# EFFECT OF HOLE NUMBER TO BEARING STRENGTH IN PIN LOADED LAMINATED COMPOSITE PLATES 

by
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February, 2006
İZMİR

# EFFECT OF HOLE NUMBER TO BEARING STRENGTH IN PIN LOADED LAMINATED COMPOSITE PLATES 

A Thesis Submitted to the<br>Graduate School and Applied Sciences of Dokuz Eylül University In Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering, Mechanics Program

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## Ms. Sc. THESIS EXAMINATION RESULT FORM

We have read the thesis, entitled "EFFECT OF HOLE NUMBER TO BEARING STRENGTH IN PIN LOADED LAMINATED COMPOSITE PLATES" completed by NUMAN TAYLAK under supervision of Prof. Dr. RAMAZAN KARAKUZU and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Supervisor

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# EFFECT OF HOLE NUMBER TO BEARING STRENGTH IN PIN LOADED LAMINATED COMPOSITE PLATES 


#### Abstract

The aim of this study is to research failure mode, failure load and bearing strength in a laminated glass-vinylester composite plate with two parallel circular holes, which are subjected to traction forces by two parallel rigid pins. The behaviour of pin loaded composite plates has been observed experimentally and numerically with different dimensions.


These are performed at three different modes; the distance from the free edge of the plate to diameter of holes (E/D) ratio (1,2,3,4,5), the distance between upper part of plate and centre of holes to diameter of holes $\mathrm{K} / \mathrm{D}$ ratio $(2,3,4)$ and the distance between two holes to diameter of holes $\mathrm{M} / \mathrm{D}$ ratio $(2,3,4,5)$. The failure analysis is performed numerically and experimentally and the orientation of fiber $\theta=0^{\circ}$ is constant during study.

Failure types and failure loads on the specimens have been determined from experimental study. In numerical study, three dimensional finite element method was used by assistance of LUSAS 13.6 finite element analysis program. In this program, maximum failure load is found with nonlinear analysis. Hashin failure criteria is used in this failure analysis. In the case of failure, appropriate properties of the nodes failed of the composite plate are reduced.

The experimental results are compared with the numerical results and it has been seen that a good agreement between experimental and numerical results.

Keywords: Composite Plate, Bearing Strength, Pin Loading

# PİM YÜKLEMELİ KOMPOZİT PLAKLARDA YATAK MUKAVEMETİNE DELİK SAYISININ ETKİSİ 

## ÖZ

Bu çalışmanın amacı iki paralel rijit pim tarafından değişken yayılı yüke maruz kalmış iki paralel delikli tabakalı glass-vinylester kompozit plaktaki, hasar modunu, hasar yükünü ve yatak mukavemetini araştırmaktır. Pim yüklü tabakalı kompozit plakanın davranışı, deneysel ve nümerik olarak farklı ölçülerde gözlemlenmiştir.

Bu çalışmalar, üç farklı şekilde gerçekleştirilmiştir. Plakanın uç kısmının deliğin çapına oranı (E/D); birden beşe kadar, plakanın üst kısmı ile deliğin merkezi arasındaki uzaklığın, deliğin çapına oranı (K/D); ikiden dörde kadar ve iki delik arasının, delik çapına oranı (M/D); ikiden beşe kadar. Hasar analizi, deneysel ve nümerik olarak gerçekleştirilmiştir ve fiber yönlendirme açısı $\theta=0$ sabittir.

Numunelerdeki hasar tipleri ve hasar yükleri deneysel çalışmalardan bulunmuştur. Nümerik çalışmada üç boyutlu sonlu eleman metodu, Lusas 13.6 sonlu eleman programı yardımıyla yapılmıştır. Bu program maksimum hasar yükünü nonlieer analizle bulur. Hasar analizinde Hashin hasar kriteri kullanılmıştır. Hasar durumunda kompozit plağın hasarlı düğümlerindeki malzeme özellikleri indirgenir.

Deneysel sonuçlar, nümerik sonuçlarla karşılaştırılmış ve aralarında iyi bir uyum olduğu gözlenmiştir.

Anahtar sözcükler : Kompozit Plak, Yatak Gerilmesi, Pimle Yükleme

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## NOMENCLATURE

| Abbreviation | Term |
| :---: | :---: |
| D | Hole diameter |
| E | End distance from the hole center |
| W | Width of the plate |
| K | Distance between two holes |
| $t$ | Thickness of the plate |
| L | Distance from hole center to fixed end |
| $a, b, c, t_{i}$ | Dimensions of Iosipescu testing specimen |
| $P$ | Tensile load |
| $P_{u l t}$ | Maximum failure load |
| $\theta$ | Fiber orientation angle |
| $E_{i j}$ | Elastic moduli in material directions |
| $G_{i j}$ | Shear moduli |
| $v_{12}$ | Poisson's ratio |
| $V_{f}$ | Fiber volume friction |
| $\sigma_{b}$ | Bearing strength |
| $X_{t}$ | Tensile strength in the fibre direction |
| $X_{c}$ | Compressive strength in the fibre direction |
| $Y_{t}$ | Tensile strength in the transverse direction |
| $Y_{\text {c }}$ | Compressive strength in the transverse direction |
| $S$ | Shearing strength |
| $\varepsilon_{i j}$ | Strains |
| $\sigma_{i j}$ | Stress |
| $\left[\bar{C}_{i j}\right]$ | Reduced-stiffness matrix |
| $\left[C_{i j}\right]$ | Inverse of compliance matrix |
| [ $S_{i j}$ ] | Compliance matrix |
| $\mathrm{u}, \mathrm{v}, \mathrm{w}$ | Displacement component |

## CHAPTER ONE

## INTRODUCTION

Composite materials are highly used in structures when high strength to weight and stiffness to weight ratios are required. Because of this, application areas of composite materials have increased in recent years. There are a lot of different techniques for joining composite members. Some of them are bonding and bolted. The use of bolted joints is promising technique since it is easier and more economical than others. It is often used due to this materials are easy for disassembly. However, mechanical fastened joints require holes to be drilled and therefore large stress zones tend to develop. Because of anisotropic and heterogeneous nature, the joint problem in composites is more difficult to analyze than the case with isotropic materials. For this reason, finding and improving to new design methods is very important to avoid cost penalties and weaknesses.

A large part of the literature published so far on mechanically fastened joints present experimental results on the effect of the dimensions, clearance between the hole and the pin, and the stacking sequence. Several numerical methods have also been investigated to predict failure of pinned joints. Most of them are reviewed in detail by Camanho \& Matthews (1997). Kretsis \& Matthews (1985) showed, using E glass fiber-reinforced plastic and carbon fiber reinforced plastic, that as the width of the specimen decreases, there is a point where the made of failure changes from one of bearing to one of tension. A similar behaviour between the end distance and the shear-out mode of failure was found. They concluded that lay-up had a great effect on both joint strength and failure mechanism. A computer code which can be used calculated the maximum load has been developed by Chang et al. (1982). In that work Yamada failure criterion has been used. Then Chang et al. (1984a) have developed their analysis to T300/T300/1034-C laminates containing a pin loaded hole or two pin loaded holes in series or in series. Chang et al (1984b) have developed a model and corresponding computer code to determine failure strength and failure mode of composite laminates containing a pin loaded hole even when the material exhibits nonlinearly elastic behaviour. Chang (1986) has carried out a study
to evaluate the effect of the assumed pin load distribution. The calculation have utilized a finite element method of stress analysis combined with the Yamata-Sun failure criterion applied along the Chang-Scott- Springer characteristic curve. A three dimensional finite element model to perform stress analysis of single and multibolted double shear lap connections of glass fiber reinforced plastic has been used by Hassan et al. (1996) with using ANSYS program.

Aktas \& Karakuzu (1999) have investigated the strength of mechanically fastened carbon fiber reinforced epoxy composite plate at the different arbitrary orientations. They have analyzed failure load and failure mode numerically and experimentally by using Tsai-Hill and fiber tensile compressive failure criteria. Icten \& Karakuzu (2002) have investigated to prediction of the behaviours of the pined joint carbon epoxy composite plates. In that work Hashin and Hoffman criteria was used to determine failure load and failure mod. Icten \& Sayman (2003) have investigated failure load and failure mod in an aluminum glass epoxy sandwich composite plate which is subjected to a traction force by a pin. Parametric studies were carried out experimentally to obtain the effects of join geometry and fiber orientation on the failure strength and failure mode. Okutan, Aslan \& Karakuzu (2001) have studied the effects of woven fiber, specimen with-to-hole diameter (W/D) and the ratio of edge distance to hole diameter (E/D) on the bearing strength of woven laminated composites. They have tested single-hole pin loaded specimens for their tensile response. They have observed failure propagation and failure type on the specimens. Gülem, Içten \& Karakuzu (2004) have investigated the bearing strength and failure analysis of woven laminated glass-vinylester. In this study, effect of holes has been investigated for the different geometries.

Kim et al (1998) have performed a progressive failure analysis to predict the failure loads of pin loaded composites. Camanho \& Matthews (1999) have improved a 3D finite element model to predict damage progression and strength of mechanically fastened joints in carbon fiber reinforced plastics. In that work Hashin failure criteria has been used to predict the failure mode.

Lessard \& Shokrieh (1995) have numerically investigated the damage modeling of pin loaded composite. In that work fiber tensile compressive shearing, matrix tensile compressive and fiber matrix shearing criterias have been used.

Hung et al (1996) have investigated failure analysis of T800/3900-2 graphiteepoxy materials by using Hashin failure criteria.

Pierron \& Cerisier (2000) have performed a numerical and experimental study to determine the bearing strength of bolted woven composite joints. Hamada \& Maekawa (1996) have investigated failure analysis of quasi isotropic carbon epoxy T300/\#2500 laminates numerically and experimentally.

Dano et al. (2000) have examined progressive failure analysis of pin loaded composite plate to predict the bearing stress pin displacement curve until joint failure occurs. In that analysis, contact between the pin and hole, progressive damage, large deformation theory and a non linear shear stress strain relationship have been investigated.

This study is concerned with the bearing strength, failure mode and failure load in pin loaded which is subjected to traction force by rigid pins glass-vinylester laminated composite plate. The failure mode and bearing strength have been examined numerically and experimentally. To determine the failure mode and failure load, a three dimensional finite element method has been used. The effects of changing the geometric parameters are observed.

## CHAPTER TWO

## MACROMECHANICAL BEHAVIOUR OF A LAMINA

### 2.1 Laminated Composite Materials

Laminated composite materials consist of layers of at least two different materials that are bonded together. Lamination is used to combine the best aspects of the constituent layers and bonding material in order to achieve a more useful constituent layers and bonding material. The properties that can be emphasized by lamination are strength, stiffness, low weight, corrosion resistance, wear resistance, beauty or attractiveness, thermal insulation, acoustical insulation, etc. (Johns, 1999)

### 2.1.1 Lamina

A lamina is a single ply in laminate, which is made up of a series of layers. The basic building block of a lamina is a lamina which is a flat arrangement of unidirectional fibers or woven in a matrix. Laminated composite materials typically have exceptional properties in the direction of the reinforcing fibers, but poor to mediocre properties to the fibers. The problem is how to obtain maximum advantage from the exceptional fiber directional properties while minimizing the effects of the low transverse properties.

A laminate consists of multiple layers of lamina with unique orientation. A typical laminate is shown in Figure 2.1. Mostly the fiber orientation of the layers is not symmetric as shown in Figure 2.1. As a result of this, the laminate may not have definable only principal directions.


Figure 2.1Construction of a laminate

### 2.2 Stress Analysis

In most cases an accurate understanding of the loads and stress levels in component operations is one of the critical involved in defining source of failure. Even though other methods of analysis may identify the origin and mode of crack propagation, stress analysis most often provides a quantitative explanation for the cause of failure. Through this analysis step, engineers involved in future or corrective redesigns are provided direct feedback regarding the actual loads experienced by the part, poor design practices and configurations, and the effectiveness of the analysis methods used in design.

Stress analysis procedures for composite materials can be relatively complex, due to the several factors. Because composite materials are fabricated by the lamination of highly anisotropic plies, a nearly infinite variety of directional module and strength can be achieved. Because of this flexibility, a different set of material properties must be considered for each failure case being examined.

### 2.2.1 Stress -Strain Relations for a Lamina

A unidirectional ply is shown in Figure 2.2, along with a coordinate system used to establish notation. Here directions 1 and 2 indicate to the fiber directions and transverse to the fibers in the plane of the ply, and direction 3 refers to the through the thickness direction.


Figure 2.2 Unidirectional fiber reinforced ply

Stress-strain relations can be expressed in matrix form as,

$$
\left\{\begin{array}{c}
\varepsilon_{1}  \tag{2.1}\\
\varepsilon_{2} \\
\varepsilon_{3} \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{array}\right\}=\left[\begin{array}{cccccc}
\frac{1}{E_{1}} & \frac{-v_{21}}{E_{2}} & \frac{-v_{31}}{E_{3}} & 0 & 0 & 0 \\
\frac{-v_{12}}{E_{1}} & \frac{1}{E_{2}} & \frac{-v_{32}}{E_{3}} & 0 & 0 & 0 \\
\frac{-v_{13}}{E_{1}} & \frac{-v_{23}}{E_{2}} & \frac{1}{E_{3}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}
\end{array}\right]\left\{\begin{array}{l}
\sigma_{1} \\
\sigma_{2} \\
\sigma_{3} \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{array}\right\}
$$

or

$$
\begin{equation*}
\{\varepsilon\}=[S]\{\sigma\} \tag{2.2}
\end{equation*}
$$

The S matrix is often called to as the compliance matrix for the lamina, or the strain-stress form of material properties with the strains as the dependent variables. It can be shown that the matrices describing the stress-strain as the dependent variables. It can be shown that the matrices describing the stress-strain relationship of an elastic material must be symmetric, so that relationships such as,

$$
\begin{equation*}
E_{1} v_{21}=E_{2} v_{12} \text { or } \frac{v_{12}}{E_{1}}=\frac{v_{21}}{E_{2}} \tag{2.3}
\end{equation*}
$$

Hold for the off-diagonal terms, so that only nine material properties are required to fully characterize the linear behaviour of a lamina in 3-D stress and strain states. The zeros in the compliance matrix reflect the fact that it is describing the stressstrain behaviour of an orthotropic material and that the description is made with respect to the principal material axes.

The stress-strain matrix can be inverted to obtain stress-strain relations:

$$
\left\{\begin{array}{c}
\sigma_{1}  \tag{2.4}\\
\sigma_{2} \\
\sigma_{3} \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{array}\right\}=\left[\begin{array}{cccccc}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{array}\right]\left\{\begin{array}{l}
\varepsilon_{1} \\
\varepsilon_{2} \\
\varepsilon_{3} \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{array}\right\}
$$

The stiffness matrix, $C_{i j}$, for an orthotropic materials in terms of the engineering constants, is obtained by inversion of the compliance matrix, $S_{i j}$. The stiffnesses in Eq. (2.3) are,

$$
\begin{array}{ll}
C_{11}=\frac{1-v_{23} v_{32}}{E_{2} E_{3} \Delta}, \quad C_{22}=\frac{1-v_{13} v_{31}}{E_{1} E_{3} \Delta}, & C_{12}=\frac{v_{21}+v_{31} v_{23}}{E_{2} E_{3} \Delta}=\frac{v_{12}+v_{32} v_{13}}{E_{1} E_{3} \Delta}, \\
C_{23}=\frac{v_{32}+v_{12} v_{31}}{E_{1} E_{3} \Delta}=\frac{v_{23}+v_{21} v_{13}}{E_{1} E_{2} \Delta}, & C_{13}=\frac{v_{31}+v_{21} v_{32}}{E_{2} E_{3} \Delta}=\frac{v_{13}+v_{12} v_{23}}{E_{1} E_{2} \Delta},
\end{array}
$$

$$
\begin{equation*}
C_{33}=\frac{1-v_{12} v_{21}}{E_{1} E_{2} \Delta}, \quad C_{44}=G_{23}, \quad C_{55}=G_{31}, \quad C_{66}=G_{12} \tag{2.5}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta=\frac{1-v_{12} v_{21}-v_{23} v_{32}-v_{31} v_{13}-2 v_{21} v_{32} v_{13}}{E_{1} E_{2} E_{3}} \tag{2.6}
\end{equation*}
$$

### 2.2.2 Stress-Strain Relations for a Lamina of Arbitrary Orientation

The stress-strain relation has been explained in the principal material coordinates on previous section. However, the principal directions of orthotrophy often do not coincide with coordinate directions that are geometrically natural to the solution of the problem. Due to this cause a method of transforming stress-strain relations from one coordinate to another system is required.

The transformation of stress matrix is,

$$
\left\{\begin{array}{l}
\sigma_{x}  \tag{2.7}\\
\sigma_{y} \\
\sigma_{z} \\
\tau_{y z} \\
\tau_{x z} \\
\tau_{x y}
\end{array}\right\}=\left[\begin{array}{cccccc}
\cos ^{2} \theta & \sin ^{2} \theta & 0 & 0 & 0 & -\sin 2 \theta \\
\sin ^{2} \theta & \cos ^{2} \theta & 0 & 0 & 0 & \sin 2 \theta \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \theta & \sin \theta & 0 \\
0 & 0 & 0 & -\sin \theta & \cos \theta & 0 \\
\sin \theta \cos \theta & -\sin \theta \cos \theta & 0 & 0 & 0 & \cos \theta-\sin ^{2} \theta
\end{array}\right]\left[\begin{array}{c}
\sigma_{1} \\
\sigma_{2} \\
\sigma_{3} \\
\tau_{23} \\
\tau_{13} \\
\tau_{12}
\end{array}\right\}
$$

$\theta$ is the angle between principal directions (1-2-3) and coordinate directions ( $x-y-$ z).

The stress-strain relation in $x-y-z$ coordinates are,

$$
\left\{\begin{array}{c}
\sigma_{x}  \tag{2.8}\\
\sigma_{y} \\
\sigma_{z} \\
\tau_{y z} \\
\tau_{x z} \\
\tau_{x y}
\end{array}\right\}=\left[\begin{array}{cccccc}
\bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 & \bar{C}_{16} \\
\bar{C}_{21} & \bar{C}_{22} & \bar{C}_{23} & 0 & 0 & \bar{C}_{26} \\
\bar{C}_{31} & \bar{C}_{32} & \bar{C}_{33} & 0 & 0 & \bar{C}_{36} \\
0 & 0 & 0 & \bar{C}_{44} & \bar{C}_{45} & 0 \\
0 & 0 & 0 & \bar{C}_{45} & \bar{C}_{55} & 0 \\
\bar{C}_{16} & \bar{C}_{26} & \bar{C}_{36} & 0 & 0 & \bar{C}_{66}
\end{array}\right]\left\{\begin{array}{l}
\varepsilon_{x} \\
\varepsilon_{y} \\
\varepsilon_{z} \\
\gamma_{y z} \\
\gamma_{x z} \\
\gamma_{x y}
\end{array}\right\}
$$

The transformed compliance coefficients $\bar{C}_{i j}$, indicated to the ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) system,

$$
\begin{aligned}
\bar{C}_{11}= & C_{11} \cos ^{4} \theta-4 C_{16} \cos ^{3} \theta \sin \theta+2\left(C_{12}+C_{66}\right) \cos ^{2} \theta \sin ^{2} \theta-4 C_{26} \cos \theta \sin ^{3} \theta \\
& +C_{22} \sin ^{4} \theta
\end{aligned}
$$

$$
\bar{C}_{12}=C_{12} \cos ^{4} \theta+2\left(C_{16}-C_{26}\right) \cos ^{3} \theta \sin \theta+\left(C_{11}+C_{12}-4 C_{66}\right) \cos ^{2} \theta \sin ^{2} \theta
$$

$$
+2\left(C_{26}-C_{16}\right) \cos \theta \sin ^{3} \theta+C_{12} \sin ^{4} \theta
$$

$\bar{C}_{13}=C_{13} \cos ^{2} \theta-2 C_{36} \cos \theta \sin \theta+C_{23} \sin ^{2} \theta$
$\bar{C}_{16}=C_{16} \cos ^{4} \theta+\left(C_{11}-C_{12}-2 C_{66}\right) \cos ^{3} \theta \sin \theta+3\left(C_{26}-C_{16}\right) \cos ^{2} \theta \sin ^{2} \theta$ $+\left(2 C_{66}+C_{12}-C_{22}\right) \cos \theta \sin ^{3} \theta-C_{26} \sin ^{4} \theta$
$\bar{C}_{22}=C_{22} \cos ^{4} \theta+4 C_{26} \cos ^{3} \theta \sin \theta+2\left(C_{12}+2 C_{66}\right) \cos ^{2} \theta \sin ^{2} \theta$ $+4 C_{16} \cos \theta \sin ^{3} \theta+C_{11} \sin ^{4} \theta$
$\bar{C}_{23}=C_{23} \cos ^{2} \theta+2 C_{36} \cos \theta \sin \theta+C_{13} \sin ^{2} \theta$
$\bar{C}_{26}=C_{26} \cos ^{4} \theta+\left(C_{12}-C_{22}+2 C_{66}\right) \cos ^{3} \theta \sin \theta+3\left(C_{16}-C_{26}\right) \cos ^{2} \theta \sin ^{2} \theta$ $+\left(C_{11}-C_{12}-2 C_{66}\right) \cos \theta \sin ^{3} \theta-C_{16} \sin ^{4} \theta$
$\bar{C}_{33}=C_{33}$

$$
\begin{align*}
& \bar{C}_{36}=\left(C_{13}-C_{23}\right) \cos \theta \sin \theta+C_{36}\left(\cos ^{2} \theta-\sin ^{2} \theta\right) \\
& \begin{aligned}
\bar{C}_{44} & =C_{44} \cos ^{2} \theta+C_{55} \sin ^{2} \theta+2 C_{45} \cos \theta \sin \theta \\
\bar{C}_{45} & =C_{45}\left(\cos ^{2} \theta-\sin ^{2} \theta\right)+\left(C_{55}-C_{44}\right) \cos \theta \sin \theta \\
\bar{C}_{55}= & C_{55} \cos ^{2} \theta+C_{44} \sin ^{2} \theta-2 C_{45} \cos \theta \sin \theta \\
\begin{array}{r}
C_{66}
\end{array} & =2\left(C_{16}-C_{26}\right) \cos 3 \sin \theta+\left(C_{11}+C_{22}-2 C_{12}-2 C_{66}\right) \cos ^{2} \theta \sin ^{2} \theta \\
& +2\left(C_{26}-C_{16}\right) \cos \theta \sin ^{3} \theta+C_{66}\left(\cos ^{4} \theta+\sin ^{4} \theta\right)
\end{aligned}
\end{align*}
$$

Note that $\mathrm{C}_{14}, \mathrm{C}_{15}, \mathrm{C}_{16}, \mathrm{C}_{24}, \mathrm{C}_{25}, \mathrm{C}_{26}, \mathrm{C}_{34}, \mathrm{C}_{35}, \mathrm{C}_{36}, \mathrm{C}_{45}, \mathrm{C}_{46}$, and $\mathrm{C}_{56}$ are zero for an orthotropic material.

The relation of stress-strain for the $x-y-z$ coordinates are,

$$
\left\{\begin{array}{l}
\varepsilon_{x}  \tag{2.10}\\
\varepsilon_{y} \\
\varepsilon_{z} \\
\gamma_{y z} \\
\gamma_{x z} \\
\gamma_{x y}
\end{array}\right\}=\left[\begin{array}{cccccc}
\bar{S}_{11} & \bar{S}_{12} & \bar{S}_{13} & 0 & 0 & \bar{S}_{16} \\
\bar{S}_{21} & \bar{S}_{22} & \bar{S}_{13} & 0 & 0 & \bar{S}_{26} \\
\bar{S}_{31} & \bar{S}_{32} & \bar{S}_{33} & 0 & 0 & \bar{S}_{36} \\
0 & 0 & 0 & \bar{S}_{44} & \bar{S}_{45} & 0 \\
0 & 0 & 0 & \bar{S}_{45} & \bar{S}_{55} & 0 \\
\bar{S}_{16} & \bar{S}_{26} & \bar{S}_{36} & 0 & 0 & \bar{S}_{66}
\end{array}\right]\left\{\begin{array}{c}
\sigma_{x} \\
\sigma_{y} \\
\sigma_{z} \\
\tau_{y z} \\
\tau_{x z} \\
\tau_{x y}
\end{array}\right\}
$$

The transformed compliance coefficients $\bar{S}_{i j}$, indicated to the ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) system,

$$
\begin{aligned}
\bar{S}_{11}= & S_{11} \cos ^{4} \theta-2 S_{16} \cos ^{3} \theta \sin \theta+\left(2 S_{12}+S_{66}\right) \cos ^{2} \theta \sin ^{2} \theta-2 S_{26} \cos \theta \sin ^{3} \theta \\
& +S_{22} \sin ^{4} \theta \\
\bar{S}_{12}= & S_{12} \cos ^{4} \theta+\left(S_{16}-S_{26}\right) \cos ^{3} \theta \sin \theta+\left(S_{11}+S_{22}-S_{66}\right) \cos ^{2} \theta \sin ^{2} \theta \\
& +\left(S_{26}-S_{16}\right) \cos \theta \sin ^{3} \theta+S_{12} \sin ^{4} \theta
\end{aligned}
$$

$$
\begin{align*}
\bar{S}_{13}= & S_{13} \cos ^{2} \theta-S_{36} \cos \theta \sin \theta+S_{23} \sin ^{2} \theta \\
\bar{S}_{16}= & S_{16} \cos ^{4} \theta+\left(2 S_{11}-2 S_{12}-S_{66}\right) \cos ^{3} \theta \sin \theta+3\left(S_{26}-S_{16}\right) \cos ^{2} \theta \sin ^{2} \theta \\
& +\left(S_{66}+2 S_{12}-2 S_{22}\right) \cos \theta \sin ^{3} \theta-S_{26} \sin ^{4} \theta \\
\bar{S}_{22}= & S_{22} \cos ^{4} \theta+2 S_{26} \cos ^{3} \theta \sin \theta+\left(2 S_{12}+S_{66}\right) \cos ^{2} \theta \sin ^{2} \theta+2 S_{16} \cos \theta \sin ^{3} \theta \\
& +S_{11} \sin ^{4} \theta \\
\bar{S}_{23}= & S_{23} \cos ^{2} \theta+S_{36} \cos \theta \sin \theta+S_{13} \sin ^{2} \theta \\
\bar{S}_{26}= & S_{26} \cos ^{4} \theta+\left(2 S_{12}-2 S_{22}+S_{66}\right) \cos ^{3} \theta \sin \theta+3\left(S_{16}-S_{26}\right) \cos ^{2} \theta \sin ^{2} \theta \\
& +\left(2 S_{11}-2 S_{12}-S_{66}\right) \cos \theta \sin ^{3} \theta-S_{16} \sin { }^{4} \theta \\
\bar{S}_{33}= & S_{33} \\
\bar{S}_{36}= & 2\left(S_{13}-S_{23}\right) \cos \theta \sin \theta+S_{36}\left(\cos ^{2} \theta-\sin ^{2} \theta\right) \\
\bar{S}_{44}= & S_{44} \cos ^{2} \theta+2 S_{45} \cos \theta \sin \theta+S_{55} \sin 2 \theta \\
\bar{S}_{45}= & S_{45}\left(\cos ^{2} \theta-\sin ^{2} \theta\right)+\left(S_{55}-S_{44}\right) \cos ^{2} \theta \sin ^{2} \theta \\
\bar{S}_{55}= & S_{55} \cos ^{2} \theta+S_{44} \sin 2 \theta-2 S_{45} \cos \theta \sin ^{2} \theta \\
\bar{S}_{66}= & S_{66}\left(\cos ^{2} \theta-\sin ^{2} \theta\right)^{2}+4\left(S_{16}-S_{26}\right)\left(\cos ^{2} \theta-\sin ^{2} \theta\right) \cos \theta \sin \theta  \tag{2.11}\\
& +4\left(S_{11}+S_{22}-2 S_{12}\right) \cos { }^{2} \theta \sin ^{2} \theta
\end{align*}
$$

### 2.3 Failure Analysis

The rapid advancement of composite materials over the past two decades has outstripped the development of appropriate failure analysis techniques. This is particularly true of the fiber composite material systems used in primary structural applications in any industries. Although some of the knowledge gained over the years in performing failure analysis on metals is applicable to composites, the fundamentally different nature of the two materials prohibits the widespread transfer of information.

Numerous failure theories have been proposed and are available to the composite structural designer. They are classified into three groups, limit or noninteractive theories (maximum stress, maximum strain); interactive theories (Tsai-Hill, TsaiWu ); and partially interactive or failure mode based theories (Hashin).

The validity and applicability of a given theory depend on the convenience of application and agreement with experimental results. The plethora of theories is accompanied by a dearth of suitable and reliable experimental data, which makes the selection of one theory over another rather difficult. Considerable effort has been devoted recently to alleviate this difficulty.

### 2.3.1 Hashin Failure Criterion

In this study, Hashin failure criteria (1980) has been used due to this criterion is a preference for use in finite element models.

Two proposals of failure criterion for fibrous composite materials that are associated with Hashin may be found in the literature. The first reference 1 of the review is known as the Hashin- Rotem criterion. This criterion predicts failure when one of the following equations is satisfied.

### 2.3.1.1 Hashin \& Rotem Criterion (1973)

Fiber Failure in Tension
$\sigma_{11}=X_{t} \quad\left(\sigma_{11}, X_{t}>0\right)$

Fiber Failure in Compression
$-\sigma_{11}=X_{t} \quad\left(\sigma_{11}<0 ; X_{c}>0\right)$

Matrix Failure in Tension
$\left(\frac{\sigma_{22}}{Y_{t}}\right)^{2}+\left(\frac{\tau_{12}}{S}\right)^{2}=1$

Matrix Failure in Compression
$\left(\frac{\sigma_{22}}{Y c}\right)^{2}+\left(\frac{\tau_{12}}{S}\right)^{2}=1$
where
$\sigma_{11}$ is the nominal stress in the lamina in the direction of the fibers.
$\sigma_{22}$ is the nominal stress in the lamina in the transverse direction to the fibers.
$\tau_{12}$ is the nominal shear stress in the plane of the lamina.
$X_{t}$ is the tensile strength of the fibers.
$Y_{t}$ is the tensile strength in the transverse direction of the fibers.
$X_{c}$ is the compressive strength of the fibers.
$Y_{c}$ is the compression strength in the transverse direction of the fibers.
S is the shear strength.

Based on observations of specimen failure in tension with different orientations of the fibers, the author of this proposal concludes that there are only two mechanisms of failure; fiber or matrix failure. With reference to the second, they do not distinguish whether the failure is exactly at the interface or inside the matrix and thus propose that both $\sigma_{2}$ and $\sigma_{12}$ contribute to the appearance of the failure (the proposal is in quadratic form).

The historical importance of this proposal is that initiates a different way of approaching the generation of composites failure criteria. The authors first set out to recognize modes of failure, then to understand the variables with these nodes and propose and interaction between them.

The idea seems adequate for the type of materials under consideration; although it may be argued that not all failure modes that can appear in fibrous composites are covered in the proposal. It is also not clear that the variables they propose for each case are the most appropriate or in what they combine them.

In 1980, Hashin re-examined the proposal and established some modifications. There are also four expressions involved in the proposal that Hashin developed for the 3D case.

Hashin Criterion 3D (1980)

Tensile Fiber Mode
$\left(\frac{\sigma_{1}}{X_{t}}\right)^{2}+\frac{1}{S^{2}}\left(\tau_{12}^{2}+\tau_{13}^{2}\right)=1$
or

$$
\begin{equation*}
\sigma_{11}=X_{t} \tag{2.17}
\end{equation*}
$$

$$
\begin{equation*}
\left|\sigma_{11}\right|=X_{c} \tag{2.18}
\end{equation*}
$$

Tensile Matrix Mode $\left(\sigma_{22}+\sigma_{33}>0\right)$

$$
\begin{equation*}
\frac{1}{Y_{T}^{2}}\left(\sigma_{22}+\sigma_{33}\right)^{2}+\frac{1}{S_{T}^{2}}\left(\tau_{23}^{2}-\sigma_{22} \sigma_{33}\right)+\frac{1}{S^{2}}\left(\tau_{12}^{2}+\tau_{13}^{2}\right)=1 \tag{2.19}
\end{equation*}
$$

Compressive Matrix Mode $\left(\sigma_{22}+\sigma_{33}\right)<0$

$$
\begin{align*}
& \frac{1}{Y_{C}}\left[\left(\frac{Y_{C}}{2 S_{T}}\right)^{2}-1\right]\left(\sigma_{2}+\sigma_{3}\right)+\frac{1}{4 S_{T}^{2}}\left(\sigma_{2}+\sigma_{3}\right)^{2}+\frac{1}{S_{T}^{2}}\left(\tau_{23}^{2}-\sigma_{2} \sigma_{3}\right)  \tag{2.20}\\
& +\frac{1}{S^{2}}\left(\tau_{12}^{2}+\tau_{13}^{2}\right)=1
\end{align*}
$$

where in addition to the previous definitions, $\mathrm{S}_{\mathrm{T}}$ represents the transverse shear strength, the allowable value of shear stress $\tau_{23}$ (the allowable value of $\tau_{13}$ is, as for $\left.\tau_{12}, S\right)$.

## CHAPTER THREE

## NUMERICAL STUDY

### 3.1 Introduction

Numerical analysis techniques, such as finite element analysis (FEA) are used extensively in the design and stress analysis of adhesively bonded and bolted structures. These techniques offer solutions to complex problems that are too difficult or impossible to resolve using analytical, closed-form solutions. Numerous FEA programs are available. These programs provide in-built constitutive models for simulating the behavior of most adhesive, allowing for non-uniform stress-strain distributions, geometric non-linearity, hydrothermal effects, elastic-plastic behavior, static and dynamic analysis, and strain rate dependence. Orthotropic element types include two dimensional solid plane stress or plain strain elements, axisymetric shell or solid elements, three dimensional solid or "brick" elements and crack tip elements. A number of automatic mesh (element) generators are available with post processing capabilities.

### 3.2 Three Dimensional Finite Element Method

In the three-dimensional finite element formulation, the displacements, traction components, and distributed body force values are the functions of the position indicated by $(x, y, z)$. The displacement vector $\mathbf{u}$ is given as (Chandrupatla, 1991)

$$
\begin{equation*}
\mathbf{u}=[u, v, w]^{T} \tag{3.1}
\end{equation*}
$$

where $u, v$ and w are the $x, y$ and $z$ components of $\mathbf{u}$, respectively. The stress and strains are given by

$$
\begin{align*}
& \sigma=\left[\sigma_{x x}, \sigma_{y y}, \sigma_{z z}, \sigma_{y z}, \sigma_{x z}, \sigma_{x y}\right]^{T} \\
& \varepsilon=\left[\varepsilon_{x x}, \varepsilon_{y y}, \varepsilon_{z z}, \gamma_{y z}, \gamma_{x z}, \gamma_{x y}\right]^{T} \tag{3.2}
\end{align*}
$$

From Figure 3.1, representing the three- dimensional problem in a general setting, the body force and traction vector are given by

$$
\begin{equation*}
\mathbf{f}=\left[f_{x}, f_{y}, f_{z}\right]^{T} \quad, \mathbf{T}=\left[T_{x}, T_{y}, T_{z}\right]^{T} \tag{3.3}
\end{equation*}
$$

The body force $\mathbf{f}$ has dimensions of force per unit volume, while the traction force $\mathbf{T}$ has dimensions of force per unit area.


Figure 3.1 Three-dimensional problem

### 3.3 The sixteen-Node Brick Element

In this study, the sixteen-node brick element was used. A typical sixteen-node brick element is shown in figure 3.1. A 3D isoparametric solid continuum element capable of modeling curved boundaries. The element is numbered according to right hand screw rule in the local $z$ direction. Freedoms of the element are $u, v, w$ at each node and node coordinates are $\mathrm{x}, \mathrm{y}, \mathrm{z}$ at each node. (Gülem, 2004)


Figure 3.2 Sixteen-node brick element

### 3.4 Modeling of the Problem in Finite Element Program

The maximum failure load values have been found by nonlinear analysis in Lusas 13.6 finite element program. First of all composite plate was modeled as a half model and symmetry boundary conditions were used to reduce to size of the model. Then mesh has been graded manually by specifying the number of elements on each of the boundary lines. After that, the surface was swept through the depth of the plate to create a volume. One element only is required through the depth of the plate the default number of mesh divisions must be set to one.

Then translation value of the surface was defined. The translation direction is Z and its value is 2.8 mm . Next we assigned the volume with mesh dataset Composite Brick (HX16L), this element descriptions are generic element type (Structural Composite), element shape (Hexahedral), interpolation order (Quadratic). After that,
the element axis of the model oriented to lie along the global X axis. The meshing half model was shown in figure 3.3.


Figure 3.3 Meshing of the half model

After completed above parts, model properties were defined. In this section of program, orthotropic material was defined, the mechanical properties of glassvinylester composite material was added to composite library, leave the units $\mathrm{N}, \mathrm{mm}$, t , C, s. 3D solid is chosen and the option to output parameters were selected for the Hashin Damage model. Mechanical properties of glass-vinylester are shown in Table 4.2.


Figure 3.4 Supported surfaces on symmetry XZ plane

After that, support conditions of the model were defined. Firstly, the bottom surfaces of the half model have been supported on symmetry XZ plane as shown in Figure in 3.4. Secondly, cylindrical axis is performed surfaces on semi cylinder. These surfaces of the plate were supported in X direction as shown in figure 3.5

Finally, tensile load was performed one by one to per unit on surface as shown in figure 3.6 and CTRL and A keys were used together whole model was selected and assigned, the element axis was selected and click OK to finish this part of the study.


Figure 3.5 Supported surfaces in X direction with respect to cylindrical


Figure 3.6 Tensile load direction

## CHAPTER FOUR

## EXPERIMENTAL STUDY

### 4.1 Problem Statement

Consider a composite rectangular plate of length $\mathrm{L}+\mathrm{E}, \mathrm{L}$ is fixed at a constant value 85 mm , width W with two hole of diameter D , the hole diameters were fixed at a constant 5 mm , the distance between holes is M , the distance from upper part of plate to centre of hole is K , as depicted in Figure 4.1. The holes are at a distance E from the free edge of the plate. Rigid pins are located at the centre of the holes. A tensile load is applied at one edge of the plate and is resisted the pins.


Figure 4.1 Geometry of a specimen

Depending on the geometry, the specimens may fail in tension, shear-out, or bearing. These three modes are shown in Figure 4.2. In real life applications, bearing
failure is usually preferred because it is not catastrophic and provides the highest joint strength.

The ratio of K/D, E/D and M/D are changed from 2 to 4,1 to5, 2 to 5 respectively. In order to find the strength of two parallel pin loaded specimens, the static bearing strength is defined as:
$\sigma_{b}=\frac{P}{2 D t}$

Table 4.1 Typical failure modes for bolted joints
Shear out

### 4.2 Manufacturing of the Specimens

Izoreel Company produced composite materials which were used in this study. Composite plate was consisted of twelve laminas. Thickness of each lamina was 0.3 mm . The woven of glass-vinylester prepress are cured about 30 minutes at $100 \mathrm{C}^{\circ}$
under 10 MPa pressure. At the end of the producing, composite plate thickness was measured as 2.8 mm . Volume fraction of the glass fiber was approximately $63 \%$.

### 4.3 Determination of Mechanical Properties

The modulus in direction of the fibers $\mathrm{E}_{1}$ and the Poisson's ratio $v_{12}$ can be characterized by means of tension tests on unidirectional coupons that instrumented with electric resistance strain gages, as depicted in Figure 4.2. One of them is placed to the fiber direction, the other in the matrix direction. The Poisson's ratio is just $v_{12}=$ $-\varepsilon_{2} / \varepsilon_{1}$, it may be noted that some nonlinearity may be observed in these tests. $\mathrm{E}_{2}$ is equal to $\mathrm{E}_{1}$ due to the woven structure.


Figure 4.2 Longitudinal tension test specimen for determination of $E_{1}$ and $v_{12}$
$\sigma_{1}=\frac{P}{A}, \quad E_{1}=\frac{\sigma_{1}}{\varepsilon_{1}}, \quad E_{2}=E_{1}$
$\mathrm{X}_{\mathrm{t}}$ is calculated by dividing the ultimate force by the cross-sectional area of the specimen.

$$
\begin{equation*}
X_{t}=\frac{P_{u l t}}{A} \tag{4.3}
\end{equation*}
$$

To find $X_{c}$, a rectangular specimen with small length whose fiber direction coincides with the loading direction is taken and it is subjected to compressive
loading Figure 4.3. $\mathrm{X}_{\mathrm{c}}$ is also calculated by dividing the ultimate force by the crosssectional area of the specimen.
$X_{c}=\frac{P_{u l t}}{A}$


Figure 4.3 Longitudinal compression test

The in-plane shear modulus can be obtained in a number of ways (Jones, 1999). One of them is to use angel ply coupons, made up of alternating layers of plies at an angle to the axis of the specimen. It will be simply stated that the stress and strain response in the axial direction of at $45^{\circ}$ laminate can be interpreted to give $\mathrm{G}_{12}$ according to the following expression:
$E_{x}=\frac{P / A}{\varepsilon_{x}}$
$G_{12}=\frac{1}{\frac{4}{E_{x}}-\frac{1}{E_{1}}-\frac{1}{E_{2}}+\frac{2 v_{12}}{E_{1}}}$

Iosipescu testing method is used to define the shear strength S Figure 4.4. The dimensions of the specimen are chosen as; $\mathrm{a}=80 \mathrm{~mm}, \mathrm{~b}=20 \mathrm{~mm}, \mathrm{c}=12 \mathrm{~mm}$ and $\mathrm{t}_{\mathrm{i}}=2.8$ mm . A compression test is applied to the specimen. In failure, S is calculated from

$$
\begin{equation*}
S=\frac{P_{\max }}{t_{i} \cdot c} \tag{3.6}
\end{equation*}
$$

where $\mathrm{P}_{\text {max }}$ is the failure force. (Gibson, 1994)


Figure 4.4 Iosipescu testing fixture
$Y_{t}$ and $Y_{c}$ are equal to, $X_{t}$ and $X_{c}$, respectively because of the woven structure. The mechanical properties of glass-vinylester composite plate which are obtained from the experimental study have been given in Table 3.1. (Gülem, 2004)

Table 4.2 Mechanical properties of glass-vinlyester composite materials

| $E_{1}=E_{2}$ <br> $(\mathrm{GPa})$ | $\mathrm{G}_{12}$ <br> $(\mathrm{GPa})$ | $v_{12}$ | $\mathrm{X}_{\mathrm{t}}=\mathrm{Y}_{\mathrm{t}}$ <br> $(\mathrm{MPa})$ | $\mathrm{X}_{\mathrm{c}}=\mathrm{Y}_{\mathrm{c}}$ <br> $(\mathrm{MPa})$ | $\mathrm{S}(\mathrm{MPa})$ | $\mathrm{V}_{\mathrm{f}}(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20.769 | 4.133 | 0.09 | 395 | 260 | 75 | 63 |

To find the failure load and the failure mode, a series of experiments were carried out. The specimens were trimmed as depicted in Figure 4.1. The effects of the pin location were studied by varying the distance from upper part of plate and centre of holes (K/D) ratio from 2 to 4, edge distance to diameter (E/D) ratio from 1 to 5 and between two holes distance to diameter (M/D) ratio from 2 to 5 , for the $0^{\circ}$ fiber orientation angle while keeping $\mathrm{D}, \mathrm{t}$ and L constant. 60 different geometries were used. All specimens were tested two times each.

The experiments were carried out in tension mode on the Tensile Machine. The lower edge of the specimen clamped and loaded from the steel pins by stretching the specimens at a ratio $0.5 \mathrm{~mm} / \mathrm{min}$ Figure 4.5. The load-pin displacement diagrams for all composite configurations were plotted.


Figure 4.5 Experimental setup for pin-joint testing

## CHAPTER FIVE

## RESULTS AND DISCUSSION

In this investigation, progressive failure analysis of pin loaded composite plates is performed to predict failures and bearing stresses and also it is compared with different geometries. The analysis takes into account contact between the pins and holes and a non-linear shear stress-strain relationship. To predict the failure, the Hashin Criteria was used for numerical analysis. After that these results have been compared with the experimentally conclusions.

In the experimental study, it is seen that load-displacement curves are linear before the initial failure. But then load decreases while deformation increases for most of the specimens. Some of them continued to keep loading on. Sometimes the highest load becomes after this event.

It is seen that when edge distance to diameter ratio (E/D), distance between holes to diameter (M/D) and distance between center of holes and upper part of plates to diameter ratio (K/D) are increased, failure load reaches higher values.

Especially when $E / D$ ratio is 1 for $M / D=2,3$ and $K / D=2$, failure load occurs lower values. In this case, plate is the weakest. E/D ratio begins to increase when M/D and $K / D$ are constant; failure load reaches the higher values. For example E/D is $4-5$ for $M / D=4-5$ and $K / D$ is 4, failure load occurs $7500-8000 \mathrm{~N}$.

In the same way, failure occurs quickly while $M / D$ ratio is $2-3$ for $E / D=1$ and $K / D=2$. When the $M / D$ ratio is increased such as 4and 5, failure load values reaches higher values.

Load-displacements curves of the experimental study are shown in Appendix A.

Bearing strength rises with increasing $M / D$ ratio, while $E / D$ and $K / D$ ratio are constant. It reaches higher values when $M / D 4,5$ for $E / D=1 K / D=2-3$.

When E/D ratio reaches 4 or 5 value, failure mode generally either bearing or bearing and shear out. It is shown in Figure 5.1 and 5.2


Figure5.1 The bearing mode for $E / D=5, M / D=5, K / D=4$


Figure5.2 The bearing and shear out mode for $E / D=2, M / D=3, K / D=3$

At low values of $\mathrm{E} / \mathrm{D}$, the failure types are shear out which are weak type of failure. This mode can be shown in Figure 5.3


Figure5.3 The shear out mode for $\mathrm{E} / \mathrm{D}=1, \mathrm{M} / \mathrm{D}=5, \mathrm{~K} / \mathrm{D}=2$

The other modes of failure of comparison with experimental and Hashin results are shown in Appendix B. 40

Figures from 5.4 to 5.15 are concerned with the bearing strength. It is clearly seen from the graphics that bearing strength is depend on $\mathrm{E} / \mathrm{D}$ ratio. Bearing strength reaches lowest values while $\mathrm{E} / \mathrm{D}=1$ for $\mathrm{M} / \mathrm{D}=2-3, K / \mathrm{D}=2-3$. As the $\mathrm{E} / \mathrm{D}$ and $\mathrm{M} / \mathrm{D}$ ratio are increases, the failure load reaches higher values.
M/D=2


$M / D=4$



Figure 5.4 The effect of $K / D$ ratio according to $E / D=1,2,3,4,5, M / D=2$, 3, 4, 5


Figure 5.5 The effect of $\mathrm{E} / \mathrm{D}$ ratio according to $\mathrm{K} / \mathrm{D}=2,3,4, \mathrm{M} / \mathrm{D}=2,3$, 4, 5
EID=1

EID=3




Figure 5.6 The effect of $M / D$ ratio according to $K / D=2,3,4, E / D=1,2$, 3, 4, 5

(a)

(b)

(c)

Figure 5.7 The effect of E/D ratio on the bearing strength for $M / D=2$ a) $K / D=2, b) K / D=3$ c) $K / D=4$

(a)

(b)

(c)

Figure 5.8 The effect of E/D ratio on the bearing strength for $M / D=3$ a) $K / D=2, b) K / D=3 c) K / D=4$


Figure 5.9 The effect of $\mathrm{E} / \mathrm{D}$ ratio on the bearing strength for $M / D=4$ a) $K / D=2, b) K / D=3$ c) $K / D=4$

(a)

(b)

(c)

Figure 5.10 The effect of E/D ratio on the bearing strength for $M / D=5$ a) $K / D=2, b) K / D=3 c$ c) $K / D=4$


Figure 5.11 The effect of M/D ratio on the bearing strength for $E / D=1$ a) $K / D=2, b) K / D=3 c$ ) $K / D=4$

(a)

(b)

(c)

Figure 5.12 The effect of M/D ratio on the bearing strength for $E / D=2 a) K / D=2, b) K / D=3 c$ ) $K / D=4$

(a)

(b)

(c)

Figure 5.13 The effect of M/D ratio on the bearing strength for $E / D=3$ a) $K / D=2, b) K / D=3 c$ ) $K / D=4$

(a)

(b)

(c)

Figure 5.14 The effect of $\mathrm{M} / \mathrm{D}$ ratio on the bearing strength for $E / D=4$ a) $K / D=2$, b) $K / D=3 c$ c) $K / D=4$

(a)

(b)

(c)

Figure 5.15 The effect of M/D ratio on the bearing strength for $E / D=5$ a) $K / D=2, b$ ) $K / D=3 c$ ) $K / D=4$

Table 5.1 Comparisons of experimental and numerical failure loads and bearings for $E / D=1$

| E/D=1 |  | Failure Load ( N ) |  | Bearing Strength (MPa) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Experimental | Hashin | Experimental | Hashin |
| K/D=2 | M/D=2 | 3163 | 5235 | 113 | 187 |
|  | M/D=3 | 3278 | 5400 | 117 | 193 |
|  | M/D=4 | 3332 | 5452 | 119 | 195 |
|  | M/D=5 | 3457 | 5578 | 123 | 199 |
| K/D=3 | M/D=2 | 3314 | 4288 | 118 | 153 |
|  | M/D=3 | 3404 | 5560 | 122 | 199 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 3874 | 5601 | 138 | 200 |
|  | M/D=5 | 3454 | 5706 | 123 | 204 |
| K/D=4 | M/D=2 | 3762 | 3912 | 134 | 140 |
|  | M/D=3 | 3737 | 5304 | 133 | 189 |
|  | M/D=4 | 3961 | 5652 | 141 | 202 |
|  | M/D=5 | 4083 | 5750 | 146 | 205 |

Table 5.2 Comparisons of experimental and numerical failure loads and bearings for $\mathrm{E} / \mathrm{D}=2$

| $E / D=2$ |  | Failure Load ( N ) |  | Bearing Strength (MPa) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Experimental | Hashin | Experimental | Hashin |
| K/D=2 | $\mathrm{M} / \mathrm{D}=2$ | 5604 | 5640 | 200 | 201 |
|  | M/D=3 | 5695 | 5684 | 203 | 203 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 5696 | 6663 | 203 | 238 |
|  | M/D=5 | 6177 | 6463 | 221 | 231 |
| K/D=3 | $\mathrm{M} / \mathrm{D}=2$ | 5625 | 5635 | 201 | 201 |
|  | M/D=3 | 5775 | 6640 | 206 | 237 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 5750 | 6912 | 205 | 247 |
|  | M/D=5 | 6851 | 6501 | 245 | 232 |
| K/D=4 | $\mathrm{M} / \mathrm{D}=2$ | 5689 | 4368 | 203 | 156 |
|  | $\mathrm{M} / \mathrm{D}=3$ | 5781 | 5676 | 206 | 203 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 6628 | 6944 | 237 | 248 |
|  | M/D=5 | 7284 | 6441 | 260 | 230 |

Table 5.3 Comparisons of experimental and numerical failure loads and bearings for $E / D=3$

| E/D=3 |  | Failure Load ( N ) |  | Bearing Strenth (MPa) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Experimental | Hashin | Experimental | Hashin |
| K/D=2 | $\mathrm{M} / \mathrm{D}=2$ | 5635 | 6105 | 201 | 218 |
|  | $\mathrm{M} / \mathrm{D}=3$ | 6273 | 7000 | 224 | 250 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 6515 | 6936 | 233 | 248 |
|  | M/D=5 | 7073 | 6850 | 253 | 245 |
| K/D=3 | $\mathrm{M} / \mathrm{D}=2$ | 6378 | 5788 | 228 | 207 |
|  | M/D=3 | 6711 | 6600 | 240 | 236 |
|  | M/D=4 | 6683 | 6912 | 239 | 247 |
|  | M/D=5 | 7159 | 6864 | 256 | 245 |
| K/D=4 | $\mathrm{M} / \mathrm{D}=2$ | 6585 | 6025 | 235 | 215 |
|  | $\mathrm{M} / \mathrm{D}=3$ | 6744 | 6540 | 241 | 234 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 7344 | 6566 | 262 | 235 |
|  | M/D=5 | 7952 | 6840 | 284 | 244 |

Table 5.4 Comparisons of experimental and numerical failure loads and bearings for $E / D=4$

| $E / D=4$ |  | Failure Load ( N ) |  | Bearing Strenth (MPa) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Experimental | Hashin | Experimental | Hashin |
| K/D=2 | $\mathrm{M} / \mathrm{D}=2$ | 6152 | 6690 | 220 | 239 |
|  | M/D=3 | 6482 | 6940 | 232 | 248 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 6585 | 6576 | 235 | 235 |
|  | M/D=5 | 7140 | 6720 | 255 | 240 |
| K/D=3 | M/D=2 | 6970 | 6376 | 249 | 228 |
|  | M/D=3 | 7026 | 6750 | 251 | 241 |
|  | M/D=4 | 7185 | 6624 | 257 | 237 |
|  | M/D=5 | 7475 | 6723 | 267 | 240 |
| K/D=4 | $\mathrm{M} / \mathrm{D}=2$ | 7137 | 6220 | 255 | 222 |
|  | M/D=3 | 7247 | 6600 | 259 | 236 |
|  | M/D=4 | 7670 | 8165 | 274 | 292 |
|  | M/D=5 | 8165 | 6690 | 292 | 239 |

Table 5.5 Comparisons of experimental and numerical failure loads and bearings for $E / D=5$

| $E / D=5$ |  | Failure Load ( N ) |  | Bearing Strenth (MPa) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Experimental | Hashin | Experimental | Hashin |
| K/D=2 | $\mathrm{M} / \mathrm{D}=2$ | 6779 | 6840 | 242 | 244 |
|  | $\mathrm{M} / \mathrm{D}=3$ | 7065 | 6900 | 252 | 246 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 6954 | 6768 | 248 | 242 |
|  | $\mathrm{M} / \mathrm{D}=5$ | 6630 | 6531 | 237 | 233 |
| K/D=3 | M/D=2 | 6931 | 6520 | 248 | 233 |
|  | M/D=3 | 6754 | 6800 | 241 | 243 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 7407 | 6570 | 265 | 235 |
|  | M/D=5 | 7176 | 6508 | 256 | 232 |
| K/D=4 | $\mathrm{M} / \mathrm{D}=2$ | 7302 | 6400 | 261 | 229 |
|  | $\mathrm{M} / \mathrm{D}=3$ | 6589 | 6720 | 235 | 240 |
|  | $\mathrm{M} / \mathrm{D}=4$ | 7688 | 6529 | 275 | 233 |
|  | M/D=5 | 7909 | 6630 | 282 | 237 |

## CHAPTER SIX

CONCLUSION

In this investigation, failure mode, maximum failure load and bearing strength in a glass vinylester composite plate with two circular and parallel holes, which is subjected to traction force by two parallel rigid pins, are performed experimentally and numerically. In numerical study, Hashin failure criteria was used to predict the maximum failure load and failure types. In the experimental study, the specimens for each E/D, M/D and K/D ratio have been tested. Experimental results concerning failure types and failure loads were obtained and compared with numerical results. It is seen that these results are close to each other. In addition the effects of geometric parameters are observed. All the numerical and experimental results which is obtained have been presented in tables and figures.

Bearing strength of the composite plate increases by going up the geometric parameters. It means that, when edge distance to diameter ratio (E/D), distance between center of the holes (M/D) and distance from center of holes to upper edge of plates (K/D) are increased, the bearing strength reaches higher values.

When $E / D$ ratio is 1 , the bearing strength is small and failure mode is generally shear out. In addition, when the E/D ratio 3,4 or 5 , the failure load reaches higher values and failure modes is bearing which is the best mode of resisting load.

While the $M / D$ and $K / D$ ratios are increased, the bearing strengths generally reach high values as E/D ratio. Failure types are bearing.

At low values of M/D, the failure types are shear out or shear out and bearing together which are weak type of failure

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## APPENDIX A


(a)

(b)

(c)

Figure A. 1 Load-displacement curves for pin-loaded glassvinylester composite plates ( $\mathrm{E} / \mathrm{D}=1,2,3,4,5 \mathrm{M} / \mathrm{D}=2$ ) a) $K / D=2, b) K / D=3, ~ c) ~ K / D=4$

(a)

(b)

(c)

Figure A. 2 Load-displacement curves for pin-loaded glassvinylester composite plates ( $\mathrm{E} / \mathrm{D}=1,2,3,4,5 \mathrm{M} / \mathrm{D}=3$ ) a) $K / D=2, b) K / D=3, ~ c) ~ K / D=4, ~ d) K / D=5$

$K / D=4 M / D=4$

(c)

Figure A. 3 Load-displacement curves for pin-loaded glassvinylester composite plates ( $\mathrm{E} / \mathrm{D}=1,2,3,4,5 \mathrm{M} / \mathrm{D}=4$ ) a) $K / D=2$, b) $K / D=3$, c) $K / D=4$

(a)

(b)
$K / D=4$ M/D=5

(c)

Figure A. 4 Load-displacement curves for pin-loaded glassvinylester composite plates ( $\mathrm{E} / \mathrm{D}=1,2,3,4,5 \mathrm{M} / \mathrm{D}=5$ ) a) $K / D=2$, b) $K / D=3$, c) $K / D=4$

(a)
$E / D=1 M / D=3$

(b)

EID=1 M/D=4

(c)


Figure A. 5 Load-displacement curves for pin-loaded glassvinylester composite plates (K/D=2, 3, $4 \mathrm{E} / \mathrm{D}=1$ ) a) $\mathrm{M} / \mathrm{D}=2$, b) $M / D=3, ~ c) ~ M / D=4, ~ d) ~ M / D=5$

(a)

(b)

(d)

Figure A. 6 Load-displacement curves for pin-loaded glassvinylester composite plates (K/D=2, 3, $4 \mathrm{E} / \mathrm{D}=2$ ) a) $\mathrm{M} / \mathrm{D}=2$,
b) $M / D=3, ~ c) ~ M / D=4, ~ d) ~ M / D=5$

(a)

(b)

(c)

EID $=3$ M/D=5

(d)

Figure A. 7 Load-displacement curves for pin-loaded glassvinylester composite plates (K/D=2, 3, $4 \mathrm{E} / \mathrm{D}=3$ ) a) $\mathrm{M} / \mathrm{D}=2$, b) $M / D=3, ~ c) ~ M / D=4 d) M / D=5$



Figure A. 8 Load-displacement curves for pin-loaded glassvinylester composite plates (K/D=2, 3, $4 \mathrm{E} / \mathrm{D}=4$ ) a) $\mathrm{M} / \mathrm{D}=2$, b) $M / D=3, ~ c) ~ M / D=4 d$ ) $M / D=5$

(a)

(b)

(d)

Figure A. 9 Load-displacement curves for pin-loaded glassvinylester composite plates (K/D=2, 3, $4 \mathrm{E} / \mathrm{D}=5$ ) a) $\mathrm{M} / \mathrm{D}=2$,
b) $M / D=3, ~ c) ~ M / D=4 M / D=5$

(a)


Figure A. 10 Load-displacement curves for pin-loaded glassvinylester composite plates (M/D=2, 3, 4,5 E/D=1) a) $K / D=2$, b) $K / D=3$, c) $K / D=4$

EID=2 K/D=2

(a)

(b)

(c)

Figure A. 11 Load-displacement curves for pin-loaded glassvinylester composite plates (M/D=2, 3, 4, $5 \mathrm{E} / \mathrm{D}=2$ ) a) $\mathrm{K} / \mathrm{D}=2$, b) $\mathrm{K} / \mathrm{D}=3$, c) $\mathrm{K} / \mathrm{D}=4$

(a)

(c)

Figure 5.12 Load-displacement curves for pin-loaded glassvinylester composite plates (M/D=2, 3, 4, $5 \mathrm{E} / \mathrm{D}=3$ ) a) $K / D=2, b) K / D=3, ~ c) ~ K / D=4$

(a)

(b)

(c)

Figure A. 13 Load-displacement curves for pin-loaded glassvinylester composite plates (M/D=2, 3, 4, $5 \mathrm{E} / \mathrm{D}=4$ ) a) $K / D=2, b) K / D=3, ~ c) K / D=4$

EJD=5 K/D=2

(a)


Figure A. 14 Load-displacement curves for pin-loaded glassvinylester composite plates ( $\mathrm{M} / \mathrm{D}=2,3,4,5 \mathrm{E} / \mathrm{D}=5$ ) a) $K / D=2, b) K / D=3, ~ c) K / D=4$

## APPENDIX B


(a)

(b)

(c)

Figure B. 1 Comparison with experimental and Hashin results a) $E / D=1 \mathrm{M} / \mathrm{D}=2 \mathrm{~K} / \mathrm{D}=2$, b) $\mathrm{E} / \mathrm{D}=1$ $M / D=2 K / D=3, c) E / D=1 M / D=2 K / D=3$

(a)

(b)

(c)

Figure B. 2 Comparison with experimental and Hashin results a) $E / D=1 M / D=3 K / D=2, b) E / D=1$ $M / D=3 K / D=3, ~ c) ~ E / D=1 M / D=3 K / D=3$

(a)

(b)

(c)

Figure B. 3 Comparison with experimental and Hashin results a) $E / D=1 \mathrm{M} / \mathrm{D}=4 \mathrm{~K} / \mathrm{D}=2$, b) $E / D=1 M / D=3 K / D=4$, c) $E / D=1 M / D=4 K / D=3$

(a)

(b)

(c)

Figure B. 4 Comparison with experimental and Hashin results a) $E / D=1 \mathrm{M} / \mathrm{D}=5 \mathrm{~K} / \mathrm{D}=2$, b) $E / D=1 M / D=5 K / D=4$, c) $E / D=1 M / D=5 K / D=3$

(a)

(b)

(c)

Figure B. 5 Comparison with experimental and Hashin results a) $\mathrm{E} / \mathrm{D}=2 \mathrm{M} / \mathrm{D}=2 \mathrm{~K} / \mathrm{D}=2$, b) $\mathrm{E} / \mathrm{D}=2 \mathrm{M} / \mathrm{D}=2 \mathrm{~K} / \mathrm{D}=3$, c) $\mathrm{E} / \mathrm{D}=2 \mathrm{M} / \mathrm{D}=2 \mathrm{~K} / \mathrm{D}=4$

(a)

(b)

(c)

Figure B. 6 Comparison with experimental and Hashin results a) $\mathrm{E} / \mathrm{D}=2 \mathrm{M} / \mathrm{D}=3 \mathrm{~K} / \mathrm{D}=2$, b) $E / D=2 M / D=3 K / D=3$, c) $E / D=2 M / D=3 K / D=4$

(a)

(b)

(c)

Figure B. 7 Comparison with experimental and Hashin results a) $\mathrm{E} / \mathrm{D}=2 \mathrm{M} / \mathrm{D}=4 \mathrm{~K} / \mathrm{D}=2$, b) $\mathrm{E} / \mathrm{D}=2 \mathrm{M} / \mathrm{D}=4 \mathrm{~K} / \mathrm{D}=3$, c) $\mathrm{E} / \mathrm{D}=2 \mathrm{M} / \mathrm{D}=4 \mathrm{~K} / \mathrm{D}=4$

(a)

(b)

(c)

Figure B. 8 Comparison with experimental and Hashin results a) $E / D=2 M / D=5 K / D=2, b) E / D=2$ $\mathrm{M} / \mathrm{D}=5 \mathrm{~K} / \mathrm{D}=3$, c) $\mathrm{E} / \mathrm{D}=2 \mathrm{M} / \mathrm{D}=5 \mathrm{~K} / \mathrm{D}=4$

(a)

(b)

(c)

Figure B. 9 Comparison with experimental and Hashin results a) $E / D=3 \mathrm{M} / \mathrm{D}=2 \mathrm{~K} / \mathrm{D}=2$, b) $\mathrm{E} / \mathrm{D}=3 \mathrm{M} / \mathrm{D}=2 \mathrm{~K} / \mathrm{D}=3$, c) $\mathrm{E} / \mathrm{D}=3 \mathrm{M} / \mathrm{D}=2 \mathrm{~K} / \mathrm{D}=4$


Figure B. 10 Comparison with experimental and Hashin results a) $E / D=3 \mathrm{M} / \mathrm{D}=3 \mathrm{~K} / \mathrm{D}=2$, b) $\mathrm{E} / \mathrm{D}=3$ $M / D=3 K / D=3$, c) $E / D=3 M / D=3 K / D=4$


Figure B. 11 Comparison with experimental and Hashin results a) $E / D=3 M / D=4 K / D=2$, b) $E / D=3$ $M / D=4 K / D=3$, c) $E / D=3 M / D=4 K / D=4$

(a)

(b)

(c)

Figure B. 12 Comparison with experimental and Hashin results a) $E / D=3 M / D=5 K / D=2, b) E / D=3$ $M / D=5 K / D=3, c) E / D=3 M / D=5 K / D=4$

(a)

(b)

(c)

Figure B. 13 Comparison with experimental and Hashin results a) $\mathrm{E} / \mathrm{D}=4 \mathrm{M} / \mathrm{D}=2 \mathrm{~K} / \mathrm{D}=2$, b) $\mathrm{E} / \mathrm{D}=4$ $M / D=2 K / D=3, c) E / D=4 M / D=2 K / D=4$


Figure B. 14 Comparison with experimental and Hashin results a) $E / D=4 M / D=3 K / D=2$, b) $E / D=4 M / D=3 K / D=3, c) E / D=4 M / D=3 K / D=4$

(b)

(c)

Figure B. 15 Comparison with experimental and Hashin results a) $E / D=4 M / D=4 K / D=2$, b) $E / D=4$ $M / D=4 K / D=3, ~ c) ~ E / D=4 M / D=4 K / D=4$

(a)

(b)

(c)

Figure B. 16 Comparison with experimental and Hashin results a) $E / D=4 M / D=5 K / D=2$, b) $E / D=4$ $M / D=5 K / D=3$, c) $E / D=4 M / D=5 K / D=4$


Figure B. 17 Comparison with experimental and Hashin results a) $E / D=5 M / D=2 K / D=2, b) E / D=5$ $M / D=2 K / D=3, c) E / D=5 M / D=2 K / D=4$

(a)

(b)


Figure B. 18 Comparison with experimental and Hashin results a) $E / D=5 \mathrm{M} / D=3 \mathrm{~K} / D=2$, b) $E / D=5$ $\mathrm{M} / \mathrm{D}=3 \mathrm{~K} / \mathrm{D}=3$, c) $\mathrm{E} / \mathrm{D}=5 \mathrm{M} / \mathrm{D}=3 \mathrm{~K} / \mathrm{D}=4$

(a)

(b)

(c)

Figure B. 19 Comparison with experimental and Hashin results a) $\mathrm{E} / \mathrm{D}=5 \mathrm{M} / \mathrm{D}=4 \mathrm{~K} / \mathrm{D}=2$, b) $\mathrm{E} / \mathrm{D}=5$ $M / D=4 K / D=3$, c) $E / D=5 M / D=4 K / D=4$


Figure B. 20 Comparison with experimental and Hashin results a) $\mathrm{E} / \mathrm{D}=5 \mathrm{M} / \mathrm{D}=5 \mathrm{~K} / \mathrm{D}=2$, b) $\mathrm{E} / \mathrm{D}=5$ $\mathrm{M} / \mathrm{D}=5 \mathrm{~K} / \mathrm{D}=3$, c) $\mathrm{E} / \mathrm{D}=5 \mathrm{M} / \mathrm{D}=5 \mathrm{~K} / \mathrm{D}=4$

