NUMERICAL SIMULATION OF A TWO DIMENSIONAL CIRCULAR REFLECTOR ANTENNA SYSTEM BY THE METHOD OF MOMENTS

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ABSTRACT

The prepared thesis is composed of a numerical simulation of a two-dimensional circular reflector antenna system by method of moments that defines the solution of the scattering problem of a two-dimensional reflector antenna system, which is designed by using a perfect conductor whose thickness is neglected. We have used the Method of Moments by discretisizing the integral equations. We do this by choosing the proper basis and testing functions for both E and H polarizations. By using complex source method (Oğuzer, 1995), we have obtained accurate simulation for this system investigated. The method is computed by the help of two boundary conditions, namely the total tangential electric field on the metallic surface vanishes and the value of the current tends to be zero on the slot part of the system.

Although in the early times, this method has been used for the problems in small size of geometries in the literature, we had a chance to solve even more larger geometries with the same method such as we take 15 times the wavelength and obtained acceptable approximations regularized for both E and H polarizations. The main advantage of this method is that it has provided us to obtain feasible results by converting the problem into a set of matrix equations instead of solving high-order integral equations.

ÖZET

Hazırlamış olduğumuz tez, kalınlığı ihmal edilmiş bir mükemmel iletkenden oluşan iki boyutlu dairesel yansıtıcı anten sistemini tanımlayan ve Moment Metodunu kullanan numerik bir çözümden oluşmaktadır. Her iki polarizasyonda (E ve H Polarizasyonları) elde etmiş olduğumuz integral denklemlerini Moment Metodunu kullanmak suretiyle uygun seçilmiş baz fonksiyonları ile genişletip, test fonksiyonları yardımıyla bilinmeyen indüksiyon akımını elde ettik. Karmaşık kaynak metodunun (Oğuzer, 1995) Moment methodu ile entegre edilmesi gerçek çözüme daha yakın sonuçlar edinmemizi sağladı. Temelde metal ve slot adını verdiğimiz iki bölümden oluşan sistemimizde Moment Metodunu işletirken sınır şartları olarak metal yüzey üzerindeki toplam teğetsel elektrik alanın ve slot kısımdaki akımın sıfır olması durumlarını kullandık.

Moment metodu, daha önce de literatürde küçük ölçüde geometriler için kullanılmış olsa da biz aynı metodu Karmaşık Kaynak Methodu ile birlikte uygulayarak daha büyük geometrilerde dahi makul sonuçlar elde etme fırsatını bulduk. Kullanmış olduğumuz yöntemin avantajı ise yüksek dereceli integral denklemleri çözmek yerine problemi matris denklem setine dönüştürerek gerçeğe yakın sonuçlar elde etmemizi sağlamış olmasıdır.

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CHAPTER ONE
INTRODUCTION

Antennas can be broadly classified according to their function as transmitting antennas, and as receiving antennas. Although the requirements, or mode of operation, are markedly different, a single antenna is used for transmitting and receiving signals simultaneously. Many of the properties of an antenna, such as its directional characteristics, apply equally to both modes of operation, this being a result of the reciprocity theorem.

1.1 Reciprocity Theorem

Reciprocity is an antenna's ability to transfer energy from the atmosphere to its receiver with the same efficiency with which it transfers energy from the transmitter into the atmosphere. Antennas form the link between transmitting and receiving equipment and the space propagation path.

A number of important consequences result from the reciprocity theorem. All practical antennas have directional patterns, that is they transmit more energy in some directions than others, and they receive more energy when pointing in some directions than others. The reciprocity theorem requires that the directional pattern for an antenna operating in the transmit mode is the same as that when operating in the receiving mode. Also there's another requirement for the reciprocity theorem is that the antenna impedance should be the same for both modes of operation.

1.2 Application Areas of Reflector Antennas

Reflectors antennas are widely used in satellite communication systems to enhance the gain of the antennas. The reflector provides a focusing mechanism which concentrates the energy in a given direction. The most commonly used form of reflector has a circular aperture.

1.2.1 System Requirements

Reflector antennas should have the following requirements in order to operate efficiently:

- high directivity: for a reflector antenna to be highly directive, it must change the normally spherical wavefront into a plane wavefront. Many highly directive microwave antennas produce a plane wavefront by using a reflector to focus the radiated energy.

- low side-lobe levels: side-lobes present the undesired radiation regions in the radiation pattern and their is low for properly designed reflector antennas systems.

- minimum propagation losses: Losses in the antenna will reduce the power available for the feeder. Receiver feeder losses will further reduce the power, so that the received power amount reaching the receiver is less than that received by the antenna. However, it can be said to be an advantage, that is, the atmospheric propagation losses are minimum for the reflector antennas.

1.2.2 Fiber Optic Systems versus Reflector Antennas

There is another type of communication system which is developing rapidly within the technology, namely fiber optic systems. The operating principle of this new system is not so difficult to understand. At the sending end there is a transmitter that serves as the origin for information being sent into the fiber line. The transmitter converts electronic signals from copper wire into corresponding light signals and passes them through the fiber cable. From there, the light conforms to the law of internal reflection, where the light exceeds the critical angle of incidence, it cannot escape the glass tube and is reflected down the core of the tube. On the receiving end of the transmission, there is a receiver that converts light back into electronic signals interpreted by the computer.

This type of systems introduces some advantages like they have:

- no electromagnetic interference
- wide bandwidth
- low loss which allows long links without repeater

- Resistance to noise as the transmission uses light rather than an electrical signal, outside noise does not effect the signal

- Security as no other communication link can be connected to it externally

Against all these advantages, reflector antenna systems also have acceptable advantages over fiber optics such as:

- low attenuation losses
- low cost of installation and maintenance
- easier to build
- not fragile
- ability to satisfy the most advantages of fiber optics by the proper design

Overall these advantages of both systems, it can be seen that although fiber optics is a newer and reliable system with respect to the latter, reflector antenna systems still have better specifications compared to fiber optics.

1.3 Parabolic Reflector Antennas

The most widely used reflector antenna systems are the ones that uses parabolic reflectors. This is the type seen in many home installations for the reception of TV signals. The term parabolic comes from which the circular aperture configuration is referred to as a paraboloidal reflector.

The main property of the paraboloidal reflector is its focusing property, normally associated with light, where parallel rays illuminating the reflector converge on a single point known as the focus, and conversely, rays originating at the focus are reflected as a parallel beam of light.

Light is a particular example of an electromagnetic wave, and the same properties apply to electromagnetic waves in general. The geometric properties of the paraboloidal reflector of interest are most easily demonstrated by means of the parabola, which is the curve traced by the reflector on any plane normal to the aperture plane. With reflector-type antennas, the feeder connecting the feed horn to the transmit/receive equipment must be kept as short as possible to minimize losses.

This is particularly important with large earth stations where the transmit power is large and very low receiver noise is required.

In order to meet the proper requirements, parabolic reflector antennas should have some specifications including the requirements for the reflector antennas, these are:

- high gain: parabolic reflectors enhance the gain of the antenna, that is the measure of how much power in dB an antenna will radiate in a certain direction with respect to that which would be radiated by a reference antenna such as an isotropic dipole.
- narrow beamwidth: Beamwidth is the angular separation between the half-power points on an antenna's radiation pattern which is the diagram indicating the intensity of radiation from a transmitting antenna or the response of a receiving antenna as a function of direction. It shows how the gain of an antenna varies with direction.

1.4 Two-Dimensional Reflector Antennas and their simulation techniques

2D reflector antennas are one type of reflector antennas whose length is too long in one dimension and therefore beamwidth belong to that dimension is narrower than the other dimension's beamwidth so these kinds of reflector antennas are used for navigation and scanning purposes. Mostly known example for its usage is in transponders used for aircrafts which determine the position of the vehicle (plane in this case). It's also used in GPS (Global Positioning System) works for military and civillian applications especially manipulated by the U.S.A.

Conditions for two dimensional curved structures can also simulate a two dimensional reflector antenna system with its directive feeder. One of the widely used technique for this solution is high frequency ray approaches like Physical and Geometrical theory of diffraction. On the other hand, one of the earliest study by MoM is performed in (Hansen, 1960) and solved moderate size geometries by using different approaches in literature. Moreover, an approximate solution is also presented for E-polarized wave about the scattering problem from a concentrially loaded slit cylinder (Mohammadian,1976). Later, a very stable MoM solution even for a narrow slit cases is formulated (Mautz et al., 1988-1989). Although this is performed for both polarizations and only for small and medium size geometries, it is also valid for cavity resonances with a high Q factor.

In addition to that, the same problem of the curved and PEC (Perfect Electrical Conductor) strip scattering, is also studied by method of regularization and especially semi-inversion by using Riemann-Hillbert Technique is used (Hashimoto, 1963), (Nosich, 1999). The circularly curved 2D surface illuminated by a directive feed antenna can also simulate the 2D reflector antenna system. Moreover, an accurate simulation of this kind of 2D antenna system is also performed (Oğuzer, 1995). Although this solution is needed some special functions like Legendre polynomials and special defined functions, it finally gives very accurate reference data.

An approximate solution is needed for this kind of 2D reflector antenna systems. A popular alternative for this is the Method of Moments (Harrington et al., 1967-1968). In this method, the integral equation is derived from boundary conditions and then it is discretized by using basis functions and tested by the same type of functions to reduce error in residue in the light of Galerkin's procedure. Finally, it is reduced to an algebraic matrix equation and solved numerically. This gives the approximate surface current density.

1.5 Aim of this work

Aim of this thesis is to find the unknown induced current and electric field of 2D scatter antenna system which is made of perfect electrical conductor whose thickness is neglected. We defined the problem in the matrix form and use the uniqueness theorem on the boundary conditions by specifying tangential components of the electric field over the surface of the antenna. Also we have modelled the parabolic reflector antenna as a circular reflector since it is valid when we take the edge angle so small. Moreover, to get more realistic results, we have used "Complex Source Method" (Oğuzer, 1995) for the line source used in the given problem. As a summary we've used the Method of Moments (MoM) by discretisizing the integral equations and convert them to a set of equations which can be solved numerically, namely matrix equations. Furthermore, this kind of combination is not presented in literature before and we have a chance to check for appropriate results with exact reference data (Oğuzer, 1995).

The following is the brief explanation about what has been mentioned in next chapters:

In Chapter 2, the derivation of the integral equations for both (E and H) polarizations are explained, starting with definition of the Maxwell's Equations, Uniqueness Theorem, Dirichlett (for E Polarization) and Neumann (for H Polarization) Boundary conditions and then as a result, obtaining the two integral equations, denoted as follows:

$$\int_M \vec{J}(\vec{r}\,')G^0(|\vec{r}-\vec{r}\,'|)dr' = -\vec{E}^{in} \qquad \text{(for E Polarization)} \qquad (1.1)$$

$$\frac{\partial \vec{H}_z^{in}}{\partial n} = -\frac{\partial}{\partial n}\int_M \vec{J}_\phi(\vec{r}\,')\frac{\partial G}{\partial n'}dr' \qquad \text{(for H Polarization)} \qquad (1.2)$$

$$\text{where } G^0 = -\frac{j}{4}H_0^{(2)}(k|\vec{r}-\vec{r}\,'|) \qquad (1.3)$$

The Method of Moments (MoM) is explained in Chapter 3 basically. In Chapter 4, the formulation of the problem solved with the MoM is defined. The numerical results obtained by using the specified method is shown in Chapter 5 with detailed explanations. After all, the aim of preparing this thesis and the comments are explained as a conclusion in Chapter 6.

CHAPTER TWO

DERIVATION OF THE INTEGRAL EQUATIONS FOR BOTH POLARIZATIONS

2.1 Maxwell's Equations

We already know that if a quantity have magnitude and direction, it is said to be a vector quantity. Generally, electric and magnetic fields are also vector quantities that all their relations and variations, including currents are manipulated by physical laws which are called Maxwell's equations. These equations can be written either in differential or integral form. Before denoting the Maxwell's equations, the following four quantities have to be known:

$\vec{E}$ = Electric field intensity (volts/meter)

$\vec{H}$ = Magnetic field intensity (amperes/meter)

$\vec{D}$ = Electric flux density (coulombs/meter square)

$\vec{B}$ = Magnetic flux density (webers/meter square)

$\vec{J}$ = Current density (amperes/meter square)

$\vec{\rho}$ = Charge density (coulomb/meter cube)

$$\nabla \times \vec{E}(\vec{r}) = -\frac{\partial \vec{B}(\vec{r})}{\partial t} \tag{2.1}$$

$$\nabla \times \vec{H}(\vec{r}) = \frac{\partial \vec{D}(\vec{r})}{\partial t} + \vec{J}(\vec{r}) \tag{2.2}$$

$$\nabla \bullet \vec{D}(\vec{r}) = \vec{\rho}(\vec{r}) \tag{2.3}$$

$$\nabla \bullet \vec{B}(\vec{r}) = 0 \tag{2.4}$$

Note that all the quantities with an arrow are vector quantities.

For sinusoidal time depencence:

$\frac{\partial}{\partial t} \rightarrow jw$ where $j^2 = -1$ and w is a real variable.

Applying this conversion, we can obtain the Maxwell's equations for time harmonic fields:

$$\nabla \times \vec{E}(\vec{r}) = -jw\vec{B}(\vec{r}) \quad (2.5)$$

$$\nabla \times \vec{H}(\vec{r}) = jw\vec{D}(\vec{r}) + \vec{J}(\vec{r}) \quad (2.6)$$

$$\nabla \bullet \vec{D}(\vec{r}) = \vec{\rho}(\vec{r}) \quad (2.7)$$

$$\nabla \bullet \vec{B}(\vec{r}) = 0 \quad (2.8)$$

Additionally, there's one more equation which is called continuity equation in time harmonic fields, that is:

$$\nabla \times \vec{J}(\vec{r}) = -jw\vec{\rho}(\vec{r}) \quad (2.9)$$

where $\vec{J}(\vec{r})$ is known as the distribution of the electric current density.

There are also integral forms of Maxwell's equations, denoted as:

$$\oint_c \vec{E} \bullet d\ell = -\oint_s \frac{\partial \vec{B}(\vec{r})}{\partial t} \bullet \partial s \quad (2.10)$$

$$\oint_c \vec{H} \bullet d\ell = \oint_s [\frac{\partial \vec{D}(\vec{r})}{\partial t} + \vec{J}(\vec{r})] \bullet \partial s \quad (2.11)$$

$$\oint_s \vec{D} \bullet ds = \oint_v \vec{\rho}_v \bullet \partial v \quad (2.12)$$

$$\oint_s \vec{B} \bullet ds = 0 \quad (2.13)$$

2.2 Boundary Conditions

Along boundaries where the media involved exhibit discontinuities in electrical properties, or there exist sources along these boundaries, the tangential Electric field vectors are continuous and the tangential magnetic field vectors are discontinuous. These behaviors across the boundaries are governed by boundary conditions.

At points of discontinuity, the behaviour of the field vectors across discontinuous boundaries must be handled by examining the field vectors themselves. The dependence of the field vectors on the electrical properties of the media along boundaries of discontinuity is always manifested. Maxwell's equations in integral form provide the most convenient formulation for derivation of the boundary conditions.

2.3 Uniqueness Theorem and Edge Condition

There are four main statements that are explained in brief as follows:

i) $\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0$ (2.14)

The expression above states that the tangential components of the electric field across an interface between two media with no impressed magnetic current densities along the boundary of the interface are continuous.

ii) $\vec{n} \times (\vec{H}_2 - \vec{H}_1) = 0$ (2.15)

Our second statement shows that the tangential components of the magnetic field across an interface between two media, neither of which is a perfect conductor, are continuous.

iii) $\vec{n} \bullet (\vec{D}_2 - \vec{D}_1) = 0$ (2.16)

The third statement above express that the normal components of the electric flux density across an interface between two media, both of which are inperfect electric conductors and where there are no sources are continuous, and this relation also holds if either or both media possess finite conductivity.

iv) $\vec{n} \bullet (\vec{B}_2 - \vec{B}_1) = 0$ (2.17)

This expression states that the normal components of the magnetic flux density across an interface between two media where there are no sources, are continuous.

Uniqueness theorem states that if a field is created by two sources namely J_i and M_i in a lossy region, is unique if one of the following criterias are specified:

a) within the region if the tangential components of the electric field,
b) within the region if the tangential components magnetic field,
c) the former over part of the boundary and latter over the rest of the boundary.

As the dissipation approaches zero, the field can be considered as the limit of the corresponding fields in that lossy medium. Therefore we use the uniqueness theorem on the boundary conditions by specifying tangential components of Electric field over the surface of the antenna.

In order to obtain a unique solution, the Helmholtz' equation has to be satisfied in every point for the field $\vec{\phi}(\vec{E},\vec{H})$.

$\nabla^2 \vec{\phi} + k^2 \vec{\phi} = \vec{\rho}$ (where $\vec{\phi}$ is the vector field) (2.18)

There are three important cases which satisfies unique solution of $\vec{\phi}(\vec{E},\vec{H})$:

i) $\vec{n}\times\vec{E}$ is specified on the surface S.

ii) $\vec{n}\times\vec{H}$ is specified on the surface S.

iii) $\vec{n}\times\vec{E}$ is specified on a part of the surface and $\vec{n}\times\vec{H}$ is specified on the rest of the surface S.

Clearly, in a problem that is composed of a two dimensional geometry with a smooth and closed surface but has a wedge type singular point, based on a differential equation, uniqueness might not obtain unless an additional requirement involving the region is included (Ziolkowski, 1987). We can show that the edge condition provides this requirement by using two discrete vector fields, denoted as $\vec{\phi_1}$ and $\vec{\phi_2}$, respectively. These both solutions satisfy the Helmholtz' equation which is:

$$\nabla^2\vec{\phi}_1 + k^2\vec{\phi}_1 = \vec{\rho} \tag{2.19}$$

We have assumed that the source $\vec{\rho}$ generate an incident plane wave e^{-jkz}. Bouwkamp has remarked that $\frac{\partial\vec{\phi}_2}{\partial z}$ is an acceptable solution for the problem. By using this remark, we can obviously write the Helmholz' equation as follows:

$$\nabla^2\frac{\partial\vec{\phi}_2}{\partial z} + k^2\frac{\partial\vec{\phi}_2}{\partial z} = \frac{\partial\vec{\rho}}{\partial z} \tag{2.20}$$

This means that the original incidence wave has been replaced by $-jke^{-jkz}$. Therefore $\frac{\partial\vec{\phi}_2}{\partial z}$ and $\vec{\phi}_1$ are solutions of the same problem, and they could coincide if uniqueness were to hold under the stated boundary conditions.

In order to show that enforcing edge condition removes this coincidence, we assume two solutions exist for the problem of interest, can be choosen namely $\vec{\phi}_a$ and $\vec{\phi}_b$, and $\vec{\phi} = \vec{\phi}_a - \vec{\phi}_b$ resolves that $\nabla^2 \vec{\phi} = 0$ on the surface S. Additionally $\vec{\phi}$ is also equals to zero on the closed contour (path) of the problem. This comes from applying Green's first theorem to $\vec{\phi}$. However, it can be only applied to a region where $\vec{\phi}$ has no singularity, and the remedy is to close the contour C with a small circle C' where it is centered on the edge with a radius r.

$$\iint_S (\vec{\phi}\nabla^2\vec{\phi} + \left|grad\vec{\phi}\right|^2)dS = \int_C \vec{\phi}\frac{\partial\vec{\phi}}{\partial n}dC \tag{2.21}$$

If we take the limit of the right side of the equation multiplied by the radius r, while r is approaching to zero, we will see that $\vec{\phi}$ has to be proportional to $r^v \sin(v\varphi)$ where $v \geq 0.5$ in order to make it equal to zero. By this way we can obtain a unique solution. That is the effect of the edge condition to the uniqueness.

As a result we can see that the in order to obtain uniqueness, the field has to be proportional with the edge parameters.

2.4 Obtaining the two integral equations: EFIE for both polarizations

In this section, derivation of the E-Field Integral Equation (EFIE) for E and H polarizations are explained in detail. In order to start the derivation of the EFIE for the E-Polarization, we need to use the boundary condition for the metal part of the system, as follows:

$$\vec{E}^{sc}_{\tan} + \vec{E}^{in}_{\tan} = 0 \tag{2.22}$$

therefore

$$\vec{E}^{sc}_{\tan} = -\vec{E}^{in}_{\tan} \tag{2.23}$$

We use the following equation which stands for the scattered E-field as the first step.

$$E_z^{sc} = \sum_n A_n H_n^{(2)}(k\rho) e^{jn\varphi} \tag{2.24}$$

By applying the following rule to the above equation,

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\mu \vec{J} \tag{2.25}$$

$$\nabla^2 \vec{E}_z + k^2 \vec{E}_z = -(-jw\mu) \vec{J}_z^{sc}(\vec{r}') \tag{2.26}$$

$$(\nabla^2 + k^2) g(|\vec{r} - \vec{r}'|) = -\zeta(\vec{r} - \vec{r}') \tag{2.27}$$

where A is the auxiliary vector potential, and $k = \sqrt{\mu\varepsilon}$, namely the wave number.

$$g(|\vec{r} - \vec{r}'|) = -(\frac{j}{4} H_0^{(2)}(k|\vec{r} - \vec{r}'|), \tag{2.28}$$

Here, $H_0^{(2)}$ is the second order Hankel function, and $\zeta(\vec{r} - \vec{r}')$ is the impulse function. $|\vec{r} - \vec{r}'|$ can also be denoted as R in equation (2.28).

As the second step, we define the line source incident field (2.29) and use the equation (2.30) which stands for the relation between the scattered E field and the incident current in the z direction,

$$E_z^{in} = -\frac{(-w\mu)}{4} H_0^{(2)}(k|\vec{r} - \vec{r}_0|) \tag{2.29}$$

$$E_z^{sc} = -jw\mu \int_C g(|\vec{r} - \vec{r}'|) \vec{J}_z^{sc}(\vec{r}') = -E_z^{in} \quad \text{(where C: contour)} \tag{2.30}$$

Since $E_z^{sc} = -E_z^{in}$

$$\int_C H_0^{(2)}(k|\vec{r}-\vec{r}'|)\vec{J}^{sc}(\vec{r}')dr' = -E_z^{in} \quad (2.31)$$

$$\int_C H_0^{(2)}(k|\vec{\rho}-\vec{\rho}'|)\vec{J}^{sc}(\vec{\rho}')d\rho' = -H_0^{(2)}(k|\vec{\rho}-\vec{\rho}_s'|) \quad (2.32)$$

Therefore we obtain the following EFIE for E-Polarization case.

$$K\int_C \vec{J}^{sc}(\vec{r}')G(\vec{r},\vec{r}')dr' = -E_z^{in} \quad (2.33)$$

where K is assumed as "1".

For the derivation of the EFIE for H-Polarization case, we use the following integral equation:

$$A = \int_C \vec{J}(\vec{r})G(\vec{r},\vec{r}')dr' \quad (2.34)$$

If we substitude the above integral to the following equality, we get:

$$\vec{H} = \nabla \times \vec{A} \quad (2.35)$$

$$= \nabla \times \int_C \vec{J}_\phi(\vec{r}')G(\vec{r},\vec{r}')dr' \quad (2.36)$$

$$= \int_C \nabla \times (\vec{J}_\phi(\vec{r}')G(\vec{r},\vec{r}'))dr' \quad (2.37)$$

for simplicity we will use single G instead of $G(\vec{r},\vec{r}')$ in the remaining calculations, and we can rewrite the inside of the integral as follows:

$$G\nabla \times \vec{J}' + \nabla G \times \vec{J}_\phi \quad (2.38)$$

Since $\nabla \times \vec{J} = 0$ our equation becomes the following:

$$\vec{H}_z = \int_C \nabla G \times \vec{J}_\phi dr' \tag{2.39}$$

$$\vec{H}_z^{sc} = \int_C \vec{J}_\phi(\vec{r}')(\frac{\partial G}{\partial n'})dr' \tag{2.40}$$

Since $\vec{E}_\phi^{sc}$ is equal to $-\frac{\partial \vec{H}_z}{\partial n'}$, we substitude the equation of $\vec{H}_z$ into this equality, we can get the following equation for $\vec{E}_\phi^{sc}$ as follows:

$$\vec{E}_\phi^{sc} = -\frac{\partial}{\partial r} \int_C \vec{J}_\phi(\vec{r}')\frac{\partial G}{\partial n'} dr' \tag{2.41}$$

As we stated in the previous derivation which is for the EFIE, the boundary condition that we have used was $\vec{E}_\phi^{sc} + \vec{E}_\phi^{in} = 0$, therefore the following equation is obtained, namely the EIFE for H-Polarization:

$$-\frac{\partial \vec{H}_z^{in}}{\partial r} = -\frac{\partial}{\partial r} \int_C \vec{J}_\phi(\vec{r}')\frac{\partial G}{\partial r'} dr \tag{2.42}$$

CHAPTER THREE
METHOD OF MOMENTS

The method of moments (MoM) is used to solve the EFIE for the unknown currents on the surfaces of the radiating elements. The Method of Moments (MoM) technique especially applies to electromagnetic radiation and scattering problems (Harrington et al., 1967-1968). MoM has been applied since 1960's, to virtually every area of electromagnetics including radiation and/or scattering by perfectly conducting and material bodies, such as thin wire antennas, aperture penetration, printed circuit structures, etc. (Hansen, 1960). In electromagnetics a moment method solution usually refers to a problem in which the MoM is used to solve a linear integral equation for a current distribution representing a body. There are basically two steps to follow in order to solve the problem with this method. The first step is to derive and obtain the referred integral equation, generally as a statement of the boundary conditions of the problem, or as a statement of the equivalence theorems used. The second step is to solve the integral equation that has been obtained, by the MoM. To be more clear, we can say that, the unknown current is expanded in terms of an appropriate set of basis functions. If N terms are retained in the expansion for the current, then there will be N number of weighted averages of the integral equations are enforced. By this time MoM is used to transform the integral equations into an order of N matrix for the N coefficients in the expansion of current. Once these coefficients are found, it means that the current will be also known which provides us to find most of the parameters of interest such as input impedance, radiation pattern in a straightforward manner (Newman, 1991). In the MoM technique, the Electric Field Integral Equation (EFIE) is typically used to mathematically define the problem and is solved for the surface currents generated on the objects of interest. These currents can then be used in radiation integrals to calculate the fields scattered by the objects.

3.1 Definition of the moment

For a given antenna structure the conductors can be broken into "segments", and the currents on the segments can then be determined. The "moment" is numerically the size of the current times the vector describing the little segment (length and orientation). One of them matches the currents at the ends of the segments. A set of "basis functions" may be assumed into which the current distributions are decomposed.

3.2 Method of Moments Procedure and Green's function

The "method of moments" starts from deriving the currents on each segment by using a coupling Green's function. This Green's function incorporates electrostatic coupling between the moments. If the spatial change of the currents is known accurately then one can compute the build up of charges at points on the structure.

There is some debate as to whether the electrostatic coupling is as accurately calculated as is the direct current coupling; and it is usual to approximate antennas having area by wire grid approximations, which also have to be chosen extremely carefully. As one always is presented with a computed result for a simulation, even if the model is in error, one can see that replacement of areas of metal by wire grids requires physical insight into the processes involved, rather than blind application of an algorithm.

The CPU time to set up and store the MoM matrix equation is proportional to N^2 while its time to solve the MoM matrix equation is proportional to N^3. As an example, in one of our solution which is for the N=250 case, the computer needed time to solve for the unknown current was about 62 hours where it needs only about 10 hours to solve for N=150. This obviously shows that the defined proportionality is valid. The current distribution may then be integrated to find the total far field (Newman, 1991).

Green's function is a kind of three dimensional version of the impulse response function familiar in linear electronic circuit analysis. One sets up a structure for the space under consideration, specifying where the excitations can be and what the boundary conditions are. One then excites the structure with a single little region of excitation, with all the other possible excitations set to zero. The Green's function is the response of all the other regions in the problem to this excitation. Since the system is assumed linear, the principle of superposition applies and the total response to an arbitrary set of excitations can be obtained for the problem by direct summation or integration over all the excitations.

3.3 Application areas

The Method of Moments (MoM) has been in use for many years for a wide variety of applications. It is very commonly used to analyze antenna structures and a number of different applications. It continues to be very popular, probably the most popular modeling technique used in electromagnetics.

The MoM technique is a frequency domain technique. It will only analyze a single frequency at a time, although most popular software codes allow the solution to iterate over a number of frequencies.

In the technique of Method of Moments used for thin-wire structures for example, the object is modelled as a structure of wires of differing radii. Conducting surfaces are modelled as a grid of wires. The radius must be such that the total surface area of the wires is the same as the total surface area of the true structure. Assumptions are made about the form the currents take on each wire which might be, for example, a polynomial with several unknown coefficients. In order to avoid ambiguity, the length of each wire must be restricted to less than 0.25 wavelengths. In practise, this length should be less than 0.1 wavelength. The solution for the coefficients on each wire is the core of the technique. Since a matrix inversion is required to obtain all the unknown coefficients, the maximum permissible number of segments of about 10000 is about the limit. This method has been used widely for HF antennas on aircraft, antenna farms on ships, tanks and many other vehicles.

3.4 Working principle

The MoM technique requires that the entire structure to be modelled, be broken down into wires and/or metal plates. Each wire is subdivided to a number of wire segments which must be small compared to the frequency's wavelength (so that the assumption of a constant value of current across that wire segment is valid). Each metal plate is subdivided into a number of surface patches, which must be small compared to the wavelength (again so the assumption of constant current is valid).

Once the model is defined, a source is imposed (a plane wave approaching, or a voltage source on one of the wire segments). The MoM technique is to determine the current on every wire segment and surface patch due to the source and all the other currents (or the other wire segments and surface patches). Once these currents are known, then the E field at any point in space is determined from the sum of all the contributions from all the wire segments and surface patches.

3.5 Advantages

Every modeling technique has some strengths and some weaknesses. Some types of models were a given technique will excel and some types of models were the same technique will have difficulty (if it is even possible to use) performing rapidly and accurately.

- MoM is a very versatile modeling technique. It is also a very intuitive technique, so users can easily understand how to use it, and know what to expect from a given model. Users can picture the RF currents on a structure and understand how they would lead to a E/H field.

- MoM models only the metal structure, and not the space around it. Therefore, long wires are easily modeled using MoM.

- Since MoM is a frequency domain technique, it can solve problems very quickly, if only one frequency is desired. If multiple frequencies are desired, then the simulation will take longer, but still solutions are often available is a short amount of time.

- MoM allows discrete components to be inserted into a model by simply defining the impedance desired on any given wire segment. This can be useful when analyzing the effect of filter location, etc.

- MoM provides high accuracy if the proper considerations taken into account. In most cases, the MoM is a direct numerical solution of the exact integral equation.

That enables one to inherent all phenomena of the problem in the integral equation, therefore they are automatically included in the MoM solution. The main requirement of the MoM that N, the number of terms must be retained in the expansion for the current in order to obtain high accuracy which is proportional to the electrical size of the body. If the number of terms are fixed to a small number where the size of the system is large, reasonable accuracy cannot be expected.

- MoM is capable of dealing with very complex structures: by using MoM, several user-oriented computer codes can be written that can treat such complex geometries as simple as a dipole or as complex as an airplane.

3.6 Disadvantages

Although MoM is very easy to use for wires and metal plates, it is very difficult to use for dielectric and special magnetic materials.

- Special solution techniques do exist to allow dielectric in a MoM solution, but these are not widely implemented and care must be taken when they are used.

- MoM assumes the current on a wire segment, or on a surface patch to be the same throughout the conductor's depth. Therefore, using MoM to determine the effect of an aperture with fields both inside and outside is difficult.

- MoM is a frequency domain technique, therefore, if a wide frequency range is desired in the solution, the simulation must be run a number of times. If the frequency step size is not sufficiently small, important effects (e.g. resonance) may be over looked.

- For the mathematical point of view, MoM does not guarentee convergence and accuracy, this means, making the number of divisions higher while decreasing the decreasing the size of the segments does not mean that the solution will be accurate or converge.

3.7 Derivation of the method

In general, the MoM is a procedure for solving a linear operator equation by transforming it into a system of simultaneous linear algebraic equations, commonly matrix equation. The description that will be given in the following, largely follows that originally presented in several studies. (Mautz et al., 1988-1989), (Newman, 1991).

The method of moments (MoM) solution procedure was first applied to electromagnetic scattering problems by Harrington. Consider a linear operator equation given by

$$AX = Y \tag{3.1}$$

where A represents the integral operator, Y is the known excitation function and X is the unknown response function to be determined. Now, let X be represented by a set of known functions, termed as basis functions or expansion functions (f_1, f_2, f_3,...,) in the domain of A as a linear combination:

$$X = \sum_{n=1}^{N} \alpha_n f_n(x) \tag{3.2}$$

where α_n's are scalar constants to be determined. Substituting eq. (3.2) into eq. (3.1), and using the linearity of A, we have

$$\sum_{n=1}^{N} \alpha_n A f_n(x) = Y \tag{3.3}$$

where the equality is usually approximate. Let (w_1, w_2, w_3,...) define a set of testing functions in the range of A. Now, multiplying eq. (3.3) with each w_j and using the linearity property of the inner product, we obtain

$$\sum_{n=1}^{N} \alpha_n < w_m, Af_n(x) >=< w_m, Y > \qquad \text{(for m = 1, 2, ... , N)} \qquad (3.4)$$

The set of linear equations represented by eq. (3.4) may be solved using simple matrix methods to obtain the unknown coefficients α_n.

The simplicity of the method lies in choosing the proper set of expansion and testing functions to solve the problem at hand. Further, the method provides a most accurate result if properly applied. While applying the method of moments to complex practical problems, the solution region, in general, is divided into. Then, one can define suitable basis and testing functions and develop a general algorithm to solve the electromagnetic problem.

3.8 Choosing the basis and testing functions

As we stated earlier, selection of the basis and testing functions are very important for our solution to be accurate and converge. Because improper selections result singularities especially at the edges of the solutions, this comes from the vanishing of the inner products of the left-hand-side and the excitation functions and the testing functions. Therefore, special care must be taken for selecting the proper functions for the specific problem. There are some several methods which are used depending on the choice of the testing functions.

In the Point Matching method, Dirac Delta functions are used as testing functions, denoted as:

$$w_m(x) = \zeta(x - x_m) \qquad (3.5)$$

where x_m's are the suitable points between the specified interval, namely the residual is forced to vanish at N different points in the specified region.

There's another method that the testing functions are choosen as pulses, denoted as:

$$w_m(x) = \begin{cases} 1, X \in R_m \\ 0, elsewhere \end{cases} \tag{3.6}$$

and force the integral of the residual function to be zero over the different subdomains of the interval. This method is called the Method of Collocation by Subdomains.

In the given problem, we've used the same equation as basis and testing, which is called Galerkin's approach. In our system, the function that has been choosen are pulse and triangle functions for E and H solutions, respectively.

3.9 Convergence of the MoM

Convergence problem of the MoM is one of the most important questions concerning this method. The main issue of interest is that does the basis function modeled for the unknown function really approaches to the exact unknown function inself as N goes to infinity? Hopelessly, very little can be said about the convergence of the MoM , in which weighting functions can be chosen differently from the basis functions, therefore it makes us decide to use Galerkin's method in our solution. However, Galerkin's method also requires that for good convergence, the operator should be positive. But special attention has to be taken that convergence does not directly means that the basis function converges to the unknown function. Because, the choice of the feasible basis functions is very difficult, such as it requires a special kind of art. Main ways of choosing the basis functions are as follows:

- taking the known properties of the unknown function f as much as possible. As an example, if f is continuous, then it is desirable and mental to choose the basis function also continuous. If f is zero at the boundaries, such as in our system, then the basis functions that are used has to be also zero at the boundaries.

- Inner products has to be evaluated with a reasonable case, ie they mustn't be close to zero which causes the MoM matrix to be singular. Otherwise the results that obtained would be unreasonable.

CHAPTER FOUR

FORMULATION OF THE PROBLEM BY METHOD OF MOMENTS

4.1 Brief definition of the system

Our system consists of two parts, which are the metallic part and the slot part. The metallic part is in a shape of a circular arc of $2\theta_{ap}$ which is far from the center in the amount of "a". The directional point source has been placed in the middle of the center and the arc and used as the feeder for the aperture.

4.2 Formulation for E-Polarization

Total Electric field is defined as the sum of incident and scattered Electric field, namely $\vec{E}^{in}$ and $\vec{E}^{sc}$ respectively, which is equal to zero on the surface of the scatterer. That is:

$$\vec{E}_{total} = \vec{E}^{in} + \vec{E}^{sc} = 0 \qquad (4.1)$$

Current on the reflector is maximum on the edges of the metal and the tangential Electric field components tend to be zero on the surface of the system. We have previously found the EFIE in Chapter 2, equation (2.33), that is:

$$-\vec{E}^{in} = \int_M \vec{J}^{sc}(\vec{r}')G(\vec{r},\vec{r}')dr' \qquad (4.2)$$

$\vec{J}_z(\vec{r})$ function can be rewritten as a series equation expanded with the basis function which is pulse function that will be explained in the last part of this section.

$$\vec{J}(\vec{r}') = \sum_{n=1}^{N} X_n f_n(\varphi') \tag{4.3}$$

The 2D Green's function can be written as follows:

$$G(\vec{r},\vec{r}') = -\frac{j}{4} H_0^{(2)}\left(2ka\sin\frac{|\varphi-\varphi'|}{2}\right) \tag{4.4}$$

$$= -\frac{j}{4} \sum_{p=-\infty}^{\infty} J_p(k_0 a) H_p^{(2)}(k_0 a) e^{jp(\varphi-\varphi')} \tag{4.5}$$

In this study, the feed antenna's radiation patterns also modelled by a Complex source point method (Oğuzer, 1995). If a line source located at the real position $\vec{r}_0$ can be converted to an imaginary position by adding a complex vector $j\vec{b}$ so a complex line source at $\vec{r}_s = \vec{r}_0 + j\vec{b}$ then radiates the following directive beam field.

$$E_z^{in}(H_z^{in}) = C.H_0^{(2)}(\mathrm{k}_0|\vec{r} - \vec{r}_s|) \tag{4.6}$$

$$= C.\sum_{n=-\infty}^{\infty} J_n(k_0 r_s) H_n^{(2)}(k_0 r) e^{jn\varphi} e^{-jn\theta_s}\,,\ \vec{r} > |\vec{r}_s| \tag{4.7}$$

$$\text{where } r_s = \sqrt{r_0^{\,2} + 2jr_0 b\cos\beta - b^2} \tag{4.8}$$

$$\text{and } \theta_s = \cos^{-1}\left(\frac{r_0 + jb\cos\beta}{r_s}\right), \mathrm{Re}(r_s) > 0 \tag{4.9}$$

The b, β, and θ_s parameters represent the complex source beamwidth and direction.

Complex source can be imagined as an aperture antenna having 2b aperture width. So imaginary region cannot be cross the PEC reflector surface (Oğuzer, 1995).

$$-\vec{E}^{in} = \sum_k J_k(k_0\vec{r}_s)H_k^{(2)}(k_0a)e^{jk(\varphi-\theta_s)} \tag{4.10}$$

Let $e_k = J_k(k_0\vec{r}_s)H_k^{(2)}(k_0a)e^{-jk\theta_s}$ (4.11)

If we substitude (4.11) into (4.10), we get the following,

$$\sum_k e_k e^{jk\varphi} = \sum_{n=1}^{N} X_n \int_{-\theta_{ap}}^{\theta_{ap}} f_n(\varphi')(-\frac{j}{4})\sum_{p=-\infty}^{\infty} J_p(k_0a)H_p^{(2)}(k_0a)e^{jp(\varphi-\varphi')}ad\phi' \tag{4.12}$$

$$\sum_{n=1}^{N} X_n(-\frac{j}{4})a\sum_{p=-\infty}^{\infty} J_p(k_0a) \int_{-\theta_{ap}}^{\theta_{ap}} f_n(\varphi')e^{-jp\varphi'}d\varphi' \tag{4.13}$$

where

$$f_n^k = \frac{1}{2\pi}\int_{\varphi_n}^{\varphi_{n+1}} f_n(\varphi')e^{-jk\varphi'}d\varphi' \tag{4.14}$$

and

$$\varphi_n = -\theta_{ap} + \frac{2\theta_{ap}}{N}(n-1) \tag{4.15}$$

$$-\sum_k e_k e^{jk\varphi} = \sum_{n=1}^{N} X_n(-\frac{j}{4})a\sum_{p=-\infty}^{\infty} J_p(k_0a)H_p^{(2)}(k_0a)\int_{\varphi_n}^{\varphi_{n+1}} f_n(\varphi')e^{-jp\varphi'}d\varphi' e^{jp\varphi} \tag{4.16}$$

$$\sum_{n=1}^{N} X_n(-\frac{j}{4})(2a\pi)\sum_{p=-\infty}^{\infty} J_p(k_0a)H_p^{(2)}(k_0a)f_n^p e^{jp\varphi} \tag{4.17}$$

Weighting (same as basis) function is added to the equation by inner product like shown below:

$$\sum_{n=1}^{N} X_n < f_m, f(g_n) > = < f_m, g > \qquad \text{where m=1,2,...,N} \tag{4.18}$$

$$= \int f_m^* .g \, d\varphi' \tag{4.19}$$

$$-\sum_k e_k \int_{-\theta_{ap}}^{\theta_{ap}} w_m(\varphi) e^{jk\varphi} d\varphi = \sum_{n=1}^{N} X_n (-\frac{ja\pi}{2}) \sum_{p=-\infty}^{\infty} J_p(k_0 a) H_p^{(2)}(k_0 a) f_n^p c' \tag{4.20}$$

$$\text{where } c' = \int_{-\theta_{ap}}^{\theta_{ap}} f_m(\varphi) e^{jn\varphi} d\varphi = 2 f_m^{*k} \pi \tag{4.21}$$

As a result, we can obtain the tangential E_{total} calculation for the E-polarization:

$$E_t = \sum_k e_k e^{jk\varphi} + \sum_{n=1}^{N} X_n (\frac{-ja\pi}{2}) \sum_{p=-\infty}^{\infty} J_p(k_0 a) H_p^{(2)}(k_0 a) f_n^p f_n^{p*} \tag{4.22}$$

If we write it in the matrix form, we have to convert the left and right hand sides into $[A_{mn}]$ and $[B_m]$ matrices by the following operations:

$$[B_m] = \sum_{p=-\infty}^{\infty} w_m^{p^*} J_p(k_0 \vec{r}_s) H_p^{(2)}(k_0 a) \tag{4.23}$$

and

$$[A_{mn}] = -\frac{ja\pi}{2} \sum_{p=-\infty}^{\infty} J_p(k_0 a) H_p^{(2)}(k_0 a) w_m^{p*} f_n^p \tag{4.24}$$

$[X_n]$ can be found from the following matrix multiplication:

$$[X_n]=[A_{mn}]^{-1}.[B_m] \qquad \text{(Note that m=n)} \qquad (4.25)$$

which is actually the unknown current on the surface of the metal part of the given problem. After obtaining the current, we can find radiation pattern and directivity of the reflector antenna system.

For the E-Solution we have used pulse function as the basis and testing functions.

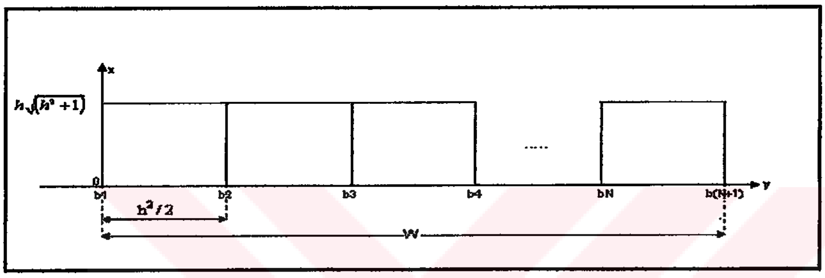

Figure 4.1 Analytical Representation of the pulse function

$b_1=0$	n=1,2,3,..., N+1
$b_2=h^2/2$	
$b_3=(h^2/2).2$	$\Delta=h = W / N$
$b_4=(h^2/2).3$	
.	$0<b<W$
.	
.	f(y') = pulse function
$b_N=(h^2/2).(N-1)$	
$b_{N+1}=(h^2/2).N$	$f_k(y')$ = fourier transform of pulse function

Fourier transform of this function will be as follows, for $k \neq 0$ and $k=0$ cases:

$$f_k(y')_{k\neq 0}=\frac{1}{2\pi}\int_{b_n}^{(b_n+\frac{h^2}{2})} h(\sqrt{h^2+1})e^{-jkb'}db' \qquad (4.26)$$

$$= \frac{h}{2\pi}(\sqrt{(h^2+1)})e^{-jkb_n}(e^{-jk\frac{h^2}{2}}-1) \tag{4.27}$$

$$f_k(y')_{k=0} = \frac{1}{2\pi}\int_{b_n}^{(b_n+\frac{h^2}{2})} h(\sqrt{h^2+1})e^0 db' \tag{4.28}$$

$$= \frac{h\sqrt{h^2+1}}{2\pi} \tag{4.29}$$

4.3 Formulation for H-Polarization

We begin the formulation by first rewriting the HFIE function, that is:

$$\frac{\partial \overrightarrow{H}_z^{in}}{\partial n} = -\frac{\partial}{\partial n}\int_M J_\phi(\vec{r}')\frac{\partial G}{\partial n'}dr' \tag{4.30}$$

where $\overrightarrow{H}_z^{in} = H_0^{(2)}(|\vec{r}-\vec{r}_s|) = \sum_n J_n(k_0\vec{r}_s)H_n^{(2)}(k_0\vec{r})e^{-in\varphi_s}$ (4.31)

As it can be seen from the above equation, complex source has been used like in the E-Polarization calculations, for better accuracy for the result of the induced current.

$$\frac{\partial \overrightarrow{H}_z^{in}}{\partial n} = k\sum_k J_k(k_0\vec{r}_s)H_k^{(2)}(k_0\vec{r}_s)e^{jk\varphi} \tag{4.32}$$

Again, as in E-polarization $\vec{J}_\phi$ can be written in terms of the expansion function which is triangle function, $w_n(\varphi')$, which is explained in the last part of this section, as follows:

$$J_\phi(\varphi) = \sum_{n=1}^{N-1} X_n w_n(\varphi') \tag{4.33}$$

$$E_{\phi}^{sc} = -\sum_{n=1}^{N-1} X_n \int_{\varphi_n}^{\varphi_{n+2}} w_n(\varphi') \frac{\partial}{\partial r} \frac{\partial}{\partial r'} G(\vec{r},\vec{r}')a \; d\varphi' \tag{4.34}$$

$$\text{where } G(\vec{r},\vec{r}') = \begin{cases} \sum_n J_n'(k_0\vec{r}')H_n^{(2)}{}'(k\vec{r})e^{in(\varphi-\varphi')}, \vec{r} > \vec{r}' \\ \sum_n J_n'(k_0\vec{r})H_n^{(2)}{}'(k\vec{r}')e^{in(\varphi-\varphi')}, \vec{r} < \vec{r}' \end{cases} \tag{4.35}$$

If we substitude eq.(4.35) into eq.(4.34), we obtain the following equation:

$$-k\sum_n J_n(k_0\vec{r}_0)H_n^{(2)}{}'(k_0\vec{r}')e^{jn\varphi} = -\sum_{n=1}^{N} X_n k^2 \int_{-\theta_{ap}}^{\theta_{ap}} w_n(\varphi') \sum_{p=-\infty}^{\infty} J_n'(k_0\vec{r}')H_n^{(2)}(k_0\vec{r}')e^{jn(\varphi-\varphi')} a d\varphi' \tag{4.36}$$

$$[B_m] = \sum_{p=-\infty}^{\infty} w_m^{p^*} J_p(k_0\vec{r}_s)H_p^{(2)}{}'(k_0 a) \tag{4.37}$$

and

$$[A_{mn}] = -\frac{jka\pi}{2} \sum_{p=-\infty}^{\infty} w_m^p J_p'(k_0 a)H_p^{(2)}{}'(k_0 a) f_n^{p*} \tag{4.38}$$

A remarkable difference of this formulation from the previous one, is that, we used triangle function as the basis and testing function, instead of using pulse function, since H-solution cannot converge as rapidly as E-solution. Although the results are not as accurate as it is in E-polarization, choosing triangle function enables us to obtain more accurate and convergent results for H-polarization.

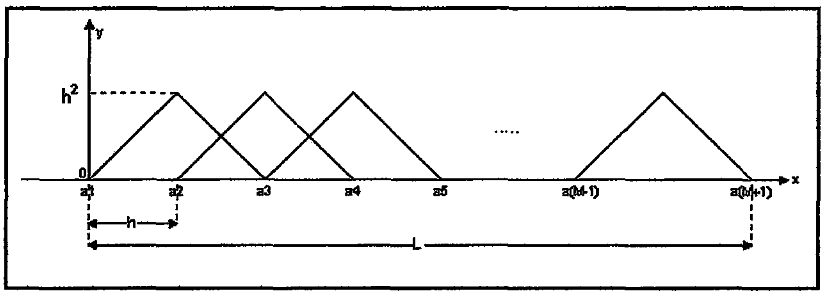

Figure 4. 2 Analytical Representation of the triangle function

Fourier transform of this function will be as follows:

$$w_n^k = \frac{1}{2\pi}\int_0^{2\pi} rect(\varphi) e^{-jk\varphi} d\varphi \tag{4.39}$$

where $rect(\varphi) = \sum_k w_n^k e^{jk\varphi}$ (4.40)

Therefore,

$$w_n^k = \frac{1}{2\Delta\pi}\left[\int_{\varphi_n}^{\varphi_{n+1}} e^{-jk\varphi} d\varphi - \int_{\varphi_{n+1}}^{\varphi_{n+2}} e^{-jk\varphi} d\varphi\right] \tag{4.41}$$

where,

$$\varphi_n = -\theta_{ap} + \Delta(n-1) \tag{4.42}$$

$$\varphi_{n+1} = -\theta_{ap} + \Delta n \tag{4.43}$$

$$\varphi_{n+2} = -\theta_{ap} + \Delta(n+1) \tag{4.44}$$

After calculating (4.41), w_n^k can be found as:

$$w_n^k = \frac{e^{jk\theta_{ap}} e^{-jk\Delta n}}{jk2\Delta\pi} \left(e^{jk\frac{\Delta}{2}} - e^{-jk\frac{\Delta}{2}} \right)^2 \tag{4.45}$$

Since $e^{jk\frac{\Delta}{2}} - e^{-jk\frac{\Delta}{2}} = 2\sin\left(k\frac{\Delta}{2}\right)$ (4.46)

Hence our equation becomes:

$$w_n^k = \frac{e^{jk\theta_{ap}} e^{-jk\Delta n}}{jk2\Delta\pi} 4\sin^2\left(k\frac{\Delta}{2}\right) \tag{4.47}$$

We can use the following relation between the triangle and pulse function in order to obtain the fourier transfrom of our triangle function:

$$\frac{\partial f_n}{d\varphi} = rect_n(\varphi) \tag{4.48}$$

$$= \sum_k w_n^k e^{jk\varphi} \tag{4.49}$$

Finally, the fourier coefficients of our n^th basis (or testing) function, namely the triangle function, denoted as f_n^k for $k \neq 0$ and $k = 0$ cases will be as follows, respectively:

$$f_n^k \big|_{k\neq 0} = \frac{e^{jk\theta_{ap}} e^{-jk\Delta n}}{k^2 2\Delta\pi} 4\sin^2\left(k\frac{\Delta}{2}\right) \tag{4.50}$$

$$f_n^k \big|_{k=0} = \frac{\Delta}{2\pi} \tag{4.51}$$

4.3 Radiation pattern and Directivity Formulations

It is denoted as Ø(φ) and equals, (for instance, written for E Polarization):

$$\phi(\varphi) = \sum_{p=-\infty}^{\infty} J_p(k_0 r_0)(j^p)e^{jp\varphi} + C\sum_{p=-\infty}^{\infty} J_p(k_0 a)(j^p)(\sum_{n=1}^{N} X_n f_n^p)e^{jp\varphi} \qquad (4.52)$$

where $C = -\dfrac{ja\pi}{2}$ (4.53)

Directivity (D) of the antenna can be found by the following formula:

$$D = \frac{2\pi|\phi(\varphi)|^2}{\int_0^{2\pi} |\phi(\varphi)|^2 d\varphi} \qquad (4.54)$$

As a results of these calculations, the induced current, tangential total E fields, and radiation patterns of the given system can be calculated by using computer programs, and each result is shown in the next chapter.

CHAPTER FIVE
NUMERICAL RESULTS

5.1 Overview

As a result of our study, acceptable results for the unknown induced current, radiation patterns, total tangential electric fields are achieved by using the specified method for the E and H solution of the system shown in Figure 5.1. We have taken N, the number of divisions on the metallic part of the system, as 100 in comparisons, and 150 for the solution of the same problem with a larger size. We also use 150 divisions when calculating the total tangential electric field for E-Polarization, and 100 for the H-Polarization as the computing time got very higher as we increase the division number, especially in H-Polarization. Moreover, we have taken "a" as ten times the wavelength for the comparisons and the calculations for the total electric field, and 15 times the wavelength for the solution of the problem with the larger size of geometry. Lastly, the aperture angle in the whole system is taken as 30 degrees for all calculations in our study.

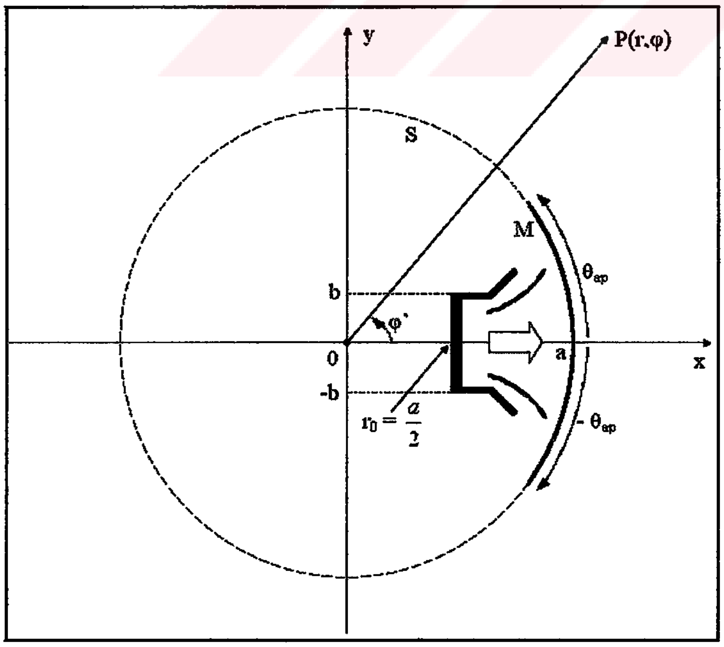

Figure 5.1 Geometry of the System

5.2 Radiation Pattern comparisons for the exact and numerical solutions

It can be seen from the following two figures that there are so small differences between the exact and numerical results for the radiation patterns for both E and H Polarization, however, between 60 and 90 degrees, results differ from each other caused by the effect of the edge conditions which is not imposed in this study, and the corresponding figures shown for the approximation for the radiation patterns for both polarizations, namely Figure 5.2, and Figure 5.3. The parameter values for the calculations are taken as equal as the ones used in the exact solutions for clear observation.

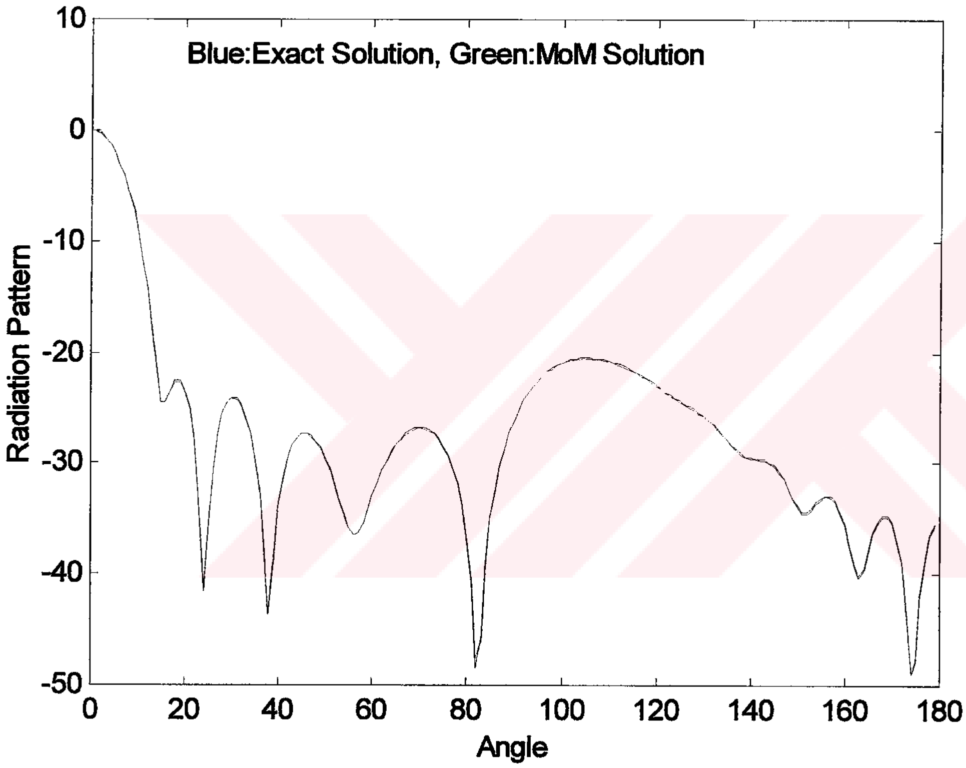

Figure 5.2 Radiation Pattern Results for E-Polarization

The difference between the exact solution and numerical solution becomes more apparent as it can be seen from Figure 5.3, which is for H-Polarization solution as the problem of convergence in the MoM which is mentioned in Chapter 3, exists even more than it does in the solution for E-Polarization. That is, solution of H-Polarization case converges slower than the solution of the E-Polarization. Moreover, the effect of edge diffraction is stronger in H-Polarization since between 140 and 160 degrees, the back-side-lobes get larger which causes this difference. However, it can be said to be a still good approximation for a numerical simulation as we have combined the MoM with the Complex Source Method by using a complex directive line source for obtaining more realistic results.

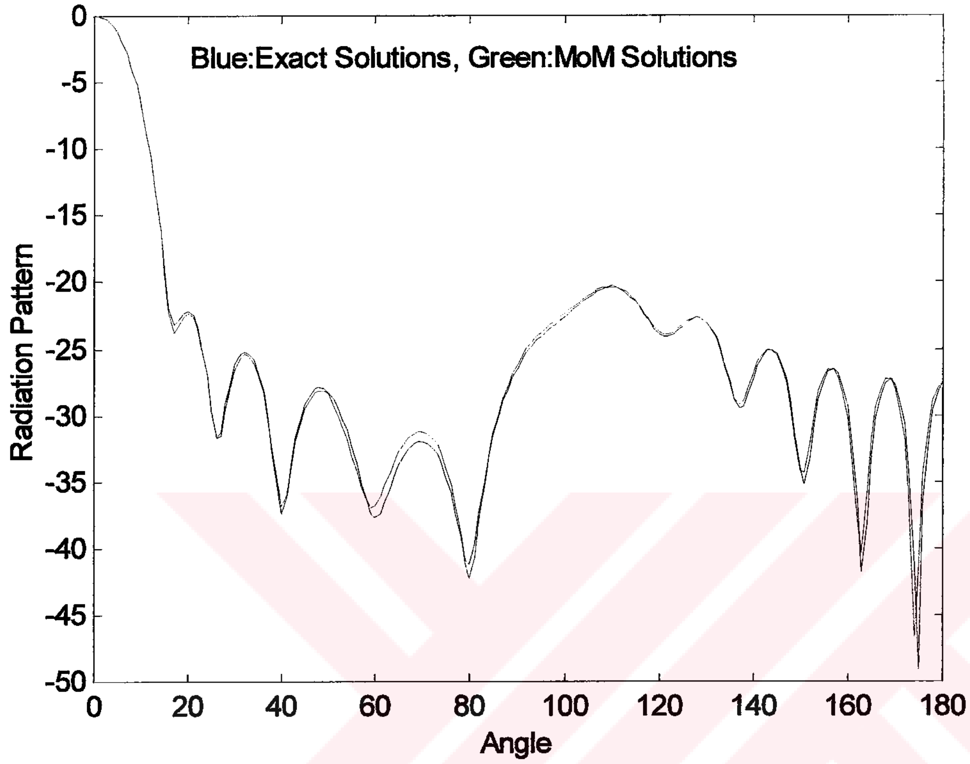

Figure 5.3 Radiation Pattern Results for H-Polarization

5.3 Solution of the problem with a larger size

We have solved the same problem for larger size, such as 150 divisions, and we saw that the amount of osscillations at the radiation pattern is increased which is caused by the increment of the size of the problem geometry. This can be seen from Figure 5.4 which is the radiation pattern for the E-Polarization case. The variation in H Solution much more exists than it does in E Solution which is again caused by the slower convergence of H Polarization case. That is, the effect of convergence becomes more important when the size of the system geometry increases, especially for H-Polarization. This can be clearly seen from the Figure 5.5, and can be compared with the previous result.

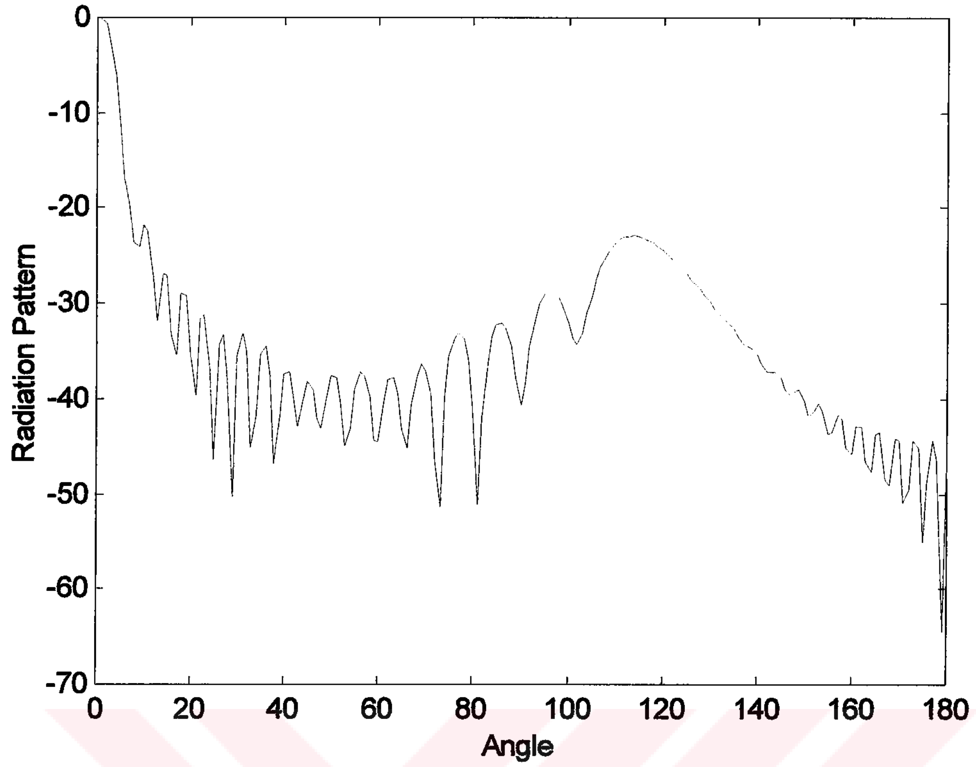

Figure 5.4 Radiation Pattern for E-Polarization for large size of geometry

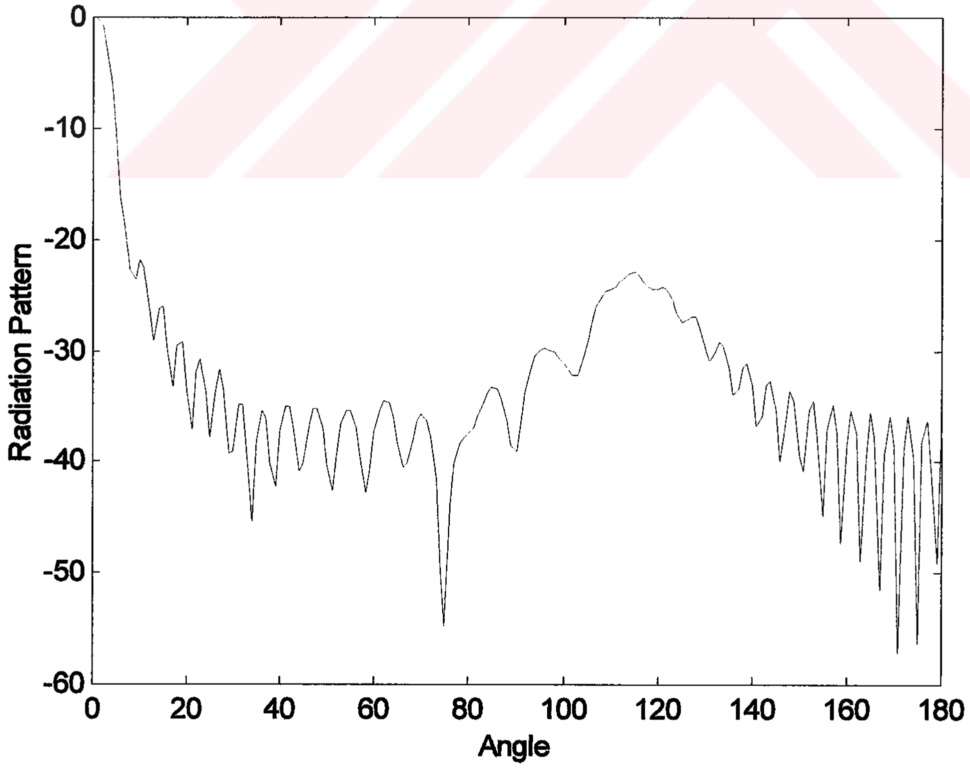

Figure 5.5 Radiation Pattern for H-Polarization for large size of geometry

5.4 Effect of convergence on the results of the induced currents

One of the most important issue in MoM, which is the convergence, can also be seen in the results of the induced current compared with the exact solution for both E and H Polarization cases. However, we should point that although we didn't impose the edge condition in this problem, we got still feasible results for the unknown induced current. As it can be seen from Figure 5.6, the numerical result with the green color is even more realistic than the other obtained with an analytic method. These results are shown below with the following two figures, namely Figure 5.6, and Figure 5.7.

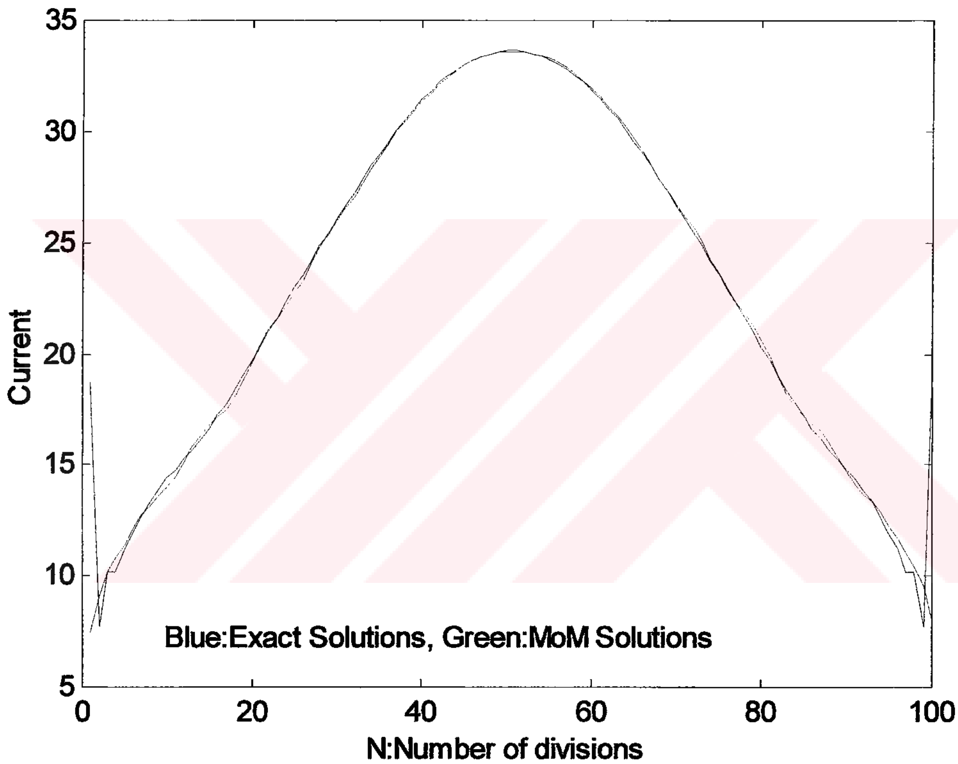

Figure 5.6 Comparison of Current Solutions for E-Polarization

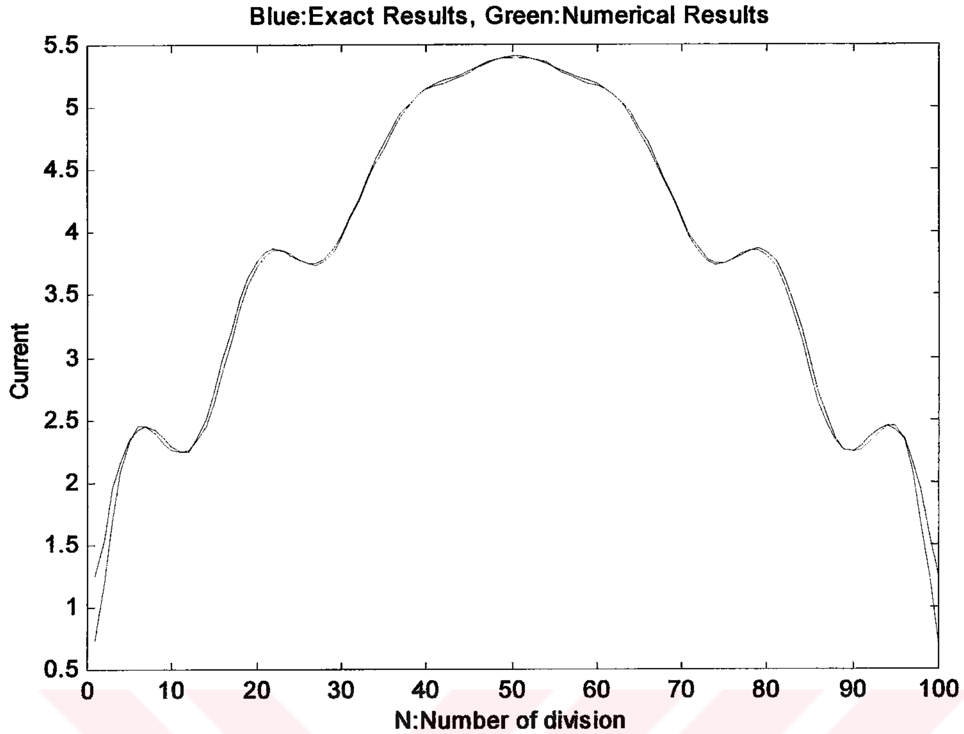

Figure 5.7 Comparison of Current Solutions for H-Polarization

5.5 Tangential Total Electric Fields

Tangential E_{total} components also ensures the boundary condition that the total tangential E field tends to be zero on the metallic part of the system as it can be seen clearly from the results that our system's boundary conditions are ensured with the obtained values on the surface of the metallic part, namely the total tangential electric fields for both E and H polarizations vanish as the sum of the incident and scattered Electric fields tends to be zero on the specified part of the antenna system. Figures 5.8 and 5.9 are the numerical solutions that we have obtained by using the induced current that we have found by MoM.

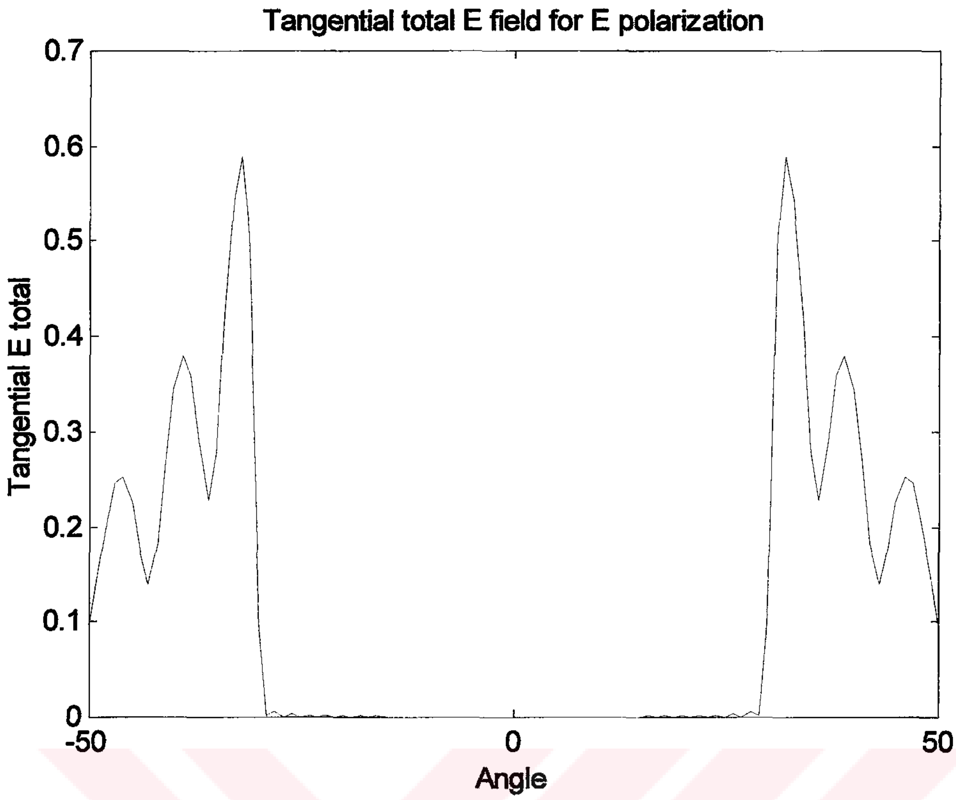

Figure 5.8 Tangential Total E field for E-Polarization

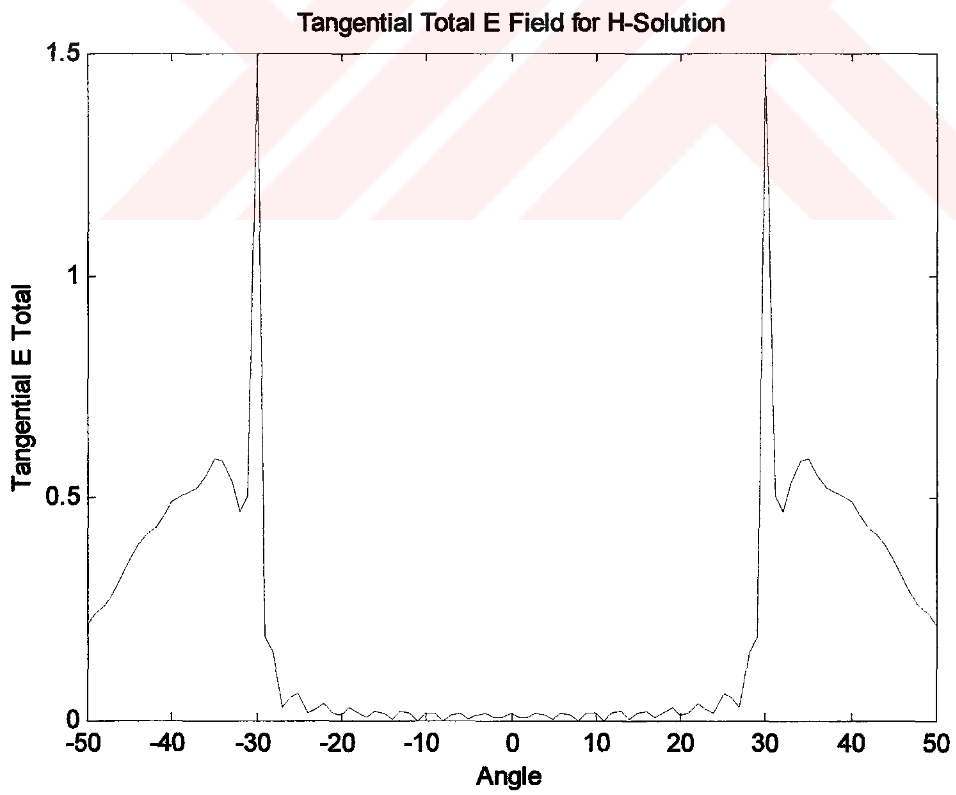

Figure 5.9 Tangential Total E field for H-Polarization

CHAPTER SIX
CONCLUSION

Scattering problem of an antenna system has been investigated before by using several methods in electromagnetics. Our goal was to solve this problem with a numerical method while trying to obtain more realistic results for a larger size of geometry than the other applications done before in the literature for problems with small geometries. Hopefully, we achieved our aim by getting even more realistic results than we expected before. Although we didn't consider the effect of edge conditions in the scattering phenomena, which is another area of study in the literature, we got even even better results than the exact solution that we compared with. This is especially very remarkable in the results of the induced currents which is mentioned in the previous chapter.

6.1 The importance of choosing proper functions

As we already stated in chapter 3 that by applying not suitable basis and testing functions, singularities can be observed in the values of the induced current on the system of interest. Therefore, testing functions has been choosen very carefully so that the inner products of these functions at the both left and right hand side of the main equation, not to get close to infinity in order to prevent singularities in the value of current induced at the edges of the reflector. In order to prevent unreasonable results, we have used triangle function as a basis and testing function in H-Polarization solution in order to resolve at least a part of the convergence problem in MoM which is already mentioned. However, we didn't have to use that function for the E-Polarization since it has no such problem with the convergence as the H-Polarization case does. Hence, using pulse function for the E-Polarization solution gives us sufficient results.

6.2 Comments on the numerical solutions obtained

As it has been predicted before, current and radiation pattern values are converged rapidly in E field solution compared to the H field solution. As we used the method of moments as the numerical solution, easier matrix equation is solved instead of solving multi-level integral equations. Therefore, as a result, numerically acceptable solutions are obtained in combination of the complex source approach with MoM. This method can be helpful for other studies in future since it's simpler and more convenient than the most of the other numerical solutions as it can be seen from the solutions and results given in the thesis.

6.3 Future work

Our problem can be also solved again by choosing the entire domain basis and testing functions. This makes the matrix size smaller, so it can be easier and faster to solve. Moreover, another study can be also worked on, that the resonance regions of the problem can be determined, and the weakness of MoM can be investigated in these regions.

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