulakis, 2006)

When correlation delay shift keying (CDSK) is compared with differential chaos shift keying (DCSK), it can be seen that CDSK does not use half of its symbol duration for carrying information so it cannot use its bandwidth efficiently. Since there is no switching, a constant transmission occurs. The delay time expressed as T_D which is shorter than the half of the symbol period, transmitted signal does not repeat itself since there is no reference part (Stavroulakis, 2006; Lau & Tse, 2003; Tam et al. 2004). CDSK demodulator structure is similar to DCSK except time delay shown in Figure 2.10. Since different chaotic signals have nonzero correlation between them, uncertainty of correlator output is increased and its bit error rate (BER) performance becomes worse than DCSK.

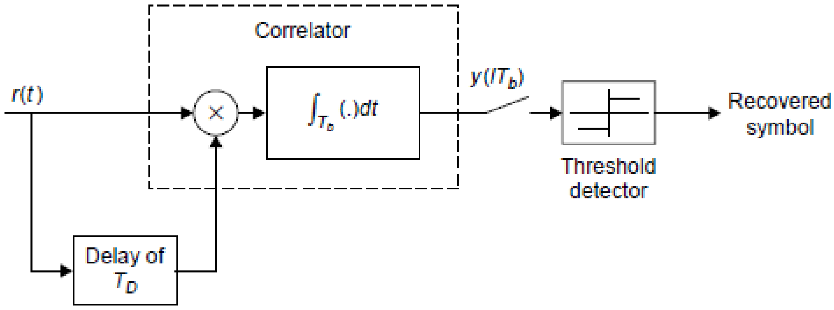

Figure 2.10 Correlation delay shift keying system demodulator (Stavroulakis, 2006)

2.2.6 Generalized Correlation Delay Shift Keying

Generalized correlation delay shift keying is classified under non-coherent chaos-based communication systems. In the non-coherent chaos based communication shown in Figure 2.11, chaotic signal does not need to be regenerated at receiver side, due to this its practically easier to implement. The chaotic signals generated at the transmitter side of GDCSK communication system use the signals' delayed versions. Those delayed chaotic signals modulated with the binary data that required to be transmitted and the delayed signal added with the original chaotic signal then the resulting signal will be transmitted. On receiver side, a simple correlator type detector can regenerate the original data. There is no switching step in GCDSK and transmission in the modulator is continuous.

GCDSK communication system is depicted in the Figure 2.11. The transmitter part consists of chaotic signal generator and $(M-1)$ delay blocks. It is generally assumed that $M > 2$ since when $M = 2$, GCDK system naturally becomes equal to CDSK system. Minimum delay is shown by L, delays of the chaotic signals are respectively $L, 3L, 5L..$ Data will be modulated with this delay sequence; thus, not with $2L, 4L, 6L$ ... delay. Spreading factor, β, is the number of transmitted signals in each bit duration; $\mathrm{k} = (\mathrm{l}-1)\beta + 1, (\mathrm{l}-1)\,\beta + 2, \ldots.\mathrm{l}\,\beta$ through that time interval, signals transmitted is s_k. If M is even, it is not modulated and if M is odd, it is modulated. Transmitted chaotic signal is obtained by adding original signal and the modulated chaotic signal.

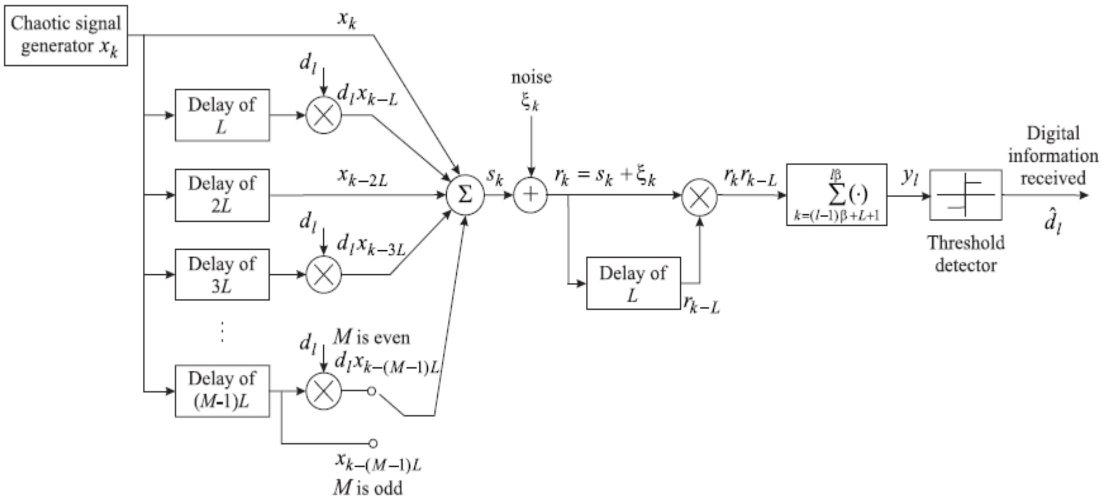

Figure 2.11 Block diagram of a GCDSK communication system (Tam et. al., 2006)

$$s_k = \sum_{m=0}^{\frac{M-2}{2}} x_{k-2mL} + d_l \sum_{m=0}^{\frac{M-2}{2}} x_{k-(2mL-1)} \quad \text{if } M \text{ is even}$$
$$s_k = \sum_{m=0}^{\frac{M-1}{2}} x_{k-2mL} + d_l \sum_{m=0}^{\frac{M-3}{2}} x_{k-(2mL+1)} \quad \text{if } M \text{ is odd} \qquad (2.14)$$

In the equations above, respectively first and second terms are shows the addition of modulated and not modulated signals. On summation block of receiver, CDSK, besides the $(\beta - L)$ (assuming $> L$) terms is similar bit CDSK demodulator only $\beta - L$ terms will be added together and useful parts of signals will be derived. Due to this, intra-signal interference components will be reduces (Wai et al. 2006).

2.3 Non-coherent Multiuser Chaotic Communication Using Chaotic Orthogonal Vectors

Although the chaotic signals are known to exhibit orthogonal behavior, this fact is not realized in practice for entire time interval. Since the bit duration T_B may vary with respect to the bit rate, an obvious question arises whether the orthogonality is ensured for any time instant t, $0 < t \leq T_b$. Consider two functions $g_1(t)$ and $g_2(t)$ obtained from the state variable of $x(t)$ of the Lorenz system

$$
\begin{aligned}
\dot{x} &= (y - x) \\
\dot{y} &= (\sigma x) - y - xz \\
\dot{z} &= xy - bz
\end{aligned}
\tag{2.15}
$$

are generated using different initial conditions. The variation of inner product can be formulized as

$$\int_0^t g_i(\tau) g_j(\tau) d\tau = G(t) \quad 0 < t \leq T_b \tag{2.16}$$

An illustration is shown in Figure 2.12 where it is clearly seen that the orthogonalization is not ensured for chosen bit duration $T_b = 0.01$ sec. In coherent and non-coherent chaotic communication systems, application of Gram-Schmidt orthogonalization procedure is described by several studies in the literature (Wren & Yang, 2010), (Venkatesh, 2012), (Abdullah & Radhi, 2015). Gram-Schmidt procedure can be applied to each chaotic signal for orthogonalization as given in (2.17)

$$\hat{x}(k)^{(p)} = \frac{x(k)^{(p)} - \sum_{q=1}^{p-1}\left[\sum_{k=1}^{\beta} x(k)^{(p)} \hat{x}(k)^{(q)}\right]\hat{x}(k)^{(q)}}{\sqrt{\sum_{k=1}^{\beta}\left[x(k)^{(p)} - \sum_{q=1}^{p-1}\left[\sum_{k=1}^{\beta} x(k)^{(p)} \hat{x}(k)^{(q)}\right]\hat{x}(k)^{(q)}\right]^2}} \tag{2.17}$$

Once the orthogonalization is applied, two chaotic signals are reshaped to behave orthogonal to each other with respect to the specified bit duration shown in Figure 2.13.

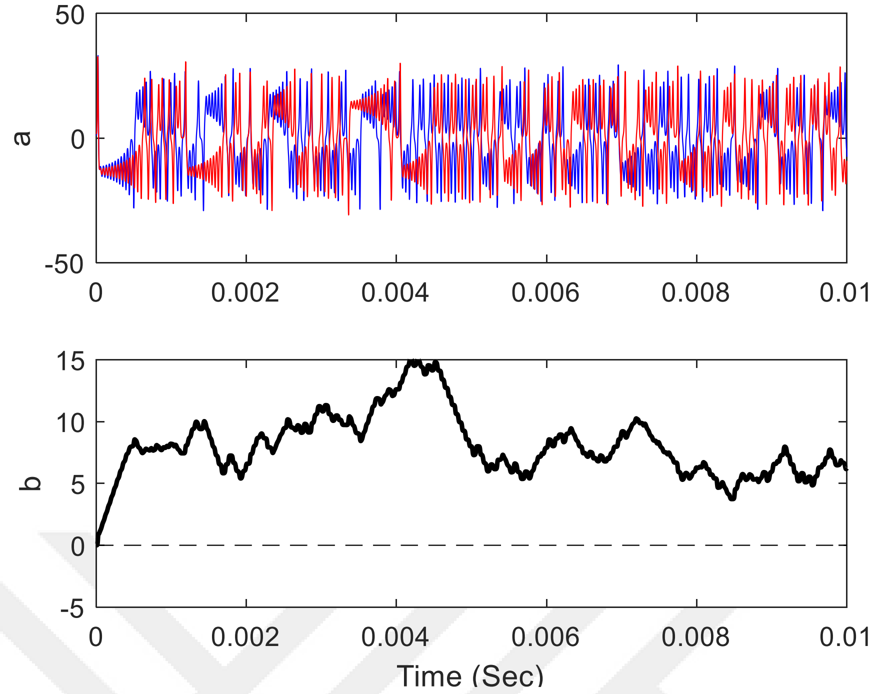

Figure 2.12 a) Two independent realizations of chaotic signals generated using different initial conditions b) Variation of inner product with respect to time

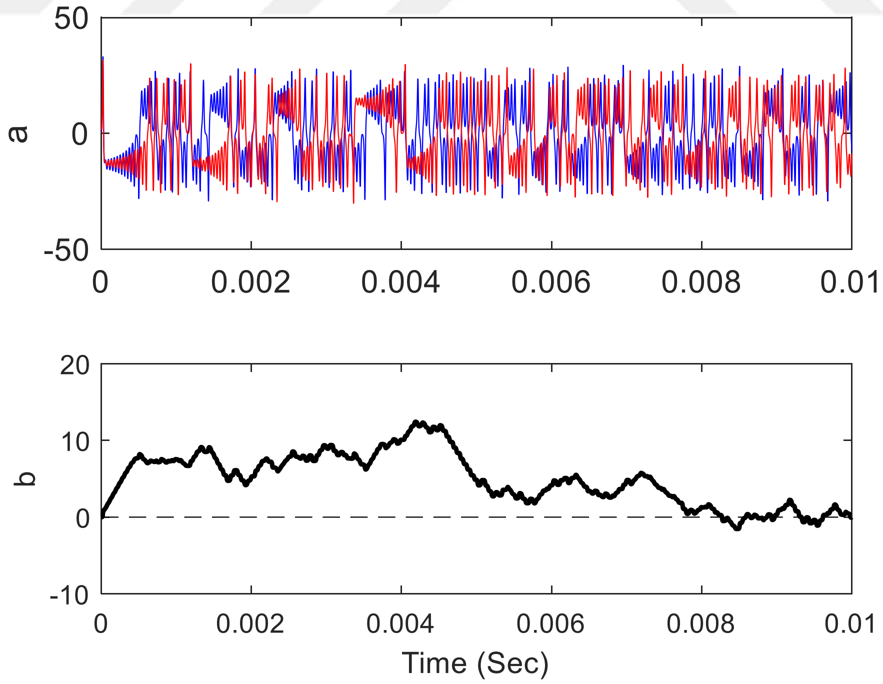

Figure 2.13 a) Two independent realizations associated with orthogonalized chaotic signals generated using different initial conditions b) Variation of inner product with respect to time

This procedure is extended to correlation delay shift keying (CDSK) as one of the non-coherent chaotic communication systems in this thesis. The chaotic signals generated for two users and carrying binary information “1” and “-1” are transmitted by User 1 and User 2, respectively. In order to enhance the signs of the autocorrelation function shown in Figure 2.14 which carry the binary information for each user, the orthogonalization method can be considered to have a potential but still needs further analysis.

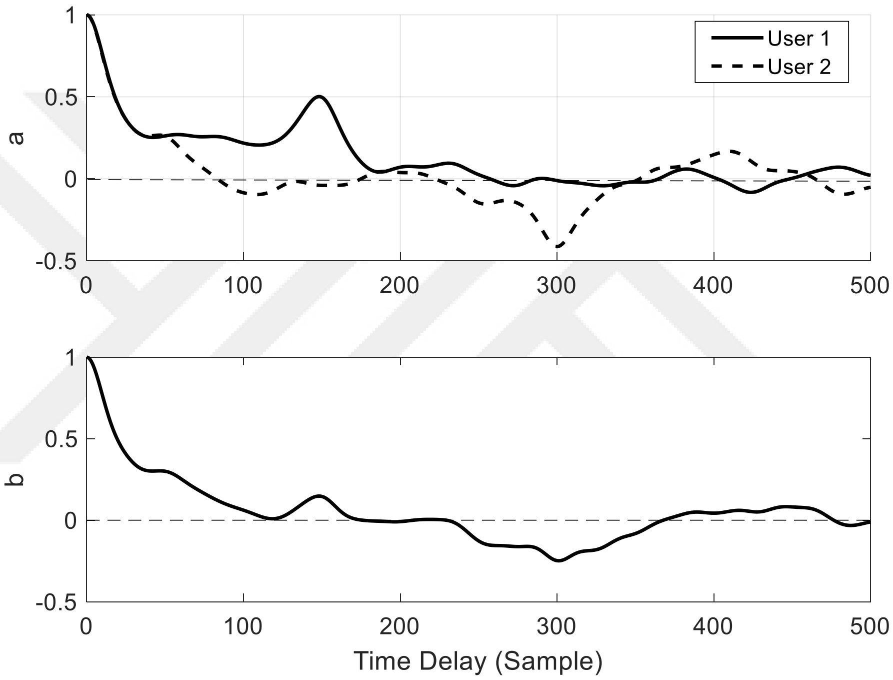

Figure 2.14 2-user CDSK communication system transmitting “1” for User 1 and transmitting “-1” for User 2 a) Autocorrelation function for each user at the transmitter. b) Autocorrelation result at the receiver

CHAPTER THREE
ALPHA- STABLE DISTRIBUTION

Gaussian distribution is common assumption to model the channel noise in digital communication systems. Basically, it is characterized by mean and variance which are finite. However, it is expressed by (Win et. al., 2009) that the channel model may exhibit impulsive behavior. It is represented by symmetric non Gaussian noise given to be $\alpha -$stable ($S\alpha S$) distribution. The infinite variance behavior of stable noise attracts a growing interest on analysis of fractional moments to provide stability. Figure 3.1 illustrates the statistical moments in this manner. Gaussian distribution is a unique member of stable distributions with finite variance. In the literature, α-stable distributions are examined in signal detection under impulsive environment (Chiang & Nikias, 1990; Tsihrintzis & Nikias, 1995), developing methods for parameter estimation (Ma & Nikias, 1995; Tsihrintzis & Nikias, 1996) and in frequency estimation of sinusoidal signal under α-stable noise (Altınkaya, et. al., 2002). The statistical derivation on noisy observations from stable distributions are analyzed by fractional lower order moments (FLOM) for p< α case (Nikias & Shao 1995; Janicki & Weron, 1993).

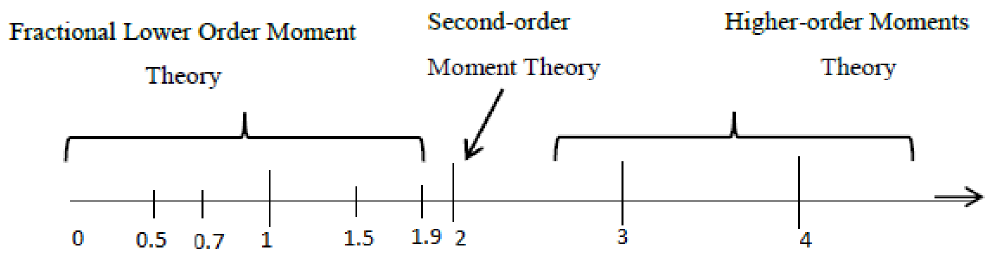

Figure 3.1 Statistical moments (Nikias & Shao 1995)

Stable distribution has a superiority due to proving an opportunity of characterizing the noise having asymmetric behavior. The impulsiveness of the noise causes the tail parts of the distributions to be heavier. That is why the noise having impulsive distribution is also called as heavy-tailed distribution.

Principally, $\alpha -$ stable distribution is analytically expressed in terms of its characteristic function as in Eq. (3.1) (Samorodnitsky & Taqqu, 1994).

$$\begin{aligned} \varphi(w) &= \exp\{-\sigma^{\alpha}\,|w|^{\alpha}\left(1 - j\beta \mathrm{sgn}(w).\tan\frac{\pi\alpha}{2}\right) + j\mu w\} \quad \text{if } \alpha \neq 1 \\ \varphi(w) &= \exp\{-\sigma\,|w|(1 + j\beta\frac{2}{\pi}\mathrm{sgn}(w)\ln|w|\,) + j\mu w\} \quad \text{if } \alpha = 1 \end{aligned} \tag{3.1}$$

where

$$\mathrm{sgn}\,(w) = \begin{cases} 1 & \text{for } w \geq 0 \\ -1 & \text{for } w < 0 \end{cases} \tag{3.2}$$

The noise parameters; characteristic exponent α, skewness β, scale σ and the shift parameter μ, are used to adjust the impulsiveness, symmetry, intensity and the location, respectively. Alternatively, the scale parameter is also expressed by an equivalent term dispersion formulated as $\gamma = \sigma^{\alpha}$. If $\beta = \mu = 0$ then the distribution is said to be symmetric. If $\beta = 0$, the distribution is said to be symmetric around μ. In this thesis study, the location parameter is not under consideration and assumed to be $\mu = 0$. The probability density function obtained from the characteristic function is defined by Eq. (3.3) (Janicki & Veron, 1993; Çek, 2010).

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi\,(w)e^{-jwx}dw \tag{3.3}$$

The stable distribution can be expressed in parameterized form $S_{\alpha}(\sigma, \beta, \mu)$ where the closed form expression of the integration given by Eq. (3.3) can only be achieved for three exceptional cases. These density functions are Gaussian $S_2(\sigma, 0, \mu)$ (Janicki & Veron, 1993),

$$f(x) = \frac{1}{2\sigma\sqrt{\pi}}e^{-\frac{(x-\mu)^2}{4\sigma^2}} \tag{3.4}$$

Cauchy distribution $S_1(\sigma, 0, \mu)$ having density function given by Eq. (3.5)

$$f(x) = \frac{2\sigma}{\pi(x-\mu)^2+4\sigma^2} \quad (3.5)$$

and Levy distribution $S_{1/2}(\sigma, 1, \mu)$ with density function given by Eq. (3.6)

$$f(x) = \left(\frac{\sigma}{2\pi}\right)^{1/2} (x-\mu)^{-3/2} e^{-\frac{\sigma}{2(x-\mu)}} \quad (3.6)$$

noting that $x \epsilon (\mu, \infty)$ and $f(x) = 0$ for $x \epsilon (-\infty, \mu]$. The density functions of α −stable distributions with respect to different noise parameters are illustrated in Figure 3.2 and Figure 3.3 respectively. Figure 3.2a indicates the distributions having increasing characteristic exponent where the tail of distribution apparently becomes heavier as seen in Figure 3.2b. On the other hand, Figure 3.3a reflects the effect of the skewness which tunes the amount of asymmetric behavior and the variation of intensity is shown in Figure 3.3b.

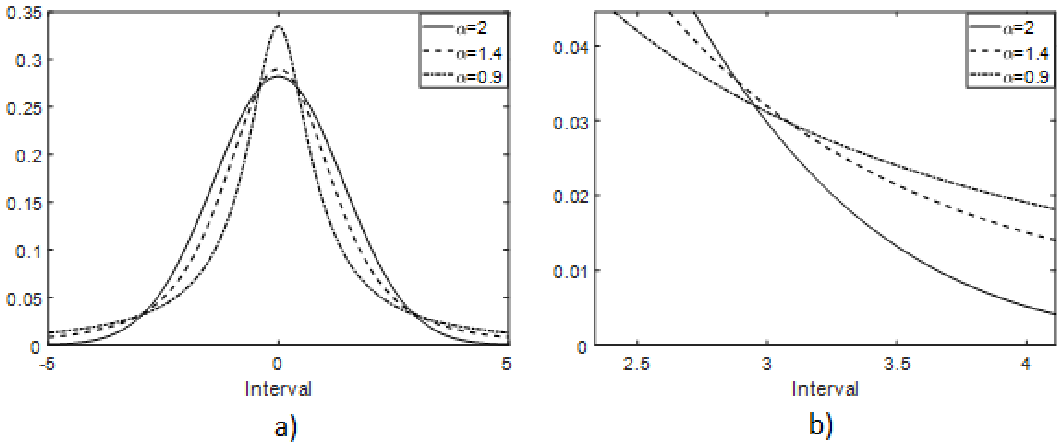

Figure 3.2 Stable density function a) Effect of characteristic exponent for $\beta = 0, \sigma = 1$, b) Density function variations at tails of distributions

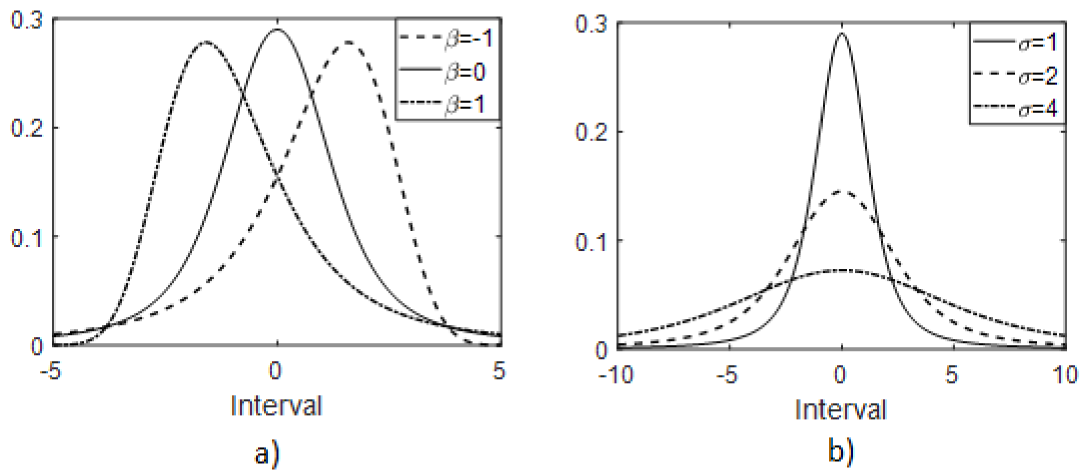

Figure 3.3 Stable density function a) Effect of skewness for $\alpha = 1.4, \sigma = 1, \mu = 0$ b) Effect of scale for $\alpha = 1.4, \beta = 0, \ \mu = 0$

The properties of stable distributions are comprehensively studied in (Samorodnitsky & Taqqu, 1994), gives information about statistical properties which are utilized to tune the signal according to the required SNR in this thesis study.

Property 1: If the independent random variables are given as $X_1 \sim S_\alpha\ (\beta_1, \mu_1, \sigma_1)$ and $X_2 \sim S_\alpha\ (\beta_2, \mu_2, \sigma_2)$ then their summation is $X_1{+}X_2 \ \sim S_\alpha\,(\beta, \mu, \sigma)$

$$\sigma = (\sigma_1^\alpha + \sigma_2^\alpha)^{1/\alpha}, \ \beta = \frac{\beta_1\sigma_1^\alpha + \beta_2\sigma_2^\alpha}{\sigma_1^\alpha + \sigma_2^\alpha}, \ \mu = \mu_1 + \mu_2$$

Property 2: If the location of X random variable is $\mu, X \sim S_\alpha\,(\beta, \mu, \sigma)$. c is a constant value and when the constant and variable are added, we get; X+c~ $S_\alpha(\beta, \mu{+}c, \sigma)$.

Property 3: Let $X \sim S_\alpha\ (\beta, \mu, \sigma)$ and c is a nonzero real constant. Then,

$$cX \sim S_\alpha\,(sign(c)\beta, c\mu, \sigma|c|) \qquad \text{if} \quad \alpha \neq 1$$

$$cX \sim S_\alpha\,(\sigma|c|, sign(c)\beta, \beta) \qquad \text{if} \quad \alpha = 1$$

Property 4: For any $0 < \alpha < 2$,

$$X_2 \sim S\alpha\ (\beta, \mu, \sigma) \quad <=> \ -X_2 \sim S\alpha\ (-\beta, \mu, \sigma)$$

The following property identifies β as a skewness parameter.

Property 5: $X \sim S_{\alpha}\ (\beta, \mu, \sigma)$ is symmetric about μ if and only if β is equal to zero.

Property 6: Let $X \sim S_{\alpha}\ (\beta, \mu, \sigma)$ random variable in $0 < \alpha < 2$ range.

$$E|X|^{p} < \infty \qquad \text{for any } 0 < p < \alpha$$

α-stable distribution, when $\alpha < 2$, has infinite second and higher order moments and also when $\alpha \leq 1$, $E|X| = \infty$.

CHAPTER FOUR
ROBUST ESTIMATORS

In most of the signal processing applications, the noise is assumed to have Gaussian distribution. However, it is reported by (Zoubir et. al., 2012) that, this common assumption is not sufficient to model the data having outlier components. If the noise distribution diverges from Gaussian distribution, it is proper to model the noise with heavy-tail distributions. The term robust is used to emphasize that the signal processing is performed for the data having outlier components and far from Gaussian distribution. The robust estimators are utilized to estimate the information under non-Gaussian noise environment having impulsive components. Conventionally, statistical signal processing is declared to depend on strong assumptions. For example, a specific parametric signal model can be derived from linearity, the probability of sensor noise distribution, independence and identical distribution of random variable, stationary estimation (Kim & Shevlyakov, 2008). Impulsive noise (heavy-tailed) that is caused the use of optimal signal processing techniques is obtained using a nominal Gaussian probability model e.g. switching transients in power lines or automobile ignition (Middleton, 1999), in radar and sonar systems as a result of natural or man-made electromagnetic and acoustic interference (Abramovich & Turcaj, 1999; Etter, 2013). Moreover, $i = b + z(x - c)$ stable distribution as heavy tailed impulsive distribution is reported to have been observed in nature (Nikias & Shao, 1995; Gonzalez & Arce, 1996; Gonzalez & Arce, 2001). Due to the necessity of analyzing data having outliers, (Huber & Ronchetti, 2009) emphasized the necessity of developing robust signal processing methods for high reliability of estimation result especially for real-life data. The most important of these methods are previously reported to be the M-estimators (Zoubir et. Al., 2012).

M-estimators have three different filters: Median filter, myriad filter and meridian filter. The parameters of the analytical expression of these filters are used to optimize the filter performance with respect to observed data (Arce, 2005).

4.1 Meridian Filter

Meridian distribution is a member of generalized Cauchy family. Meridian filtering corresponds to maximum likelihood estimation under meridian statistics. Potentially, meridian filter may exhibit more robust behavior than myriad and median filters (Aysal & Barner, 2007; Stork, 2010). Consider N independent samples of an observation. In Eq. (4.1), " the ML estimate of location $y[n]$ " or "sample meridan" is given as

$$y[n] = \underset{\beta}{\operatorname{argmin}}\left[\sum_{\mathrm{i}=1}^{N} \log\left(\delta+(|x_i - \beta|)\right)\right] \tag{4.1}$$

In this equation, common scale parameter "δ" is named as medianity parameter. Because meridian filter free-tunable parameter "δ" contains, the meridian estimation in characteristic can be changed depending on this parameter (Aysal & Barner, 2007).

4.2 Myriad Filter

Myriad filter can be considered like a high efficient filtering framework under impulsive distribution, especially $\alpha -$ stable distribution. Myriad filter have the important optimality property in α-stable family and it is also called as myriad smoother (Arce, 2005).

Myriad filter characteristics are controlled by the linearity parameter, K. Sample sets observed at the filter input are $x_1[n], x_2[n], x_3[n], \ldots, x_{\mathrm{N}}[n]$, and K has the positive values only. Output of the running myriad smoother filter at a time instant n is given in Eq. (4.2)

$$Y_K[n] = MYRIAD\left(K;\ X_1, X_2, X_3, \ldots\ldots, X_N(n)\right) \tag{4.2}$$

$$Y_K[n] = \underset{\beta}{argmin} \prod_{i=1}^{N} [K^2 + (X_i(n) - \beta)^2]$$

Free-tunable parameter, K, of the myriad filter is the linearity parameter and adjustment of this parameter is comprehensively explained by (Arce, 2005). Taking into account the $\alpha -$stable distributions a proper selection of the parameter K is suggested as given in Eq. (4.3)

$$K = \sqrt{\frac{\alpha}{2-\alpha}} \tag{4.3}$$

Note that filter behavior converges to linear estimator as K grows larger. It becomes more robust against impulsive noise at it converges to lower linear values.

4.2 Median Filter

In many fields of signal processing, linear filters can be used to suppress unwanted components of discrete time signal sequence inputs. Though, in some fields, linear filters might be insufficient. Nonlinear or adaptive filters are preferred to suppress impulsive noise components. One of this nonlinear filters is the Median filter, which is also called as Running Median Smoother. To define a Running Median Smoother, $\{X(.)\}$ defined with a discrete time sequence as in the following and this sequence is windowed. Observation window centered at n (Arce, 2005) is given as

$$X(n) = [X(n - N_L),\ldots,X(n),\ldots,X(n + N_R)]^T \tag{4.4}$$

The parameters N_L and N_R are left and right window lengths from the temporal center n of the window. The overall window size is $N = N_L + N_R + 1$ and in most cases window is considered to be symmetric $(N_L = N_R = N_1)$. The filter output $Y(\cdot)$ is obtained from filter input $X(\cdot)$ at time instant n as

$$Y(n) = MEDIAN[X(n - N_1),\ldots,X(n),\ldots,X(n + N_1)] \tag{4.5}$$
$$Y(n) = MEDIAN[X_1(n),\ldots,X_N(n)]$$

where the sorted samples are expressed in the form $X_i(n) = X(n - N_1 - 1 + i)$ for $i = 1, \cdots, N$. Hence, the filter output for sorted data can be given as (Arce, 2005)

$$
\begin{aligned}
Y(n) &= X_{\frac{(N+1)}{2}}(n) && \text{if N is odd} \\
Y(n) &= \frac{\left(X_{\frac{(N)}{2}}(n) + X_{\left(\frac{N}{2}\right)+1}(n)\right)}{2} && \text{if N is even}
\end{aligned}
\tag{4.6}
$$

Median operation is nonlinear, the running median smoother does not have the superposition property and impulse response of median smoother is always zero. Median smoother can be characterized as statistical or deterministic (Arce, 2005).

CHAPTER FIVE
CHAOTIC COMMUNICATION UNDER IMPULSIVE NOISE

One of the disadvantages of the chaotic systems in communication is the sensitivity against the channel noise. Due to the effect on synchronization error, the noise in the channel degrades the error performance of the coherent chaotic communication systems. Recently, the noise reduction is proposed to increase the performance of DCSK systems under additive white Gaussian noise and Rayleigh fading channels (Zhou et. al., 2018). Rather than generating conventional DCSK signal, it is introduced an noise robust DCSK approach which does not require reference signal and data transmission rate could be increased (Liu et. al., 2018). In spite of the existence of other wide variety of studies, there does not exist a research on chaotic communication under impulsive noise. Since the noncoherent chaotic communication systems are more convenient to use to discard synchronization error and also to increase the data transmission rate, the most widely used method in the literature such as differential chaotic shift keying is employed to analyze the impulsive noise behavior. With this motivation, Figure 5.1 illustrates chaotic signal taken from Rössler system together with impulsive noise and filtered signal with respect to each robust estimator. One should take into account that the filtering performance strongly depends on the chosen filter length. The symmetric total window length W and corresponding mean square error as a function of filter length for M point observation is determined by

$$MSE(W) = \frac{1}{M}\sum_{m=1}^{M}\left(y(m) - s_W^{est}(m)\right)^2 \tag{5.1}$$

where $s_W^{est}(\cdot)$ is the robust estimator result obtained from median, myriad or meridian filter.

Variation of mean square error with respect to selection of window length is expressed in Figure 5.2 where one can clearly see that the specified filter length can increase or decrease filtering performance.

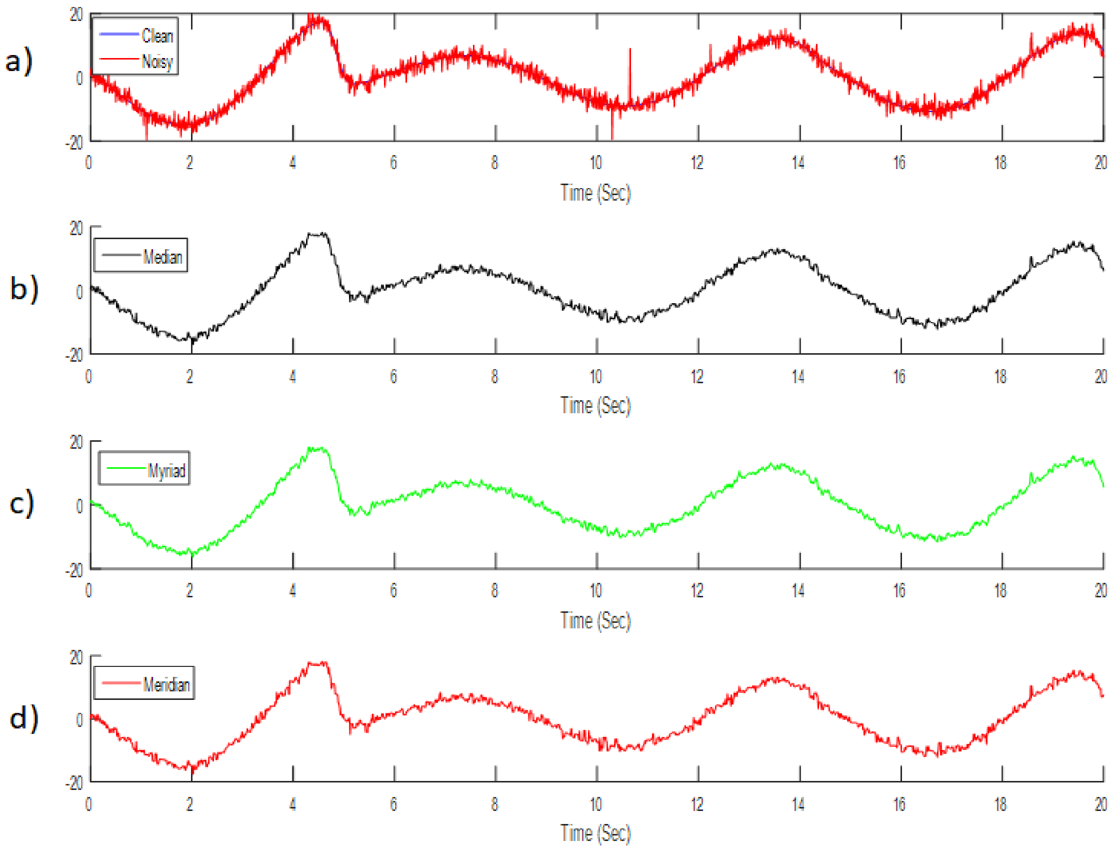

Figure 5.1 Chaotic signal from Rössler system a) Clean signal and noise b) Median estimator result c) Myriad estimator result d) Meridian estimator result

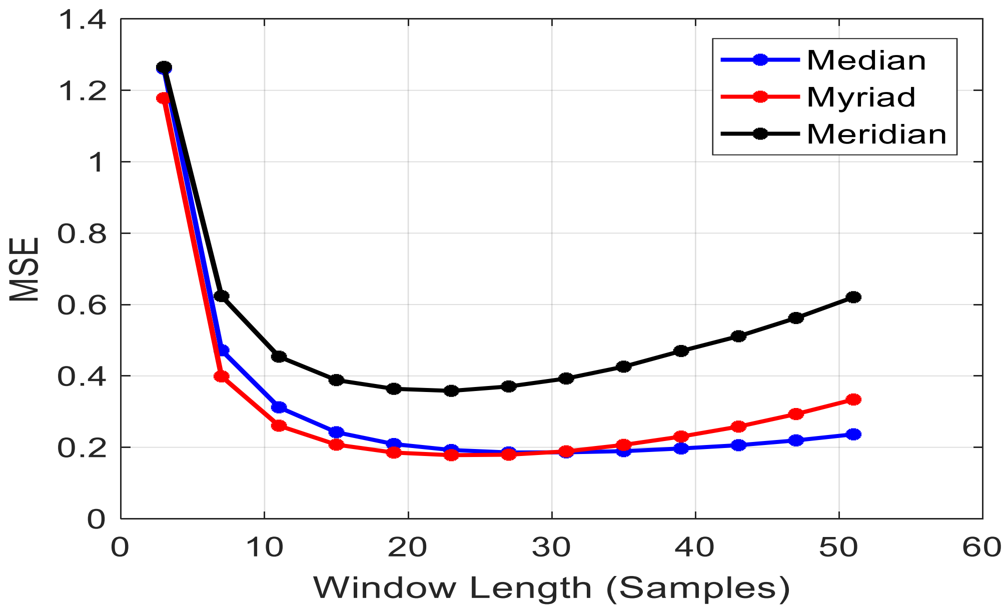

Figure 5.2 Mean square error of estimator results with respect to filter window lengths. (Characteristic exponent $\alpha = 1.5$)

Once the filtering process is performed, probability of error is approximated by Monte Carlo simulations with respect to varying characteristic exponent. Since the noise has infinite variance, conventional SNR can be expressed in terms of generalized signal to noise ratio $(GSNR)$ (Sureka et. al., 2013)

$$GSNR = 10\log\frac{P}{\sigma} \tag{5.2}$$

where σ is the $S\alpha S$ channel noise intensity and P is the power of chaotic signal within bit duration. For the sake of simplicity, the noise intensity is accepted as $\sigma = 1$ and the chaotic signal is multiplied by a scale factor in order to tune the GSNR value. The mean power of chaotic signal is then reduced to $P = 10^{GSNR/10}$. In order to represent the effect of window length, the bit error rate (BER) simulations using median filter in Figure 5.3 are illustrated with respect to different α values for 10^4 bits and the length of the chaotic signal is taken as 1000 samples for window lengths of 3 samples and 11 samples in Figure 5.3a and Figure 5.3b, respectively.

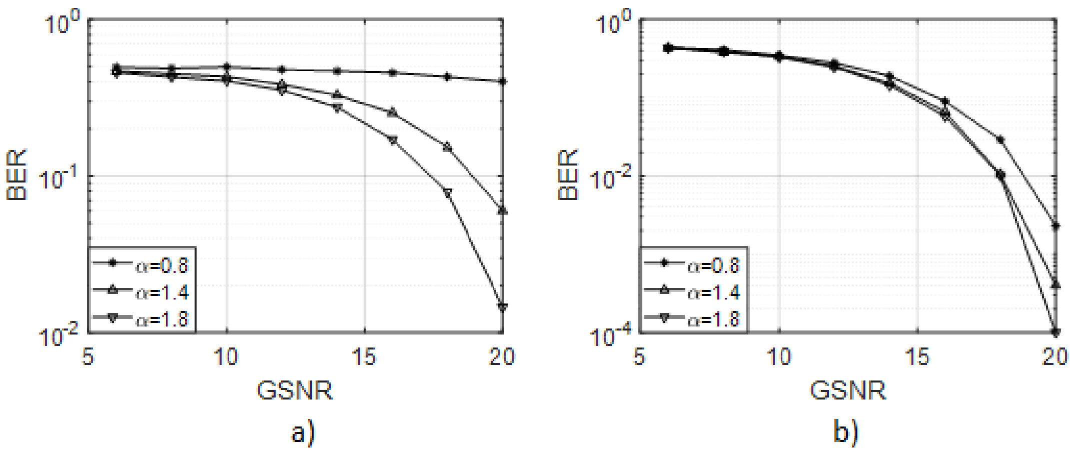

Figure 5.3 Bit error rate performance of the DCSK communication system with respect to channel noise having different impulsiveness parameter in Median filter with filter length having a) 3 samples b) 11 samples

One can clearly see that the error performance degrades in Figure 5.3a and Figure 5.3b when the impulsiveness increases i.e., characteristic exponent decreases for both selection of window lengths. In Figure 5.3a the selection of short window length compared with Figure 5.3b results in distinct but poorer error performances. Increasing the window length improves the error performance, however, the impulsiveness of the noise does not affect the results significantly.

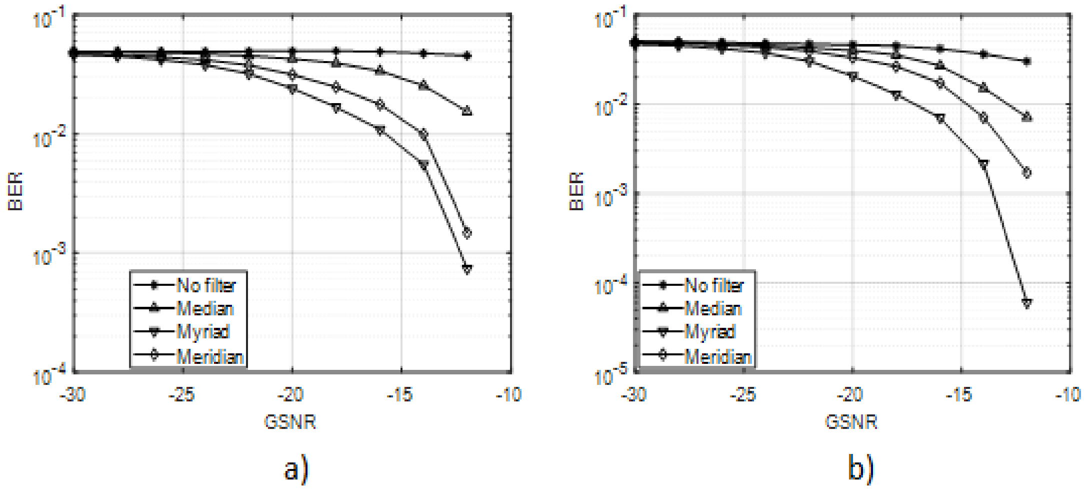

Figure 5.4 BER performances of DCSK system under impulsive noise using robust estimators a) $\alpha = 1.4$ b) $\alpha = 1.8$

The comparison of all estimators for fixed characteristic exponent is shown in Figure 5.4a and Figure 5.4b. It is seen that filter performance for myriad filter is slightly better when the characteristic exponent increases. Therefore, the selection of the estimator window length has a vital importance and affects the BER results dramatically.

Although robust estimators are proper candidates for filtering impulsive noise on observations, the conventional noise reduction approach based on sliding window is valid for location estimator. If the received signal has a time varying nature, this procedure should be improved. In a recent study by (Yang et. al., 2018), the myriad filter is modified to apply for noise filtering on sinusoidal signal where the signal samples apart from periodic interval constructed a new signal vector to be filtered by robust estimator and this procedure is repeated for every time instants taken from same periodic lengths.

As one of the contributions in this thesis, the aforementioned scheme is applied to generate chaotic communication signal. A certain length of the chaotic signal is repeated and the same points are considered to generate the vector to be analyzed for noise reduction. Figure 5.5 illustrates five repetition of a certain time segment of chaotic signal and the same points are represented as the input points of the robust estimator.

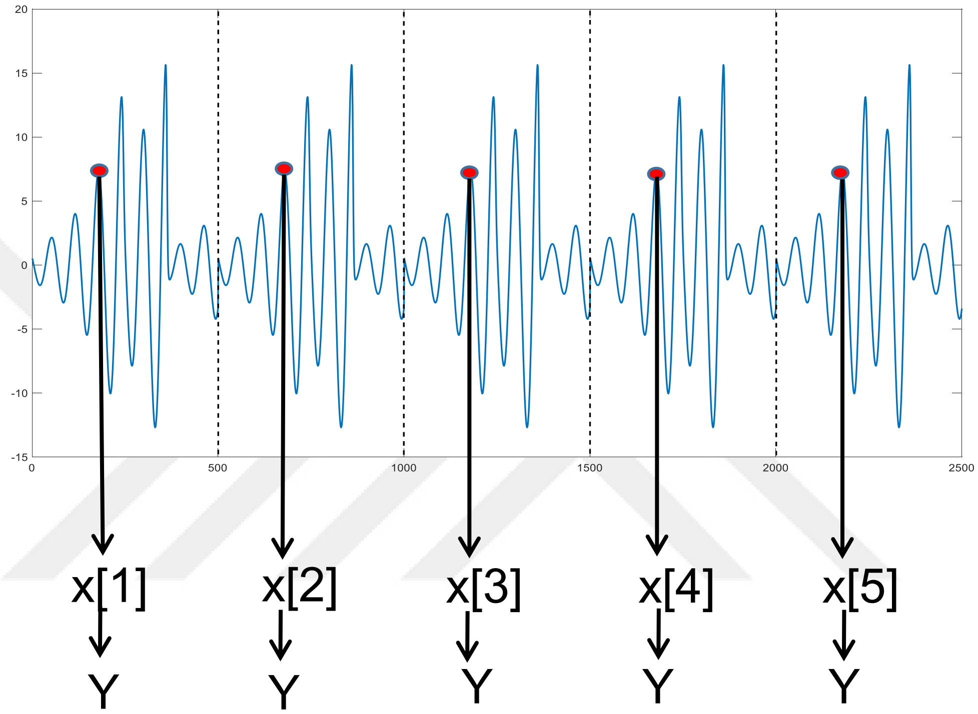

Figure 5.5 Illustration of chaotic signal augmented five times and the sample representation of selected points as the input $X[i], i = 1, \cdots 5$. The output Y carries the filtering result which is the new value of these points

The performance of this approach is shown in Figure 5.6 in an extremely impulsive noise environment $(\alpha = 0.8)$ where the chaotic signal is repeated 100 times. It is seen that the clean signal can be estimated even if the impulsiveness of the noise is prevents the information bearing signal to be observed. In order to provide more clear appearance, Figure 5.7 represents only the clean and estimated signal together. This method is considered to have a strong potential to use robust estimators for chaotic communication under non-Gaussian noise channels.

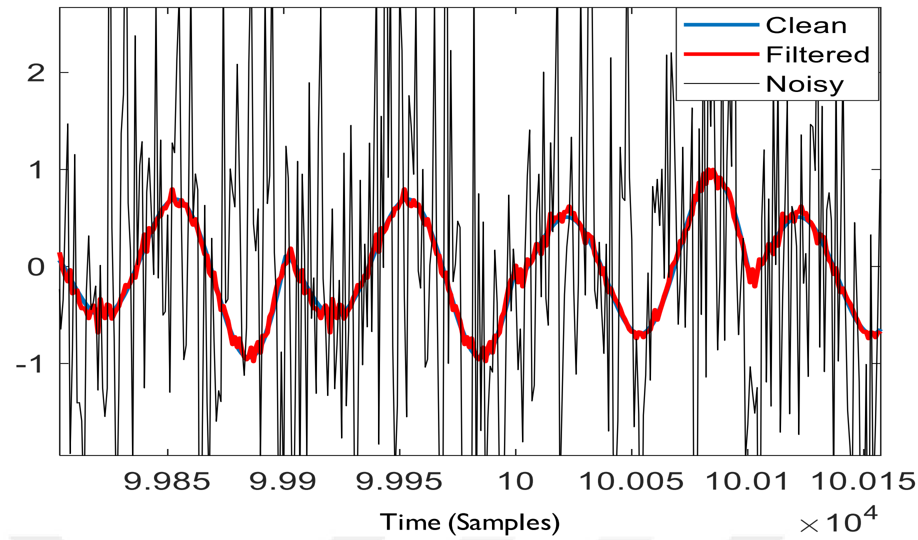

Figure 5.6 Noise filtering performance in time domain for the proposed method. The clean signal, estimated signal and noisy signal are given together within a certain time interval. (α = 0.8)

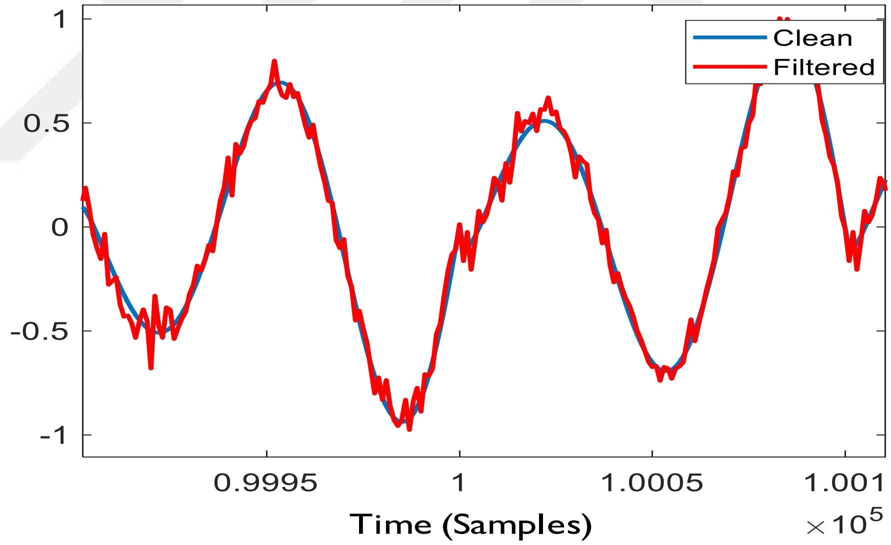

Figure 5.7 Noise filtering performance in time domain for the proposed method. Only the clean signal, estimated signal are given together within a certain time interval. (α = 0.8)

CHAPTER SIX
CONCLUSION

The thesis is primary based on analysis of chaotic communication under non-Gaussian noise environment. Due to the broadband nature of chaotic signals, a band pass filter cannot be used since there exists a notable information loss at the receiver. The adaptive filter utilization such as least mean square, recursive least square and Kalman filters is invalid since the channel noise is considered to be non-Gaussian. Particle filters can be a proper candidate to obtain noise free data but prior probability density cannot be known in advance for practical applications especially for unknown channel structure. Therefore robust estimators can be considered to perform filtering in noisy chaotic signal at the receiver.

The non-Gaussian noise is expressed by $\alpha -$stable distribution due to the common channel noise modeling given in the literature. Since it is difficult to provide synchronization of chaotic systems under noise contamination, rather than coherent chaotic communication which requires strong synchronization between transmitter and receiver, non-coherent chaotic communication system such as differential chaos shift keying (DCSK) is investigated under $(S\alpha S)$ noise. The proposed receiver structure utilizes robust estimators such as median, myriad and meridian estimators. These are conventionally used to determine constant value from observations corresponding to location under noise and not designed for filtering time varying signals. Nevertheless, an obvious idea is to implement filtering process by selecting an appropriate window length which directly affects the performance sliding this window along the observed signal at the receiver. The optimum estimator length is determined by mean square error with respect to the selection of varying estimator length. However, it is also observed that the optimum filter length depends on the length of the chaotic signal within single bit duration. For fixed signal duration, increasing estimator window length yields better probability of error performance although the sensitivity of the estimator with respect to the varying impulsiveness is lost. When the estimator length is decreased the filtering performance in terms of bit error rate becomes more distinct under different noise impulsiveness, but also decreases the noise performance at the same time. All of the estimator performances become worse for increased impulsiveness i.e., decreased characteristic exponent, as expected.

As another contribution in the thesis study, the robust estimators are redesigned to receive some specified samples of the chaotic signals as the input and correspondingly, more valid approach is introduced in filtering of signals having strong time varying nature especially for non-coherent chaotic communication applications. If the amount of repetition is sufficiently large, the signal can be filtered to recover the message under impulsive noise channels.

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