BRUSHLESS SYNCHRONOUS GENERATOR WITH WOUND FIELD AND ITS MODELING

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by
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İZMİR

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ABSTRACT

In this thesis, in order to analyze a brushless synchronous generator with a wound field, a mathematical model is obtained by using the generalized machine theory. The model is solved by using a dedicated FORTRAN program. Also the parameters of the machine are measured in the laboratory.

The stator of the generator is connected to the 3-phase uncontrolled rectifier. The output of the rectifier is connected to a battery through a current limiting resistor. The whole system is also modeled in rotor reference frame. The result of the computer program provides the estimated value of stator currents, electromagnetic torque, angular velocity of the rotor, input and output power, efficiency and rotor angle.

Keywords: Brushless synchronous generator with wound field, homopolar synchronous machine

ÖZET

Bu tezde, alan sargılı fırçasız senkron jeneratörü analiz etmek için genel makine teorisi kullanılarak matematiksel modeli elde edildi. Bu model FORTRAN programı kullanılarak çözüldü. Ayrıca makinenin parametreleri laboratuarda ölçüldü.

Jeneratörün statoru 3-fazlı kontrolsüz doğrultucuya bağlandı. Doğrultucunun çıkışı akım sınırlayıcı rezistans üzerinden aküye bağlandı. Tüm sistem rotor referans frame' de modellendi. Bilgisayar programının sonucu, stator akımları, elektromanyetik tork, rotorun açısal hızı, giriş ve çıkış gücü, verimi ve rotor açısının değerlerini tahmin etmemizi sağladı.

Anahtar sözcükler: Ala sargılı fırçasız senkron jeneratör, tek yönü kutuplandırılmış senkron makine

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CHAPTER ONE
INTRODUCTION

1. Introduction

An AC generator converts the mechanical power to ac electric power. The principle of ac machine operation is that if a balanced three-phase set of currents, flows in a three-phase-balanced winding, then it will produce a rotating magnetic field at constant magnitude and speed. When the rotor having magnetic poles rotates at the same speed with the rotating magnetic field, the electromagnetic torque is developed.

Although the field winding in an ordinary synchronous generator is located on the rotor and the field winding of a brushless synchronous generator is located in the stator, both machines share the same terminal characteristics, and therefore can be described with the same lumped parameters (Tsao et al., 2002).

A brushless synchronous generator (BSG) has many advantages, listed in table1.1.

Table 1.1 Brushless Synchronous Generator Features

BSG Advantage	Why BSG provides this advantage.
Improved efficiency	Eliminate brush voltage drop and brush-friction.
Reduced electrical noise	Eliminate arcing from brushes to commutator.
Reduced acoustic noise	Eliminate brush bounce, especially at high speeds.
Reduced debris	Eliminate brush wear
Increased speed range	Remove mechanical limitations imposed by brush / commutator interface.
Reduced size due to superior thermal characteristics	Brushless generators have windings on the stator, brush generators have windings on rotor. Windings are the main heat generators on brushless generators and it is easier to remove heat from the stator than from the rotor, if stator is the outer part of the machine.
Reduced maintenance	Eliminate brush wear
Reduced weight and size	Eliminate commutator, brush, and brush holder

Some advantages directly impact overall machine cost. For example, lighter generators require less structural support. Higher efficiency generators require a smaller power stage or may eliminate the need for cooling fans. Relocating the generator without concern for brush debris may reduce the cost of mechanical structures.

Other advantages are important because they add value to the machine. For example eliminating brush noise may take the machine run more quietly. Eliminating brushes increases reliability and can reduce the machine's scheduled maintenance cost. Other features such as improved efficiency, elimination of brush debris or reduced electrical noise may all be important beyond direct cost impact (Ellis, G., 1996). The general view of a brushless synchronous generator is given in Figure 1.1.

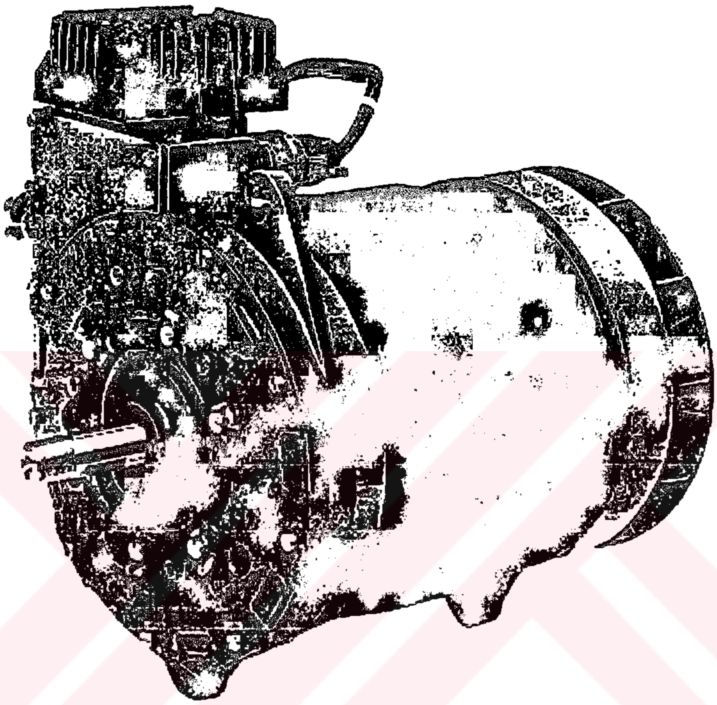

Figure 1.1 Brushless synchronous generator

High efficiency, low rotor losses, and a robust rotor structure are key requirements for the brushless synchronous generator.

The rotor must be able to withstand the stresses of spinning at high-speed. This requirement makes the use of permanent magnets on the rotor difficult because of their brittleness and low strength. Permanent magnets are further limited by their temperature restrictions. Another important consideration is that rotor structures with multiple parts are more difficult to balance and are more likely to have unwanted vibration modes in the target operating speed range.

Although not widely used in practice, brushless synchronous generators have been implemented for a variety of applications. They are sometimes referred to as 'homopolar inductor alternator/motors' (He, J.,& Lin, F., 1995)-(Siegl, M.,& Kotrba, V, 1991), or simply as 'homopolar generators' (Rao et al., 2000)-(Ichikawa et al., 1999). The defining feature of these generators is the homopolar d-axis magnetic field created by a field winding (He, J.,& Lin, F., 1995)- (Siegl, M.,& Kotrba, V, 1991)-(Ichikawa et al., 1999)-(Hippner, M.,& Harley, R.G., 1992), permanent magnets, or a combination of permanent magnets and windings (Rao et al., 2000). The principle is the same as in a traditional synchronous generator, with which the brushless synchronous generator has similar terminal characteristics. However, in the case of the brushless synchronous generator the field winding is fixed to the stator and generally encircles the rotor rather than being placed on the rotor. The field winding and the magnetizing flux path in the present motor design are shown schematically in Figure 1.2. Note that the rotor pole faces on the upper part of the rotor are offset from the pole faces on the lower part (see Figure 1.3).

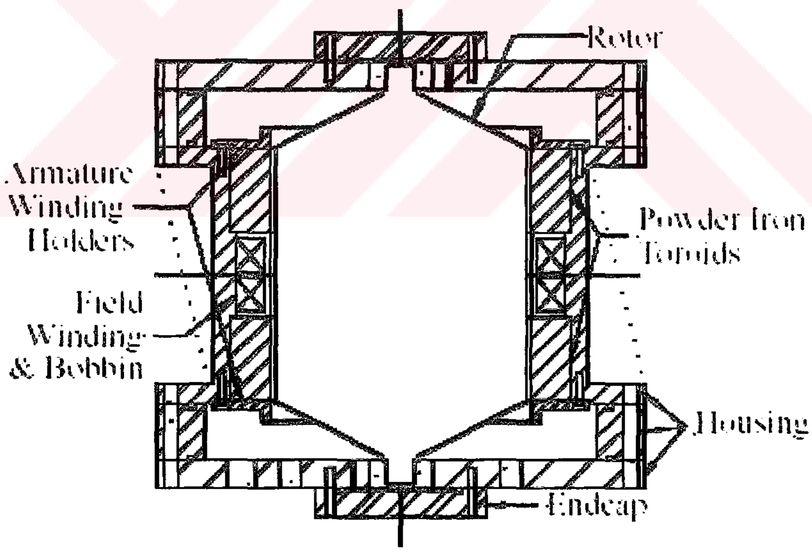

Figure 1.2 Cut-away view of brushless synchronous generator with arrows indicating magnetizing flux path

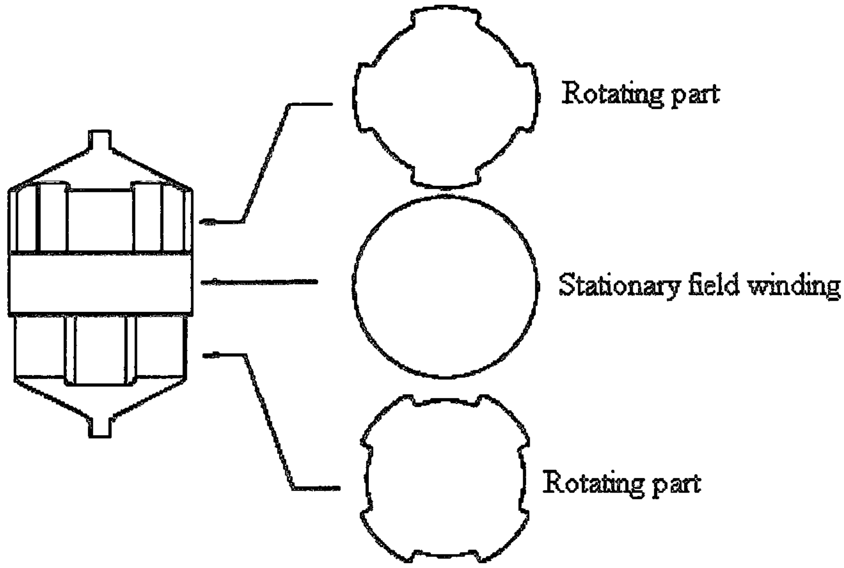

Figure 1.3 View of rotor structure

There are several advantages to have the field winding in the stator. One of them is elimination of slip rings and greatly simplified rotor construction, making it practical to construct the rotor from a single piece of high strength steel. This feature makes brushless synchronous generators very attractive for high speed operation; a single piece steel rotor is used in the design presented here and in (He, J.,& Lin, F., 1995)-(Hippner, M.,& Harley, R.G., 1992). The other rotor designs feature laminations (Ichikawa et al., 1999), permanent magnets (Rao et al., 2000), or other non-magnetic structural elements to increase strength and reduce windage losses (Siegl, M.,& Kotrba, V, 1991).

The content of this thesis can be summarized briefly as follows:

Chapter 2 focuses on brushless synchronous generator model. Firstly, from three phase to two phase and from αβf to dqf transformation in arbitrary reference frame, synchronously rotating reference frame and rotor reference frame are described. Then stator voltages and currents are given in arbitrary reference frame and rotor reference frame. Finally, the mathematical models of brushless synchronous generator and rectifier load are combined.

The chapter 3 of this thesis presents the measurements of brushless synchronous generator parameters. For this reason, open circuit test, short circuit test, and run down test have been done in order to calculate stator winding self inductance, mutual inductance, field winding inductance, stator winding resistance and moment of inertia constant.

Chapter 4 focuses on simulation result. The mathematical model of brushless synchronous generator with rectifier load is solved in FORTRAN program by using 4^{th} order Runge-Kutta integration routine. And power, torque, and current graphics of brushless synchronous generator are drawn by using Matlab program.

Chapter 5 is concerned with the conclusions.

CHAPTER TWO

BRUSHLESS SYNCHRONOUS GENERATOR MODELING

2. Brushless Synchronous Generator Modeling

The electrical and electromechanical behavior of brushless synchronous machine can be predicted from the non-linear set of equations which describe this machine. The solution of these equations can be used to predict the performance of the machine.

In this chapter, the voltage and electromagnetic torque equations are first established in machine variables. Reference frame theory is then used to establish the machine equations in arbitrary reference frame, synchronously rotating reference frame and in the rotor reference frame (Park's equations). The torque-load angle equation of the generator with rectifier is obtained at synchronously rotating reference frame. Transient analysis of the system is carried out at rotor reference frame. The equations which describe the steady state behavior are also derived in the frame. Computer traces are given to illustrate the dynamic behavior of brushless synchronous generators.

2.1. Mathematical Modeling of Brushless Synchronous Generator

2.1.1. Voltage Equations in Machine Variables

A 12 pole, 3-phase, wye connected brushless synchronous generator is shown in Figure 1.1. The stator windings are identical with N_s equivalent turns and resistance R_s.

Since the brushless synchronous machine is operated as a generator it is convenient to assume that the direction of positive stator current is out of the terminals. With this convention the voltage equations in generator variables may be expressed in matrix form as (Krause et al., 1995).

$$V_{abcs} = -R_s I_{abcs} + p\lambda_{abcs} \tag{2.1}$$

$$V_f = R_f I_f + p\lambda_f \tag{2.2}$$

In the above equations the s and f subscripts denote variables associated with the stator and field windings, respectively.

The flux linkage equations are

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda_f \end{bmatrix} = \begin{bmatrix} L_s & L_{sf} \\ (L_{sf})^T & L_f \end{bmatrix} \begin{bmatrix} -I_{abcs} \\ I_f \end{bmatrix} \tag{2.3}$$

Where L_s is given as

$$L_s = \begin{bmatrix} L_{ls} + L_A & -\frac{1}{2}L_A & -\frac{1}{2}L_A \\ -\frac{1}{2}L_A & L_{ls} + L_A & -\frac{1}{2}L_A \\ -\frac{1}{2}L_A & -\frac{1}{2}L_A & L_{ls} + L_A \end{bmatrix} \tag{2.4}$$

The inductance matrices L_{sf} and L_f may then be expressed

$$L_{sf} = \begin{bmatrix} L_{sf}\sin\theta_r \\ L_{sf}\sin\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{sf}\sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \tag{2.5}$$

$$L_f = [L_{lf} + L_{mf}] \tag{2.6}$$

Whereupon it is easy to show that

$$L_{sf} = \left(\frac{N_f}{N_s}\right)\left(\frac{2}{3}\right)L_m \text{ and } L_{mf} = \left(\frac{N_f}{N_s}\right)^2\left(\frac{2}{3}\right)L_m \tag{2.7}$$

In the above equations the leakage inductances are denoted with l in the subscript. The subscript sf denotes mutual inductance between stator and field windings.

The flux linkage, referred to stator, may now be written

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda'_f \end{bmatrix} = \begin{bmatrix} L_s & L'_{sf} \\ (L'_{sf})^T & L'_f \end{bmatrix}\begin{bmatrix} -I_{abcs} \\ I'_f \end{bmatrix} \tag{2.8}$$

Where

$$L'_{sf} = \begin{bmatrix} L_m\sin\theta_r \\ L_m\sin\left(\theta_r - \frac{2\pi}{3}\right) \\ L_m\sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \tag{2.9}$$

$$L'_f = [L'_{lf} + L_m] \tag{2.10}$$

The voltage equations expressed in terms of generator variables referred to the stator side are

$$\begin{bmatrix} V_{abcs} \\ V'_f \end{bmatrix} = \begin{bmatrix} R_s + pL_s & pL'_{sf} \\ \frac{2}{3}p(L'_{sf})^T & R'_f + pL'_f \end{bmatrix} \begin{bmatrix} -I_{abcs} \\ I'_f \end{bmatrix} \tag{2.11}$$

2.1.2. Three Phase / Two Phase Transformation in Arbitrary Reference Frame

Figure 2.1 shows an electric machine with a generic smooth rotor. This is a 2-pole, 3-phase machine; each phase is represented as a concentrated coil. Each coil has two sides, labeled here in italics (e.g. a and a'). A small circle in a coil side indicates that the positive sense of current is out of the page, while a cross in a coil side indicates positive sense into the page. The axis associated with a coil, labeled in bold typeface, is defined by the right-hand rule, following the sense of the coil.

It is common in the analysis of electric machines to imagine electrical variables such as voltage, current, and flux as rotating vectors in 2-dimensional space. The peak value of current in a coil occurs when the current vector is aligned with the coil axis. Similarly, peak voltage across the coil and peak flux linking the coil occurs when the voltage and flux vectors, respectively, are aligned with the coil axis.

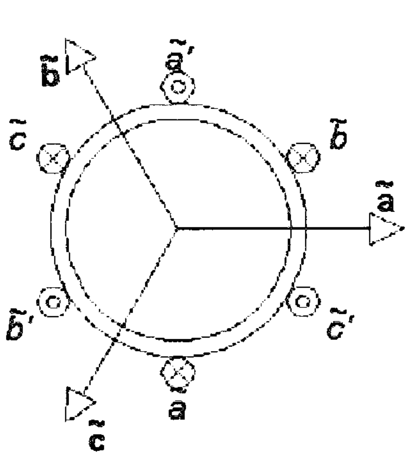

Figure 2.1 A 3-phase machine

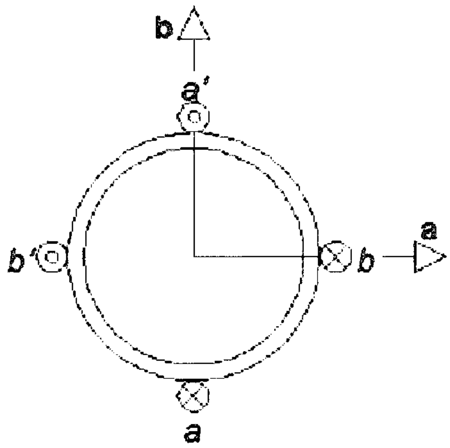

Figure 2.2 A 2-phase machine

Note, however, that the three axes defined by the windings in Figure 2.1 are obviously linearly. Thus it is useful to perform a linear transformation of variables from three phases to two. Figure 2.2 shows the hypothetical machine after a 3-to-2 phase stationary transformation.

The 3-to-2 phase transformation is,

$$f_{\alpha\beta 0} = C_1 f_{abcs} \quad \text{And} \quad f_{abcs} = C_1^{-1} f_{\alpha\beta 0} \tag{2.12}$$

Where

$$C_1 = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{2.13}$$

$$C_1^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix} \tag{2.14}$$

In the above the C_1 and C_1^{-1} are the transformation matrices from a-b-c axis to α-β axis and from α-β axis to a-b-c axis respectively. The 3-to-2 phase transformation is applied on the variables as follows;

$$V_{abcs} = -R_s I_{abcs} + p\lambda_{abcs} \tag{2.15}$$

$$C_1^{-1} V_{\alpha\beta0} = -R_s C_1^{-1} I_{\alpha\beta0} + p\left[C_1^{-1}\lambda_{\alpha\beta0}\right] \tag{2.16}$$

$$V_{\alpha\beta0} = -R_s C_1 C_1^{-1} I_{\alpha\beta0} + C_1 p\left[C_1^{-1}\lambda_{\alpha\beta0}\right] \tag{2.17}$$

$$V_{\alpha\beta0} = -R_s I_{\alpha\beta0} + C_1 p\left[C_1^{-1}\right]\lambda_{\alpha\beta0} + C_1 C_1^{-1} p\left[\lambda_{\alpha\beta0}\right] \tag{2.18}$$

$$V_{\alpha\beta0} = -R_s I_{\alpha\beta0} + p\lambda_{\alpha\beta0} \tag{2.19}$$

And the field winding in the αβ reference frame is represented below;

$$V_f' = R_f' I_f' + p\lambda_f' \tag{2.20}$$

The flux linkages may now be written in compact form as follows;

$$\begin{bmatrix} \lambda_{\alpha\beta0} \\ \lambda_f' \end{bmatrix} = \begin{bmatrix} C_1 L_s C_1^{-1} & C_1 L_{sf}' \\ \frac{2}{3}(L_{sf}')^T C_1^{-1} & L_f' \end{bmatrix} \begin{bmatrix} -I_{\alpha\beta0} \\ I_f' \end{bmatrix} \tag{2.21}$$

In explicit form,

$$C_1 L_s C_1^{-1} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} L_{ls}+L_A & -\frac{1}{2}L_A & -\frac{1}{2}L_A \\ -\frac{1}{2}L_A & L_{ls}+L_A & -\frac{1}{2}L_A \\ -\frac{1}{2}L_A & -\frac{1}{2}L_A & L_{ls}+L_A \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix} \quad (2.22)$$

$$C_1 L_s C_1^{-1} = \begin{bmatrix} L_{ls}+\frac{3}{2}L_A & 0 & 0 \\ 0 & L_{ls}+\frac{3}{2}L_A & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \quad (2.23)$$

The magnetizing inductance is defined as

$$L_m = \frac{3}{2}L_A \quad (2.24)$$

Let us define, L_s

$$L_{ls} + L_m = L_s \quad (2.25)$$

Then

$$L_{s\alpha} = C_1 L_s C_1^{-1} = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \quad (2.26)$$

And then

$$C_1 L'_{sf} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} L_m \sin\theta_r \\ L_m \sin\left(\theta_r - \frac{2\pi}{3}\right) \\ L_m \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \tag{2.27}$$

$$\Rightarrow \quad L'_{sf\alpha} = \begin{bmatrix} L_m \sin\theta_r \\ L_m \cos\theta_r \\ 0 \end{bmatrix} \tag{2.28}$$

And lastly

$$\frac{2}{3}(L'_{sf})^T C_1^{-1} = \frac{2}{3}\left[L_m \sin\theta_r \quad L_m \sin\left(\theta_r - \frac{2\pi}{3}\right) \quad L_m \sin\left(\theta_r + \frac{2\pi}{3}\right) \right]\begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$$

(2.29)

$$\Rightarrow \quad (L'_{sf\alpha})^T = [L_m \sin\theta_r \quad L_m \cos\theta_r \quad 0] \tag{2.30}$$

$$L'_f = [L'_f] \qquad \text{where } L'_f = L'_{lf} + L_m \tag{2.31}$$

Then the resultant flux linkage matrix may be expressed as

$$\begin{bmatrix} \lambda_\alpha \\ \lambda_\beta \\ \lambda_0 \\ \lambda'_f \end{bmatrix} = \begin{bmatrix} L_s & 0 & 0 & L_m \sin\theta_r \\ 0 & L_s & 0 & L_m \cos\theta_r \\ 0 & 0 & L_{ls} & 0 \\ L_m \sin\theta_r & L_m \cos\theta_r & 0 & L'_f \end{bmatrix}\begin{bmatrix} -I_\alpha \\ -I_\beta \\ -I_0 \\ I'_f \end{bmatrix} \tag{2.32}$$

Therefore,

$$\lambda_\alpha = -L_s I_\alpha + L_m \sin\theta_r \, I'_f \tag{2.33}$$

$$\lambda_\beta = -L_s I_\beta + L_m \cos\theta_r \, I'_f \tag{2.34}$$

$$\lambda_0 = -L_{ls} I_0 \tag{2.35}$$

$$\lambda'_f = -L_m \sin\theta_r \, I_\alpha - L_m \cos\theta_r \, I_\beta + L'_f I'_f \tag{2.36}$$

Then the resultant voltage equations in the $\alpha\beta$ reference frame may be expressed as

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \\ V'_f \end{bmatrix} = \begin{bmatrix} -R_s & 0 & 0 & 0 \\ 0 & -R_s & 0 & 0 \\ 0 & 0 & -R_s & 0 \\ 0 & 0 & 0 & R'_f \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \\ I_0 \\ I'_f \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_\alpha \\ \lambda_\beta \\ \lambda_0 \\ \lambda'_f \end{bmatrix} \tag{2.37}$$

Hence,

$$V_\alpha = -R_s I_\alpha + p\lambda_\alpha \tag{2.38}$$

$$V_\beta = -R_s I_\beta + p\lambda_\beta \tag{2.39}$$

$$V_0 = -R_s I_0 + p\lambda_0 \tag{2.40}$$

$$V'_f = R'_f I'_f + p\lambda'_f \tag{2.41}$$

The basic expression for the torque is shown in the below;

$$T_e = \frac{P}{2}\frac{1}{2} I^T_{\alpha\beta f} \frac{dL}{d\theta} I_{\alpha\beta f} \tag{2.42}$$

Where the P is the number of pole,

$$T_e = \frac{P}{2}\frac{1}{2}\begin{bmatrix} I_\alpha & I_\beta & I'_f \end{bmatrix}\begin{bmatrix} 0 & 0 & L_m\cos\theta_r \\ 0 & 0 & -L_m\sin\theta_r \\ L_m\cos\theta_r & -L_m\sin\theta_r & 0 \end{bmatrix}\begin{bmatrix} I_\alpha \\ I_\beta \\ I'_f \end{bmatrix} \quad (2.43)$$

$$T_e = \frac{P}{2}L_m I'_f\left(I_\alpha\cos\theta_r - I_\beta\sin\theta_r\right) \quad (2.44)$$

2.1.3. αβf / qdf Transformation in Arbitrary Reference Frame

In the previous section the impedance matrix has been obtained for the non-salient pole synchronous machine in the αβ reference frame. When the result is subjected to the phase transformation C_2, the 3-phase variables of stationary circuit elements are transformed to the arbitrary reference frame.

The αβ to qdf transformation is

$$f_{qdf} = C_2 f_{\alpha\beta 0f} \quad \text{and} \quad f_{\alpha\beta 0f} = C_2^{-1} f_{qdf} \quad (2.45)$$

Where

$$C_2 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.46)$$

$$C_2^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.47)$$

Where $\theta = \int_0^t \omega(\xi)d\xi + \theta_0$ and ω is the speed of reference frame.

The transformation from α-β axis to the q-d axis, which is rotating with an angular velocity of ω, can be obtained by rotating the α-β axis with ωt.

In the above the C_2 and C_2^{-1} are the transformation matrices. The αβ-to-qd phase transformation for the voltage equations are

$$V_{\alpha\beta 0} = -R_s I_{\alpha\beta 0} + p\lambda_{\alpha\beta 0} \tag{2.48}$$

$$C_2^{-1} V_{qd0s} = -R_s C_2^{-1} I_{qd0s} + p\left[C_2^{-1}\lambda_{qd0s}\right] \tag{2.49}$$

$$V_{qd0s} = -R_s C_2 C_2^{-1} I_{qd0s} + C_2 p\left[C_2^{-1}\lambda_{qd0s}\right] \tag{2.50}$$

$$V_{qd0s} = -R_s I_{qd0s} + C_2 p\left[C_2^{-1}\right]\lambda_{qd0s} + C_2 C_2^{-1} p\left[\lambda_{qd0s}\right] \tag{2.51}$$

$$\Rightarrow \quad V_{qd0s} = -R_s I_{qd0s} + C_2 p\left[C_2^{-1}\right]\lambda_{qd0s} + p\left[\lambda_{qd0s}\right] \tag{2.52}$$

$$\Rightarrow \quad V_{qd0s} = -R_s I_{qd0s} + \omega\lambda_{dq0s} + p\left[\lambda_{qd0s}\right] \tag{2.53}$$

The resultant stator and field voltage equations in arbitrary reference frame is

$$V_{qd0s} = -R_s I_{qd0s} + \omega\lambda_{dq0s} + p\lambda_{qd0s} \tag{2.54}$$

$$V_f' = R_f' I_f' + p\lambda_f' \tag{2.55}$$

Where

$$\left(\lambda_{dq0s}\right)^T = \left[\lambda_{ds} \quad -\lambda_{qs} \quad \lambda_{0s}\right] \tag{2.56}$$

For a linear magnetic system, the flux linkage equations may be expressed with the transformation of the stator variables to the arbitrary reference frame incorporated

$$\begin{bmatrix} \lambda_{qd0s} \\ \lambda'_f \end{bmatrix} = \begin{bmatrix} C_2 L_{s\alpha} C_2^{-1} & C_2 L'_{sf\alpha} \\ \frac{2}{3}(L'_{sf\alpha})^T C_2^{-1} & L'_f \end{bmatrix} \begin{bmatrix} -I_{qd0s} \\ I'_f \end{bmatrix} \tag{2.57}$$

It can be shown that all terms of the inductance matrix as shown above

$$C_2 L_{s\alpha} C_2^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.58}$$

$$\Rightarrow\ L_s = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \tag{2.59}$$

$$C_2 L'_{sf\alpha} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_m \sin\theta_r \\ L_m \cos\theta_r \\ 0 \end{bmatrix} \tag{2.60}$$

$$\Rightarrow\quad L'_{sf} = \begin{bmatrix} L_m \sin(\theta_r - \theta) \\ L_m \cos(\theta_r - \theta) \\ 0 \end{bmatrix} \tag{2.61}$$

$$\frac{2}{3}(L'_{sf\alpha})^T C_2^{-1} = \frac{2}{3}\begin{bmatrix} L_m \sin\theta_r & L_m \cos\theta_r & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.62}$$

$$\Rightarrow\quad (L'_{sf})^T = \begin{bmatrix} L_m \sin(\theta_r - \theta) & L_m \cos(\theta_r - \theta) & 0 \end{bmatrix} \tag{2.63}$$

$$\Rightarrow \; L'_f = [L'_f] \tag{2.64}$$

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda'_f \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m \sin(\theta_r - \theta) \\ 0 & L_s & L_m \cos(\theta_r - \theta) \\ L_m \sin(\theta_r - \theta) & L_m \cos(\theta_r - \theta) & L'_f \end{bmatrix} \begin{bmatrix} -I_{qs} \\ -I_{ds} \\ I'_f \end{bmatrix} \tag{2.65}$$

Therefore,

$$\lambda_{qs} = -L_s I_{qs} + L_m \sin(\theta_r - \theta) I'_f \tag{2.66}$$

$$\lambda_{ds} = -L_s I_{ds} + L_m \cos(\theta_r - \theta) I'_f \tag{2.67}$$

$$\lambda'_f = -L_m \sin(\theta_r - \theta) I_{qs} - L_m \cos(\theta_r - \theta) I_{ds} + L'_f I'_f \tag{2.68}$$

The voltage equation is given below;

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V'_f \end{bmatrix} = \begin{bmatrix} -R_s & 0 & 0 \\ 0 & -R_s & 0 \\ 0 & 0 & R'_f \end{bmatrix} \begin{bmatrix} I_{qs} \\ I_{ds} \\ I'_f \end{bmatrix} + \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda'_f \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda'_f \end{bmatrix} \tag{2.69}$$

Hence,

$$V_{qs} = -R_s I_{qs} + \omega \lambda_{ds} + p\lambda_{qs} \tag{2.70}$$

$$V_{ds} = -R_s I_{ds} - \omega \lambda_{qs} + p\lambda_{ds} \tag{2.71}$$

$$V'_f = R'_f I'_f + p\lambda'_f \tag{2.72}$$

If the flux linkage equation are substituted into the above voltage equations

$$V_{qs} = (-R_s - pL_s)I_{qs} - \omega L_s I_{ds} + (\omega L_m Cos(\theta_r - \theta) + pL_m Sin(\theta_r - \theta))I'_f \tag{2.73}$$

$$V_{ds} = (-R_s - pL_s)I_{ds} + \omega L_s I_{qs} + (-\omega L_m Sin(\theta_r - \theta) + pL_m Cos(\theta_r - \theta))I'_f \tag{2.74}$$

$$V'_f = -pL_m Sin(\theta_r - \theta)I_{qs} - pL_m Cos(\theta_r - \theta)I_{ds} + (R'_f + pL'_f)I'_f \tag{2.75}$$

Let us define, E'_{xf} as follows;

$$E'_{xf} = \frac{X_m}{R'_f} V'_f \tag{2.76}$$

Therefore, substituting (2.75) into (2.76)

$$E'_{xf} = -\frac{X_m}{R'_f}(pL_m Sin(\theta_r - \theta))I_{qs} - \frac{X_m}{R'_f}(pL_m Cos(\theta_r - \theta))I_{ds} + \frac{X_m}{R'_f}(R'_f + pL'_f)I'_f \tag{2.77}$$

Then the voltage equation matrix is

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ E'_{xf} \end{bmatrix} = \begin{bmatrix} -R_s - pL_s & -\omega L_s & \omega L_m Cos(\theta_r - \theta) + pL_m Sin(\theta_r - \theta) \\ \omega L_s & -R_s - pL_s & -\omega L_m Sin(\theta_r - \theta) + pL_m Cos(\theta_r - \theta) \\ -\frac{X_m}{R'_f}(pL_m Sin(\theta_r - \theta)) & -\frac{X_m}{R'_f}(pL_m Cos(\theta_r - \theta)) & \frac{X_m}{R'_f}(R'_f + pL'_f) \end{bmatrix} \begin{bmatrix} I_{qs} \\ I_{ds} \\ I'_f \end{bmatrix}$$

(2.78)

The expression for the electromagnetic torque in terms of arbitrary reference frame variables is that;

$$T_e = \left(\frac{P}{2}\right)\left[(K_s)^{-1} I_{qd0s}\right]^T \left\{-\frac{1}{2}\frac{d}{d\theta}[L_s](K_s)^{-1} I_{qd0s} + \frac{d}{d\theta}[L'_{sf}]I_{qdf}\right\} \tag{2.79}$$

Where $K_s = C_2 C_1$

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)L_m I_f' \left[I_{qs}\cos(\theta_r - \theta) - I_{ds}\sin(\theta_r - \theta)\right] \tag{2.80}$$

2.1.4. Synchronously Rotating Reference Frame

Let us define $\delta=\theta_r-\theta$; then flux linkage equations (2.66),(2.67) and (2.68) are;

$$\lambda_{qs} = -L_s I_{qs} + L_m \sin\delta\, I_f' \tag{2.81}$$

$$\lambda_{ds} = -L_s I_{ds} + L_m \cos\delta\, I_f' \tag{2.82}$$

$$\lambda_f' = -L_m \sin\delta\, I_{qs} - L_m \cos\delta\, I_{ds} + L_f' I_f' \tag{2.83}$$

Then we get ds and qs axis currents;

$$I_{ds} = -\frac{\lambda_{ds}}{L_s} + \frac{L_m}{L_s}\cos\delta\, I_f' \tag{2.84}$$

$$I_{qs} = -\frac{\lambda_{qs}}{L_s} + \frac{L_m}{L_s}\sin\delta\, I_f' \tag{2.85}$$

If we substitute $\omega=\omega_e$ in the voltage equations (2.70),(2.71) and we substitute current equations (2.84), (2.85) in this voltage equations, the flux linkage equations are;

$$\frac{d}{dt}\lambda_{ds} = -\frac{R}{L_s}\lambda_{ds} + \omega_e\lambda_{qs} + \frac{RL_m}{L_s}I_f'\cos\delta + V_{ds} \tag{2.86}$$

$$\frac{d}{dt}\lambda_{qs} = -\frac{R}{L_s}\lambda_{qs} - \omega_e\lambda_{ds} + \frac{RL_m}{L_s}I_f'\sin\delta + V_{qs} \tag{2.87}$$

The voltage equation is

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V'_f \end{bmatrix} = \begin{bmatrix} -R_s & 0 & 0 \\ 0 & -R_s & 0 \\ 0 & 0 & R'_f \end{bmatrix} \begin{bmatrix} I_{qs} \\ I_{ds} \\ I'_f \end{bmatrix} + \begin{bmatrix} 0 & \omega_e & 0 \\ -\omega_e & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda'_f \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda'_f \end{bmatrix} \tag{2.88}$$

Therefore,

$$V_{qs} = -R_s I_{qs} + \omega_e \lambda_{ds} + p\lambda_{qs} \tag{2.89}$$

$$V_{ds} = -R_s I_{ds} - \omega_e \lambda_{qs} + p\lambda_{ds} \tag{2.90}$$

$$V'_f = R'_f I'_f + p\lambda'_f \tag{2.91}$$

If the flux linkage equations are substituted into the above voltage equations

$$V_{qs} = (-R_s - pL_s) I_{qs} - \omega_e L_s I_{ds} + (\omega_e L_m \mathrm{Cos}\delta + pL_m \mathrm{Sin}\delta) I'_f \tag{2.92}$$

$$V_{ds} = (-R_s - pL_s) I_{ds} + \omega_e L_s I_{qs} + (-\omega_e L_m \mathrm{Sin}\delta + pL_m \mathrm{Cos}\delta) I'_f \tag{2.93}$$

$$V'_f = -pL_m \mathrm{Sin}\delta\, I_{qs} - pL_m \mathrm{Cos}\delta\, I_{ds} + (R'_f + pL'_f) I'_f \tag{2.94}$$

Hence,

$$E'_{xf} = -\frac{X_m}{R'_f}(pL_m \mathrm{Sin}\delta) I_{qs} - \frac{X_m}{R'_f}(pL_m \mathrm{Cos}\delta) I_{ds} + \frac{X_m}{R'_f}(R'_f + pL'_f) I'_f \tag{2.95}$$

Then the result voltage equation is

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ E'_{xf} \end{bmatrix} = \begin{bmatrix} -R_s - pL_s & -\omega_e L_s & \omega_e L_m \mathrm{Cos}(\delta) + pL_m \mathrm{Sin}(\delta) \\ \omega_e L_s & -R_s - pL_s & -\omega_e L_m \mathrm{Sin}(\delta) + pL_m \mathrm{Cos}(\delta) \\ -\frac{X_m}{R'_f}(pL_m \mathrm{Sin}\delta) & -\frac{X_m}{R'_f}(pL_m \mathrm{Cos}\delta) & \frac{X_m}{R'_f}(R'_f + pL'_f) \end{bmatrix} \begin{bmatrix} I_{qs} \\ I_{ds} \\ I'_f \end{bmatrix}$$

(2.96)

The electromagnetic torque equation (2.97) can be obtained from equation (6.5-35) of (Krause, P.C.,& Wasynczuk, S., 1989). And we substitute equations (2.81) and (2.82) into the torque equation, and we get;

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_{ds} I_{qs} - \lambda_{qs} I_{ds}\right) \tag{2.97}$$

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left[\left(-L_s I_{ds} + L_m \cos\delta\, I'_f\right) I_{qs} - \left(-L_s I_{qs} + L_m \sin\delta\, I'_f\right) I_{ds}\right] \tag{2.98}$$

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left[L_m I'_f I_{qs} \cos\delta - L_m I'_f I_{ds} \sin\delta\right] \tag{2.99}$$

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left[L_m I'_f \left(I_{qs} \cos\delta - I_{ds} \sin\delta\right)\right] \tag{2.100}$$

Where the P is the pole number.

2.1.5. Voltage Equations in Arbitrary Reference Frame

Although the transformation equations are valid regardless of the waveform of the variables, it is instructive to consider the characteristics of the transformation when the 3-phase system is symmetrical and the voltages form a balanced 3-phase set of abc sequence as given in the below. A balanced 3-phase set is generally defined as a set of equal-amplitude sinusoidal quantities which are displaced by 120 degree. Since the sum of this set is zero, the 0s variables are zero.

$$V_{as} = \sqrt{2}V\mathrm{Cos}(\omega_e t) \tag{2.101}$$

$$V_{bs} = \sqrt{2}V\mathrm{Cos}(\omega_e t - \frac{2\pi}{3}) \tag{2.102}$$

$$V_{cs} = \sqrt{2}V\mathrm{Cos}(\omega_e t + \frac{2\pi}{3}) \tag{2.103}$$

$$V_{\alpha\beta o} = C_1 V_{abcs} \tag{2.104}$$

$$\begin{bmatrix} V_\alpha \\ V_\beta \\ V_o \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \sqrt{2}V\mathrm{Cos}(\omega_e t) \\ \sqrt{2}V\mathrm{Cos}(\omega_e t - \frac{2\pi}{3}) \\ \sqrt{2}V\mathrm{Cos}(\omega_e t + \frac{2\pi}{3}) \end{bmatrix} \tag{2.105}$$

$$V_\alpha = \sqrt{2}V\mathrm{Cos}(\omega_e t) \tag{2.106}$$

$$V_\beta = -\sqrt{2}V\mathrm{Sin}(\omega_e t) \tag{2.107}$$

$$V_{qds} = C_2 V_{\alpha\beta} \tag{2.108}$$

$$\begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} = \begin{bmatrix} \mathrm{Cos}\theta & -\mathrm{Sin}\theta \\ \mathrm{Sin}\theta & \mathrm{Cos}\theta \end{bmatrix}\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \tag{2.109}$$

$$\begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} = \begin{bmatrix} \mathrm{Cos}\theta & -\mathrm{Sin}\theta \\ \mathrm{Sin}\theta & \mathrm{Cos}\theta \end{bmatrix}\begin{bmatrix} \sqrt{2}V\mathrm{Cos}(\omega_e t) \\ -\sqrt{2}V\mathrm{Sin}(\omega_e t) \end{bmatrix} \tag{2.110}$$

The voltage equations in arbitrary reference frame are as follows;

$$V_{qs} = \sqrt{2}V\mathrm{Cos}(\omega_e t - \theta) \tag{2.111}$$

$$V_{ds} = -\sqrt{2}V\mathrm{Sin}(\omega_e t - \theta) \tag{2.112}$$

If we substitute $\theta=\omega t$ and $\omega=\omega_e$ in the equations (2.109) and (2.110), then we get the voltages in synchronously rotating reference frame.

$$V_{ds} = -\sqrt{2}V\mathrm{Sin}(\omega_e t - \omega_e t) \tag{2.113}$$

$$\Rightarrow \quad V_{ds} = 0 \tag{2.114}$$

$$V_{qs} = \sqrt{2}V\mathrm{Cos}(\omega_e t - \omega_e t) \tag{2.115}$$

$$\Rightarrow \quad V_{qs} = \sqrt{2}V \tag{2.116}$$

2.1.6. Machine Equations in Rotor Reference Frame

R. H. Park was the first to incorporate a change of variables in the analysis of synchronous machine. He transformed the stator variables to the rotor reference frame which eliminates the time-varying inductances in the voltage equations. In this section the speed of the arbitrary reference frame equal to the rotor speed ($\omega=\omega_r$). Thus

$$V^r_{qd0s} = -R_s I^r_{qd0s} + \omega_r \lambda^r_{dqs} + p\lambda^r_{qd0s} \tag{2.117}$$

$$V'^r_f = R'_f I'^r_f + p\lambda'^r_f \tag{2.118}$$

Where

$$\left(\lambda_{dqs}^{r}\right)^{T}=\left[\lambda_{ds}^{r}\quad -\lambda_{qs}^{r}\quad 0\right] \tag{2.119}$$

For a magnetically linear system, the flux linkages may be expressed in the rotor reference frame by setting $\theta=\theta_r$ and

$$\begin{bmatrix}\lambda_{qs}^{r}\\ \lambda_{ds}^{r}\\ \lambda_{0s}^{r}\\ \lambda_{f}^{\prime r}\end{bmatrix}=\begin{bmatrix}L_s & 0 & 0 & 0\\ 0 & L_s & 0 & L_m\\ 0 & 0 & L_{ls} & 0\\ 0 & L_m & 0 & L_f^{\prime}\end{bmatrix}\begin{bmatrix}-I_{qs}^{r}\\ -I_{ds}^{r}\\ -I_{0s}^{r}\\ I_{f}^{\prime r}\end{bmatrix} \tag{2.120}$$

Thus the resultant flux linkage equations are obtained as follows by omitting the zero sequence of the flux linkage

$$\lambda_{qs}^{r}=-L_sI_{qs}^{r} \tag{2.121}$$

$$\lambda_{ds}^{r}=-L_sI_{ds}^{r}+L_mI_f^{\prime r} \tag{2.122}$$

$$\lambda_{f}^{\prime r}=-L_mI_{ds}^{r}+L_f^{\prime}I_f^{\prime r} \tag{2.123}$$

The voltage equation is

$$\begin{bmatrix}V_{qs}^{r}\\ V_{ds}^{r}\\ V_{0s}^{r}\\ V_{f}^{\prime r}\end{bmatrix}=\begin{bmatrix}-R_s & 0 & 0 & 0\\ 0 & -R_s & 0 & 0\\ 0 & 0 & -R_s & 0\\ 0 & 0 & 0 & R_f^{\prime}\end{bmatrix}\begin{bmatrix}I_{qs}^{r}\\ I_{ds}^{r}\\ I_{0s}^{r}\\ I_{f}^{\prime r}\end{bmatrix}+\begin{bmatrix}0 & \omega_r & 0 & 0\\ -\omega_r & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\lambda_{qs}^{r}\\ \lambda_{ds}^{r}\\ \lambda_{0s}^{r}\\ \lambda_{f}^{\prime r}\end{bmatrix}+\frac{d}{dt}\begin{bmatrix}\lambda_{qs}^{r}\\ \lambda_{ds}^{r}\\ \lambda_{0s}^{r}\\ \lambda_{f}^{\prime r}\end{bmatrix} \tag{2.124}$$

Voltage equations are often written in expended form thus

$$V_{qs}^{r} = -R_s I_{qs}^{r} + \omega_r \lambda_{ds}^{r} + p\lambda_{qs}^{r} \tag{2.125}$$

$$V_{ds}^{r} = -R_s I_{ds}^{r} - \omega_r \lambda_{qs}^{r} + p\lambda_{ds}^{r} \tag{2.126}$$

$$V_f'^{r} = R_f' I_f'^{r} + p\lambda_f'^{r} \tag{2.127}$$

If the flux linkage equations are substituted into the above voltage equations

$$V_{qs}^{r} = \left(-R_s - pL_s\right) I_{qs}^{r} - \omega_r L_s I_{ds}^{r} + \omega_r L_m I_f'^{r} \tag{2.128}$$

$$V_{ds}^{r} = \left(-R_s - pL_s\right) I_{ds}^{r} + \omega_r L_s I_{qs}^{r} + pL_m I_f' \tag{2.129}$$

$$V_f'^{r} = -pL_m I_{ds}^{r} + \left(R_f' + pL_f'\right) I_f'^{r} \tag{2.130}$$

Hence,

$$E_{xf}'^{r} = -\frac{X_m}{R_f'}\left(pL_m\right) I_{ds}^{r} + \frac{X_m}{R_f'}\left(R_f' + pL_f'\right) I_f'^{r} \tag{2.131}$$

The voltage equations for the machine are given in terms of currents and flux linkages. As we have pointed out earlier the current and flux linkages are related and both cannot be independent or state variables. We will need to express the voltage equations in terms of either currents or flux linkages when formulating the system and simulating in a computer program.

If we select the currents as independent variables the flux linkages are replaced by currents and the voltage equations, then

$$\begin{bmatrix} V_{qs}^{r} \\ V_{ds}^{r} \\ E_{xf}^{\prime r} \end{bmatrix} = \begin{bmatrix} -R_s - pL_s & -\omega_r L_s & \omega_r L_m \\ \omega_r L_s & -R_s - pL_s & pL_m \\ 0 & -\frac{X_m}{R_f^{\prime}}(pL_m) & \frac{X_m}{R_f^{\prime}}\left(R_f^{\prime} + pL_f^{\prime}\right) \end{bmatrix} \begin{bmatrix} I_{qs}^{r} \\ I_{ds}^{r} \\ I_f^{\prime r} \end{bmatrix} \tag{2.132}$$

The electromagnetic torque equation can be obtained from equations (2.121) and (2.122);

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_{ds}^{r} I_{qs}^{r} - \lambda_{qs}^{r} I_{ds}^{r}\right) \tag{2.133}$$

$$Te = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(L_m I_{qs}^{r} I_f^{\prime r}\right) \tag{2.134}$$

2.1.7. Stator Currents in Rotor Reference Frame

The procedure, as shown in the below, is used to obtain stator phase currents I_{as}, I_{bs}, I_{cs} in the arbitrary reference frame. Firstly, rotor reference frame to arbitrary reference frame the transformation of quadrature and direct axis currents is carried out;

$$\begin{bmatrix} I_{qs} \\ I_{ds} \end{bmatrix} = \begin{bmatrix} \mathrm{Cos}(\theta - \theta_r) & -\mathrm{Sin}(\theta - \theta_r) \\ \mathrm{Sin}(\theta - \theta_r) & \mathrm{Cos}(\theta - \theta_r) \end{bmatrix} \begin{bmatrix} I_{qs}^{r} \\ I_{ds}^{r} \end{bmatrix} \tag{2.135}$$

$$I_{qs} = I_{qs}^{r}\mathrm{Cos}(\theta - \theta_r) - I_{ds}^{r}\mathrm{Sin}(\theta - \theta_r) \tag{2.136}$$

$$I_{ds} = I_{qs}^{r}\mathrm{Sin}(\theta - \theta_r) + I_{ds}^{r}\mathrm{Cos}(\theta - \theta_r) \tag{2.137}$$

Then qd0/abc transformation of currents is carried out by using this equations as shown in the below,

$$I_{abcs} = K_s^{-1} I_{qd0} \tag{2.138}$$

$$\begin{bmatrix} I_{as} \\ I_{bs} \\ I_{cs} \end{bmatrix} = \begin{bmatrix} \text{Cos}\theta & \text{Sin}\theta & 1 \\ \text{Cos}\left(\theta - \frac{2\pi}{3}\right) & \text{Sin}\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \text{Cos}\left(\theta + \frac{2\pi}{3}\right) & \text{Sin}\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \begin{bmatrix} I_{qs} \\ I_{ds} \end{bmatrix} \tag{2.139}$$

$$\begin{bmatrix} I_{as} \\ I_{bs} \\ I_{cs} \end{bmatrix} = \begin{bmatrix} \text{Cos}\theta & \text{Sin}\theta & 1 \\ \text{Cos}\left(\theta - \frac{2\pi}{3}\right) & \text{Sin}\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \text{Cos}\left(\theta + \frac{2\pi}{3}\right) & \text{Sin}\left(\theta - \frac{2\pi}{3}\right) & 1 \end{bmatrix} \begin{bmatrix} I_{qs}^r \text{Cos}(\theta - \theta_r) - I_{ds}^r \text{Sin}(\theta - \theta_r) \\ I_{qs}^r \text{Sin}(\theta - \theta_r) + I_{ds}^r \text{Cos}(\theta - \theta_r) \end{bmatrix} \tag{2.140}$$

The procedure, as shown in the below, is used to obtain stator phase voltages V_{as}, V_{bs}, V_{cs} in the arbitrary reference frame. Firstly, rotor reference frame to arbitrary reference frame transformation of quadrature and direct axis currents is carried out;

$$\begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} = \begin{bmatrix} \text{Cos}(\theta - \theta_r) & -\text{Sin}(\theta - \theta_r) \\ \text{Sin}(\theta - \theta_r) & \text{Cos}(\theta - \theta_r) \end{bmatrix} \begin{bmatrix} V_{qs}^r \\ V_{ds}^r \end{bmatrix} \tag{2.141}$$

$$V_{qs} = V_{qs}^r \text{Cos}(\theta - \theta_r) - V_{ds}^r \text{Sin}(\theta - \theta_r) \tag{2.142}$$

$$V_{ds} = V_{qs}^r \text{Sin}(\theta - \theta_r) + V_{ds}^r \text{Cos}(\theta - \theta_r) \tag{2.143}$$

Then qd0/abc transformation of voltages is carried out by using this equations as shown in the below,

$$V_{abcs} = K_s^{-1} V_{qd0} \tag{2.144}$$

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} = \begin{bmatrix} Cos\theta & Sin\theta & 1 \\ Cos\left(\theta - \frac{2\pi}{3}\right) & Sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ Cos\left(\theta + \frac{2\pi}{3}\right) & Sin\left(\theta - \frac{2\pi}{3}\right) & 1 \end{bmatrix} \begin{bmatrix} V_{qs} \\ V_{ds} \end{bmatrix} \tag{2.145}$$

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} = \begin{bmatrix} Cos\theta & Sin\theta & 1 \\ Cos\left(\theta - \frac{2\pi}{3}\right) & Sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ Cos\left(\theta + \frac{2\pi}{3}\right) & Sin\left(\theta - \frac{2\pi}{3}\right) & 1 \end{bmatrix} \begin{bmatrix} V_{qs}^{r} Cos(\theta - \theta_r) - V_{ds}^{r} Sin(\theta - \theta_r) \\ V_{qs}^{r} Sin(\theta - \theta_r) + V_{ds}^{r} Cos(\theta - \theta_r) \end{bmatrix} \tag{2.146}$$

2.2. Mathematical Model of Brushless Synchronous Generator with Rectifier Load

2.2.1. Mathematical Model of Brushless Synchronous Generator with Rectifier Load without l_f

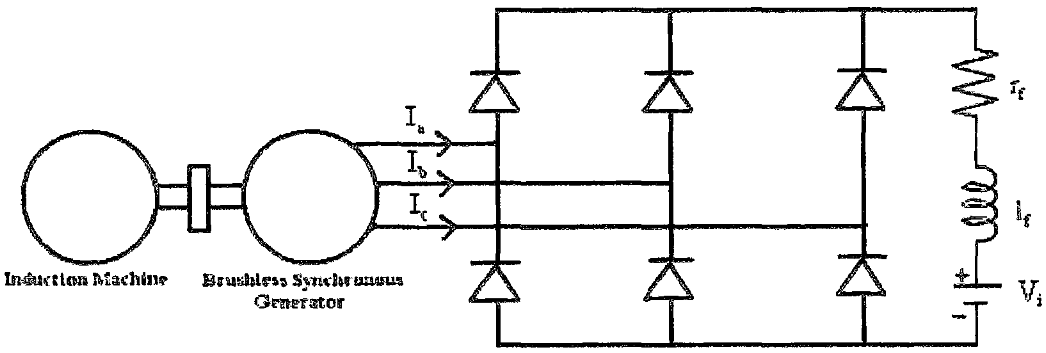

Figure 2.3 Brushless synchronous generator with rectifier load

The configuration of the brushless synchronous generator with rectifier is shown schematically in Figure 2.3.

A well-known approach in the dynamic modeling of such systems is a detailed description and simulation of the switching process and exact calculation of the voltage and current waveforms. These models are nonlinear and describe the cyclic behavior.

In this study the dynamic model is based on the average value of real power which is same at the input and output of rectifier.

Also, the overlap angle is neglected and the input power factor of rectifier is assumed to be unity. The rectifier input currents are assumed to be sinusoidal.

$$V_{dc} = l_f p I_{dc} + r_f I_{dc} + V_i \tag{2.147}$$

$$V_{dc} = \frac{3\sqrt{3}}{\pi} V_s \cos\alpha_1 \quad (\alpha_1 = 0) \tag{2.148}$$

$$V_s = \left(V'^{r2}_{qs} + V'^{r2}_{ds}\right)^{1/2} \tag{2.149}$$

As shown in the above equations, V_{dc} is the output of the rectifier voltage and r_f, l_f, and V_i are the load resistance, inductance , and power supply respectively. The real power at the input and output of rectifier is that;

$$\frac{3}{2}\left(V^r_{qs} I^r_{qs} + V^r_{ds} I^r_{ds}\right) = V_{dc} I_{dc} \qquad (\text{Real Power}) \tag{2.150}$$

And the reactive power input for rectifier is that;

$$V^r_{ds} I^r_{ds} - V^r_{qs} I^r_{ds} = 0 \qquad (\text{Reactive Power Output for Rectifier}) \tag{2.151}$$

And then;

$$\frac{3\sqrt{3}}{\pi}V_s = l_f pI_{dc} + r_f I_{dc} + V_i \tag{2.152}$$

Hence, the voltage equation in the rotor reference frame may be rearranged as follows;

$$\begin{bmatrix} V_{qs}^r \\ V_{ds}^r \\ E_{xf}'^r \end{bmatrix} = \begin{bmatrix} -R_s & -\omega_r L_s & \omega_r L_m \\ \omega_r L_s & -R_s & 0 \\ 0 & 0 & X_m \end{bmatrix} \begin{bmatrix} I_{qs}^r \\ I_{ds}^r \\ I_f'^r \end{bmatrix} + \begin{bmatrix} -L_s & 0 & 0 \\ 0 & -L_s & L_m \\ 0 & \dfrac{-X_m L_m}{R_f'} & \dfrac{X_m}{R_f'} L_f' \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_{qd}^r \\ I_{ds}^r \\ I_f'^r \end{bmatrix}$$

(2.153)

$$X_m = \omega_b L_m \tag{2.154}$$

$$\begin{bmatrix} \dfrac{dI_{qs}^r}{dt} \\ \dfrac{dI_{ds}^r}{dt} \\ \dfrac{dI_f'^r}{dt} \end{bmatrix} = \begin{bmatrix} -L_s & 0 & 0 \\ 0 & -L_s & L_m \\ 0 & -\dfrac{X_m L_m}{R_f'} & \dfrac{X_m}{R_f'} L_f' \end{bmatrix}^{-1} \left\{ \begin{bmatrix} V_{qs}^r \\ V_{ds}^r \\ E_{xf}'^r \end{bmatrix} - \begin{bmatrix} -R_s & -\omega_r L_s & \omega_r L_m \\ \omega_r L_s & -R_s & 0 \\ 0 & 0 & X_m \end{bmatrix} \begin{bmatrix} I_{qs}^r \\ I_{ds}^r \\ I_f'^r \end{bmatrix} \right\}$$

(2.155)

$$\begin{bmatrix} -L_s & 0 & 0 \\ 0 & -L_s & L_m \\ 0 & -\dfrac{X_m L_m}{R_f'} & \dfrac{X_m L_{fd}'}{R_f'} \end{bmatrix}^{-1} = \begin{bmatrix} -\dfrac{1}{L_s} & 0 & 0 \\ 0 & -\dfrac{L_f'}{L_s L_f' - L_m^2} & \dfrac{L_m R_f'}{X_m\left(L_s L_f' - L_m^2\right)} \\ 0 & -\dfrac{L_m}{L_s L_f' - L_m^2} & \dfrac{L_s R_f'}{X_m\left(L_s L_f' - L_m^2\right)} \end{bmatrix}$$

(2.156)

$$\begin{bmatrix} \frac{dI_{qs}^{r}}{dt} \\ \frac{dI_{ds}^{r}}{dt} \\ \frac{dI_{f}^{\prime r}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{L_s} & 0 & 0 \\ 0 & -\frac{L_f'}{L_s L_f' - L_m^2} & \frac{L_m R_f'}{X_m \left(L_s L_f' - L_m^2\right)} \\ 0 & -\frac{L_m}{L_s L_f' - L_m^2} & \frac{L_s R_f'}{X_m \left(L_s L_f' - L_m^2\right)} \end{bmatrix} \begin{bmatrix} V_{qs}^{r} + R_s I_{qs}^{r} + \omega_r L_s I_{ds}^{r} - \omega_r L_m I_f^{\prime r} \\ V_{ds}^{r} - \omega_r L_s I_{qs}^{r} + R_s I_{ds}^{r} \\ E_{xf}' - X_m I_f^{\prime r} \end{bmatrix} \tag{2.157}$$

The main eight equations are;

$$\frac{dI_{qs}^{r}}{dt} = -\frac{1}{L_s}\left(V_{qs}^{r} + R_s I_{qs}^{r} + \omega_r L_s I_{ds}^{r} - \omega_r L_m I_f^{\prime r}\right) \tag{2.158}$$

$$\frac{dI_{ds}^{r}}{dt} = \left(-\frac{L_f'}{L_s L_f' - L_m^2}\right)\left(V_{ds}^{r} - \omega_r L_s I_{qs}^{r} + R_s I_{ds}^{r}\right) + \left(\frac{L_m R_f'}{X_m \left(L_s L_f' - L_m^2\right)}\right)\left(E_{xf}' - X_m I_f^{\prime r}\right) \tag{2.159}$$

$$\frac{dI_{f}^{\prime r}}{dt} = \left(-\frac{L_m}{L_s L_f' - L_m^2}\right)\left(V_{ds}^{r} - \omega_r L_s I_{qs}^{r} + R_s I_{ds}^{r}\right) + \left(\frac{L_s R_f'}{X_m \left(L_s L_f' - L_m^2\right)}\right)\left(E_{xf}' - X_m I_f^{\prime r}\right) \tag{2.160}$$

$$\frac{3}{2}\left(V_{qs}^{r} I_{qs}^{r} + V_{ds}^{r} I_{ds}^{r}\right) = \frac{3\sqrt{3}}{\pi}\sqrt{V_{qs}^{r\,2} + V_{ds}^{r\,2}}\, I_{dc} \tag{2.161}$$

$$V_{ds}^{r} I_{qs}^{r} - V_{qs}^{r} I_{ds}^{r} = 0 \tag{2.162}$$

$$\frac{dI_{dc}}{dt} = \frac{1}{l_f}\left\{\frac{3\sqrt{3}}{\pi}\sqrt{V_{qs}^{r\,2} + V_{ds}^{r\,2}} - r_f I_{dc} - V_i\right\} \tag{2.163}$$

$$\frac{d\omega_r}{dt} = -\frac{1}{J}\left(T_e - T_L\right) \tag{2.164}$$

Stator voltage equation are described again to solve R variable, which identify as a load instead of r_f, l_f, and V_i.

$$V_{qs}^r = RI_{qs}^r \tag{2.165}$$

$$V_{ds}^r = RI_{ds}^r \tag{2.166}$$

$$\Rightarrow \frac{3}{2}\left(RI_{qs}^{r\;2} + RI_{ds}^{r\;2}\right) = \frac{3\sqrt{3}}{\pi}\sqrt{V_{qs}^{r\;2} + V_{ds}^{r\;2}}\, I_{dc} \quad \text{(Real Power)} \tag{2.167}$$

$$I_{dc} = \frac{\frac{3\sqrt{3}}{\pi}\sqrt{V_{qs}^{r\;2} + V_{ds}^{r\;2}} - V_i}{r_f} \quad \text{(dc link inductance } l_f \text{ is neglected)} \tag{2.168}$$

$$\Rightarrow \frac{3}{2}R\left(I_{qs}^{r\;2} + I_{ds}^{r\;2}\right) = \frac{3\sqrt{3}}{\pi}R\sqrt{I_{qs}^{r\;2} + I_{ds}^{r\;2}}\left(\frac{\frac{3\sqrt{3}}{\pi}R\sqrt{I_{qs}^{r\;2} + I_{ds}^{r\;2}} - V_i}{r_f}\right) \tag{2.169}$$

$$\Rightarrow r_f\left(I_{qs}^{r\;2} + I_{ds}^{r\;2}\right)\frac{3}{2} = \left(\frac{3\sqrt{3}}{\pi}\right)^2 R\left(I_{qs}^{r\;2} + I_{ds}^{r\;2}\right) - \frac{3\sqrt{3}}{\pi}V_i\sqrt{I_{qs}^{r\;2} + I_{ds}^{r\;2}} \tag{2.170}$$

$$\Rightarrow R = \frac{\frac{3}{2}\left(I_{qs}^{r\;2} + I_{ds}^{r\;2}\right)r_f + \frac{3\sqrt{3}}{\pi}V_i\sqrt{I_{qs}^{r\;2} + I_{ds}^{r\;2}}}{\left(\frac{3\sqrt{3}}{\pi}\right)^2\left(I_{qs}^{r\;2} + I_{ds}^{r\;2}\right)} \tag{2.171}$$

$$\Rightarrow R = \frac{\frac{3}{2}r_f}{\left(\frac{27}{\pi^2}\right)} + \frac{\left(\frac{3\sqrt{3}}{\pi}V_i\right)}{\left(\frac{27}{\pi^2}\right)}\frac{1}{\sqrt{I_{qs}^{r\;2} + I_{ds}^{r\;2}}} \tag{2.172}$$

$$\Rightarrow \text{ if } V_i = 0 \tag{2.173}$$

The result R variable is;

$$\Rightarrow R = \frac{\frac{3}{2} r_f}{\left(\frac{3\sqrt{3}}{\pi}\right)^2} \tag{2.174}$$

$$\Rightarrow R = \frac{\pi^2}{18} r_f \tag{2.175}$$

2.2.2. Mathematical Model of Brushless Synchronous Generator with Rectifier Load with l_f and V_i

The derivation of the dc current of the output rectifier is given in the below;

$$\frac{dI_{dc}}{dt} = \frac{1}{l_f}\left[\frac{3\sqrt{3}}{\pi}\sqrt{V_{qs}^{r\ 2} + V_{ds}^{r\ 2}} - r_f I_{dc} - V_i\right] \ (\text{KVL}) \tag{2.176}$$

$$V_{qs}^r = RI_{qs}^r \qquad V_{ds}^r = RI_{ds}^r \tag{2.177}$$

$$\frac{3}{2}\left(RI_{qs}^{r\ 2} + RI_{ds}^{r\ 2}\right) = \frac{3\sqrt{3}}{\pi} R \sqrt{I_{qs}^{r\ 2} + I_{ds}^{r\ 2}}\, I_{dc} \tag{2.178}$$

$$\frac{3}{2}\left(I_{qs}^{r\ 2} + I_{ds}^{r\ 2}\right) = \frac{3\sqrt{3}}{\pi} \sqrt{I_{qs}^{r\ 2} + I_{ds}^{r\ 2}}\, I_{dc} \tag{2.179}$$

$$\Rightarrow I_{dc} = \frac{\pi}{3\sqrt{3}} \frac{3}{2} \sqrt{I_{qs}^{r\ 2} + I_{ds}^{r\ 2}} \tag{2.180}$$

Derivative of the dc current is taken in order to find the R variable.

$$\frac{dI_{dc}}{dt} = \frac{1}{l_f}\left[\frac{3\sqrt{3}}{\pi} R\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2} - r_f I_{dc} - V_i\right] \text{ (KVL)} \tag{2.181}$$

$$\frac{dI_{dc}}{dt} = \frac{1}{l_f}\left[\frac{3\sqrt{3}}{\pi} R\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2} - r_f \frac{\pi}{3\sqrt{3}}\frac{3}{2}\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2} - V_i\right] \tag{2.182}$$

$$\frac{dI_{dc}}{dt} = \frac{\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2}}{l_f}\left[\frac{3\sqrt{3}}{\pi} R - \frac{\pi}{3\sqrt{3}}\frac{3}{2} r_f - \frac{V_i}{\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2}}\right] \tag{2.183}$$

$$I_{dc} = \frac{\pi}{3\sqrt{3}}\frac{3}{2}\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2} \qquad K = \frac{\pi}{3\sqrt{3}}\frac{3}{2} \tag{2.184}$$

$$\Rightarrow \ I_{dc} = K\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2} \tag{2.185}$$

$$\frac{dI_{dc}}{dt} = K\frac{1}{2}\left({I^r_{qs}}^2 + {I^r_{ds}}^2\right)^{-1/2} 2\left(I_{qs}\frac{dI^r_{qs}}{dt} + I_{ds}\frac{dI^r_{ds}}{dt}\right) \tag{2.186}$$

$$I^r_{qs}\frac{dI^r_{qs}}{dt} + I^r_{ds}\frac{dI^r_{ds}}{dt} = \frac{\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2}}{K}\left(\frac{dI_{dc}}{dt}\right) \tag{2.187}$$

Substitute KVL for $\frac{dI_{dc}}{dt}$

$$I^r_{qs}\frac{dI^r_{qs}}{dt} + I^r_{ds}\frac{dI^r_{ds}}{dt} = \frac{\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2}}{K}\frac{1}{l_f}\left(\frac{3\sqrt{3}}{\pi} R\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2} - r_f\frac{\pi}{3\sqrt{3}}\frac{3}{2}\sqrt{{I^r_{qs}}^2 + {I^r_{ds}}^2} - V_i\right)$$

(2.188)

$$\Rightarrow I_{qs}^{r}\frac{dI_{qs}^{r}}{dt}+I_{ds}^{r}\frac{dI_{ds}^{r}}{dt}=\frac{1}{K}\frac{1}{l_f}\left(\frac{3\sqrt{3}}{\pi}R\left(I_{qs}^{r\ 2}+I_{ds}^{r\ 2}\right)-r_f\frac{\pi}{3\sqrt{3}}\frac{3}{2}\left(I_{qs}^{r\ 2}+I_{ds}^{r\ 2}\right)-V_i\sqrt{I_{qs}^{r\ 2}+I_{ds}^{r\ 2}}\right)$$

(2.189)

$$\Rightarrow R=\left(I_{qs}^{r}\frac{dI_{qs}^{r}}{dt}+I_{ds}^{r}\frac{dI_{ds}^{r}}{dt}\right)\frac{1}{I_{qs}^{r\ 2}+I_{ds}^{r\ 2}}l_f\frac{\pi^2}{18}+r_f\frac{\pi^2}{18}+\frac{V_i}{\sqrt{I_{qs}^{r\ 2}+I_{ds}^{r\ 2}}}\frac{\pi}{3\sqrt{3}} \tag{2.190}$$

Derivative of the stator currents I_{qs} and I_{ds} are substituted into the above equation,

$$\frac{dI_{qs}^{r}}{dt}=-\frac{1}{L_s}\left(V_{qs}^{r}+R_sI_{qs}^{r}+\omega_rL_sI_{ds}^{r}-\omega_rL_mI_f'^{r}\right) \tag{2.191}$$

$$\frac{dI_{ds}^{r}}{dt}=\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)\left(V_{ds}^{r}-\omega_rL_sI_{qs}^{r}+R_sI_{ds}^{r}\right)+\left(\frac{L_mR_f'}{X_m\left(L_sL_f'-L_m^2\right)}\right)\left(E_{xf}-X_mI_f'^{r}\right)$$

(2.192)

$$\frac{dI_{qs}^{r}}{dt}=-\frac{1}{L_s}\left(RI_{qs}^{r}\right)-\frac{1}{L_s}\left(R_sI_{qs}^{r}+\omega_rL_sI_{ds}^{r}-\omega_rL_mI_f'^{r}\right) \tag{2.193}$$

$$\frac{dI_{ds}^{r}}{dt}=\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)\left(RI_{ds}^{r}\right)+\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)\left(-\omega_rL_sI_{qs}^{r}+R_sI_{ds}^{r}\right)+\left(\frac{L_mR_f'}{L_sL_f'-L_m^2}\right)\left(E_{xf}'-X_mI_f'^{r}\right)$$

(2.194)

$$R=\left[I_{qs}^{r}\left\{-\frac{1}{L_s}RI_{qs}^{r}-\frac{1}{L_s}\left[R_sI_{qs}^{r}+\omega_rL_sI_{ds}^{r}-\omega_rL_mI_f'^{r}\right]\right\}+I_{ds}^{r}\left\{\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)RI_{ds}^{r}+\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)\left(-\omega_rL_sI_{qs}^{r}+R_sI_{ds}^{r}\right)\right.\right.$$
$$\left.\left.+\left(\frac{L_mR_f'}{L_sL_f'-L_m^2}\right)\left(E_{xf}'-X_mI_f'^{r}\right)\right\}\right]\frac{1}{{I_{qs}^{r}}^2+{I_{ds}^{r}}^2}l_f\frac{\pi^2}{18}+r_f\frac{\pi^2}{18}+\frac{V_i}{\sqrt{{I_{qs}^{r}}^2+{I_{ds}^{r}}^2}}\frac{\pi}{3\sqrt{3}} \tag{2.195}$$

$$R=\left[-\frac{1}{L_s}R{I_{qs}^{r}}^2-\frac{I_{qs}^{r}}{L_s}\left(R_sI_{qs}^{r}+\omega_rL_sI_{ds}^{r}-\omega_rL_mI_f'^{r}\right)+\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)R{I_{ds}^{r}}^2+\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)I_{ds}^{r}\left(-\omega_rL_sI_{qs}^{r}+R_sI_{ds}^{r}\right)\right.$$
$$\left.\left(\frac{L_mR_f'}{L_sL_f'-L_m^2}\right)I_{ds}^{r}\left(E_{xf}'-X_mI_f'^{r}\right)\right]\frac{1}{{I_{qs}^{r}}^2+{I_{ds}^{r}}^2}l_f\frac{\pi^2}{18}+r_f\frac{\pi^2}{18}+\frac{V_i}{\sqrt{{I_{qs}^{r}}^2+{I_{ds}^{r}}^2}}\frac{\pi}{3\sqrt{3}} \tag{2.196}$$

$$R+\frac{1}{L_s}R{I_{qs}^{r}}^2\frac{1}{{I_{qs}^{r}}^2+{I_{ds}^{r}}^2}l_f\frac{\pi^2}{18}-\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)R{I_{ds}^{r}}^2\frac{1}{{I_{qs}^{r}}^2+{I_{ds}^{r}}^2}l_f\frac{\pi^2}{18}=\frac{1}{{I_{qs}^{r}}^2+{I_{ds}^{r}}^2}l_f\frac{\pi^2}{18}\left[-\frac{I_{qs}^{r}}{L_s}\left(R_sI_{qs}^{r}+\omega_rL_sI_{ds}^{r}-\omega_rL_mI_f'^{r}\right)\right.$$
$$\left.+\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)I_{ds}^{r}\left(-\omega_rL_sI_{qs}^{r}+R_sI_{ds}^{r}\right)+\left(\frac{L_mR_f'}{L_sL_f'-L_m^2}\right)I_{ds}^{r}\left(E_{xf}'-X_mI_f'^{r}\right)\right]+r_f\frac{\pi^2}{18}+\frac{V_i}{\sqrt{{I_{qs}^{r}}^2+{I_{ds}^{r}}^2}}\frac{\pi}{3\sqrt{3}} \tag{2.197}$$

$$
\begin{aligned}
&R\left[1+\frac{1}{L_s}{I_{qs}^{r}}^{2}\frac{1}{{I_{qs}^{r}}^{2}+{I_{ds}^{r}}^{2}}l_f\frac{\pi^2}{18}-\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right){I_{ds}^{r}}^{2}\frac{1}{{I_{qs}^{r}}^{2}+{I_{ds}^{r}}^{2}}l_f\frac{\pi^2}{18}\right]=r_f\frac{\pi^2}{18}+\frac{V_i}{\sqrt{{I_{qs}^{r}}^{2}+{I_{ds}^{r}}^{2}}}\frac{\pi}{3\sqrt{3}}+\frac{1}{{I_{qs}^{r}}^{2}+{I_{ds}^{r}}^{2}}l_f\frac{\pi^2}{18}\\
&\left[-\frac{I_{qs}^{r}}{L_s}\left(R_sI_{qs}^{r}+\omega_rL_sI_{ds}^{r}-\omega_rL_mI_f'^{r}\right)+\left(-\frac{L_f'}{L_sL_f'-L_m^2}\right)I_{ds}^{r}\left(-\omega_rL_sI_{qs}^{r}+R_sI_{ds}^{r}\right)+\left(\frac{L_mR_f'}{L_sL_f'-L_m^2}\right)I_{ds}^{r}\left(E_{xf}'-X_mI_f'^{r}\right)\right]
\end{aligned}
\tag{2.198}
$$

The result R variable is that,

$$
\begin{aligned}
R=&\left\{\frac{\pi^2}{18}l_f\frac{1}{{I_{qs}^{r}}^{2}+{I_{ds}^{r}}^{2}}\left[-\frac{I_{qs}^{r}}{L_s}\left(R_sI_{qs}^{r}+\omega_rL_sI_{ds}^{r}-\omega_rL_mI_f'^{r}\right)-\frac{L_f'}{L_sL_f'-L_m^2}I_{ds}^{r}\left(-\omega_rL_sI_{qs}^{r}+R_sI_{ds}^{r}\right)+\frac{L_mR_f'}{X_m\left(L_sL_f'-L_m^2\right)}I_{ds}^{r}\left(E_{xf}'-X_mI_f'^{r}\right)\right]\right.\\
&\left.r_f\frac{\pi^2}{18}+\frac{\pi}{3\sqrt{3}}\frac{V_i}{\sqrt{{I_{qs}^{r}}^{2}+{I_{ds}^{r}}^{2}}}\right\}\Bigg/\left(1+\frac{{I_{qs}^{r}}^{2}}{L_s}\frac{1}{{I_{qs}^{r}}^{2}+{I_{ds}^{r}}^{2}}l_f\frac{\pi^2}{18}+\frac{L_f'}{L_sL_f'-L_m^2}\frac{{I_{ds}^{r}}^{2}}{{I_{qs}^{r}}^{2}+{I_{ds}^{r}}^{2}}l_f\frac{\pi^2}{18}\right)
\end{aligned}
\tag{2.199}
$$

2.3. Torque-Load Angle Equation of Generator with Rectifier at Steady State

In order to obtain torque equation, equation (2.73) and (2.74) are used and $\delta=\theta_r-\theta$ is substituted into this voltage equations;

$$V_{qs} = (-R_s - pL_s)I_{qs} - \omega L_s I_{ds} + (\omega L_m Cos(\theta_r - \theta) + pL_m Sin(\theta_r - \theta))I'_f \qquad (2.200)$$

$$V_{ds} = (-R_s - pL_s)I_{ds} + \omega L_s I_{qs} + (-\omega L_m Sin(\theta_r - \theta) + pL_m Cos(\theta_r - \theta))I'_f \qquad (2.201)$$

For steady state conditions, the differential portions, V_{ds} and I_{ds} become zero and the quadrature axis current is obtained from above equation;

$$\Rightarrow \quad \omega L_s I_{qs} = \omega L_m Sin\delta\, I'_f \qquad (2.202)$$

$$\Rightarrow \quad I_{qs} = \frac{L_m}{L_s} Sin\delta\, I'_f \qquad (2.203)$$

Then the above equation is substituted in to the below torque equation then;

$$Te = \frac{3}{2}\frac{P}{2} L_m I'_f I_{qs} Cos\delta \qquad (2.204)$$

$$Te = \frac{3}{2}\frac{P}{2} L_m I'_f \frac{L_m}{L_s} Sin\delta\, Cos\delta\, I'_f \qquad (2.205)$$

Then the trigonometric relation $Sin2\delta = 2Sin\delta\, Cos\delta$ is carried out to the above torque equation;

$$Te = \frac{1}{2}\frac{3}{2}\frac{P}{2}\frac{L_m^2}{L_s} I'^2_f Sin2\delta \qquad (2.206)$$

The electromagnetic torque equation at steady state can be obtained, when the equation (3.22) is substituted into (2.206).

$$Te = \frac{1}{2}\frac{3}{2}\frac{P}{2}\frac{E_f^2}{\omega^2 L_s}\text{Sin}2\delta \tag{2.207}$$

$$Te = \frac{1}{2}\frac{3}{2}\frac{P}{2}\frac{E_f^2}{\omega X_s}\text{Sin}2\delta \tag{2.208}$$

Where $E_f = \omega L_m I_f'$

CHAPTER THREE

MEASUREMENT OF BRUSHLESS SYNCHRONOOS GENERATOR PARAMETERS

3. Measurement of Brushless Synchronous Generator Parameters

The model of the machine given in (2.132) can be solved numerically if the following parameters are known;

1. Stator winding self inductance, L_s
2. Stator winding mutual inductance, L_m
3. Field winding inductance, L_f
4. Stator winding resistance, R_s
5. Field winding resistance, R_f

This section describes the tests for determining these quantities.

3.1. Tests on the Machine

In this machine, there are 6 poles at the upper and 6 poles at the lower parts of the rotor, with the lower poles rotated 30° with respect to the upper poles. The center portion of the rotor is cylindrical, and the field winding encircles this portion of the rotor. The 6 upper poles are all the same magnetic polarity (N), and the flux returns down through the back core iron to the lower set of poles (S). The machine has 12 poles and no saliency assumed. Therefore the direct axis reactance is equal to quadratic reactance [Tsao et al., 2002].

3.1.1. Open Circuit Test

The first step in the process is to perform the open circuit test on the brushless synchronous generator. During this test, the generator is run at the rated speed, the terminals are left open-circuited, and the field current is set to zero. Then the field current is gradually increased in steps, and the terminal voltage is measured at each step. With the terminals open, I_A=0, so the terminal voltage is equal to back emf. This plot is called open circuit characteristic (OCC) of a generator. With this characteristic, it is possible to find the internal generated voltage of the generator for any given field current.

From open-circuit test, as shown in Figure 3.1, field current versus line-to-neutral phase voltage values are obtained;

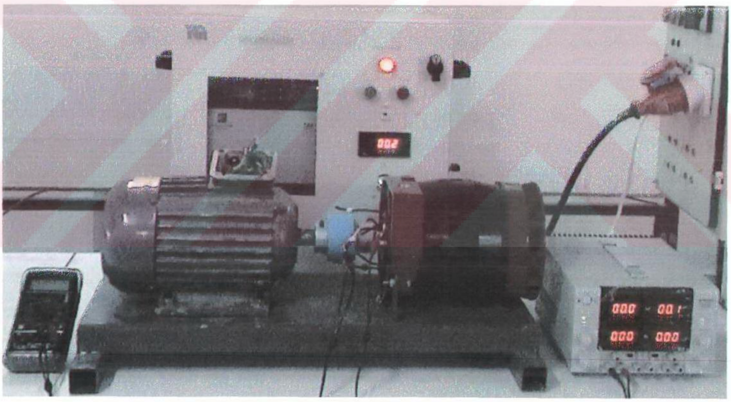

Figure 3.1 Open-circuit test

If(Field Current)(A)	Vac(Phase Voltage)(Line-to-Neutral)(V)
0.01	0.07
0.24	1.83
0.4	2.65
0.5	3.22
0.6	4.07
0.7	4.68
0.8	5.3
0.9	5.9
1	6.70
1.2	8.35
1.3	9.05
1.41	10.03
1.5	10.61
1.61	11.39
1.71	12.24
1.8	12.87
1.9	13.85
2	14.45
2.15	15.44
2.41	17.18
2.6	17.47
2.9	20.55
3.15	21.70
4	24.82

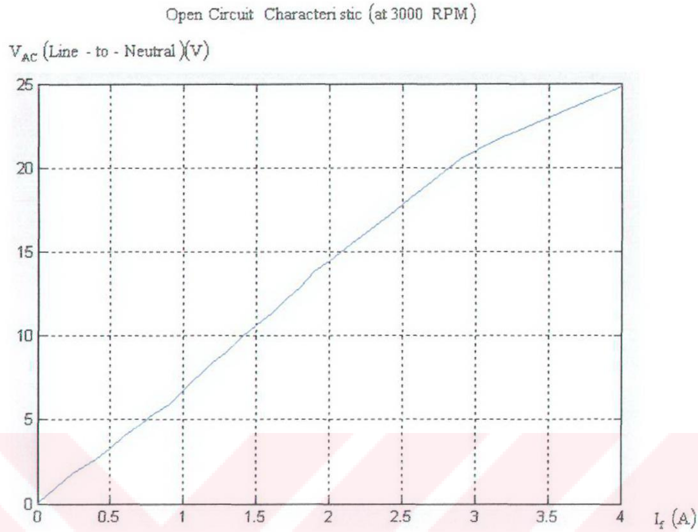

Figure 3.2 Open circuit characteristic

3.1.2. Short Circuit Test

If the armature terminals of a brushless synchronous generator, which is being driven as a generator at synchronous speed, are short-circuited through suitable ammeter, as shown in Figure 3.3, and if the field current is gradually increased until the armature current has reached a maximum safe value, then data can be obtained from which the short-circuit armature current can be plotted against the field current. This relation is known as the short-circuit characteristic.

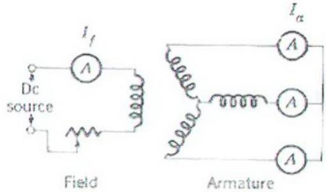

Figure 3.3 Connections for short-circuit test

From short-circuit test, as shown in Figure 3.4, field current versus phase current values are obtained;

Figure 3.4 Short-circuit Test

Field Current (I_F)(A)	Phase Current (I_{A-C})(V)
0	0
1.69	20
2.54	30

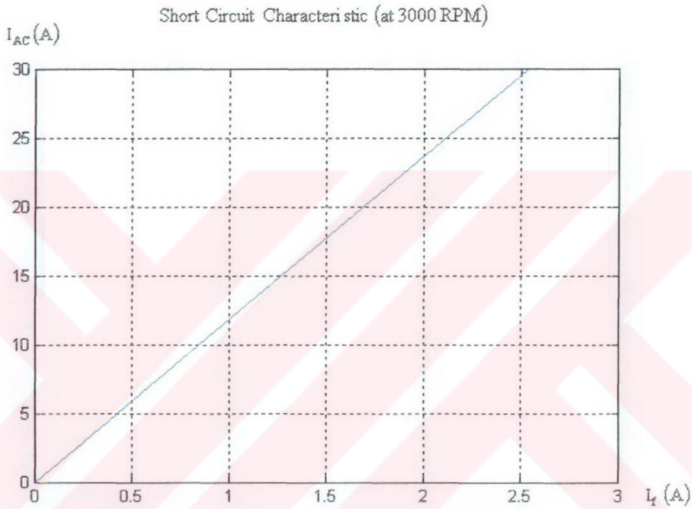

Figure 3.5 Short circuit characteristic

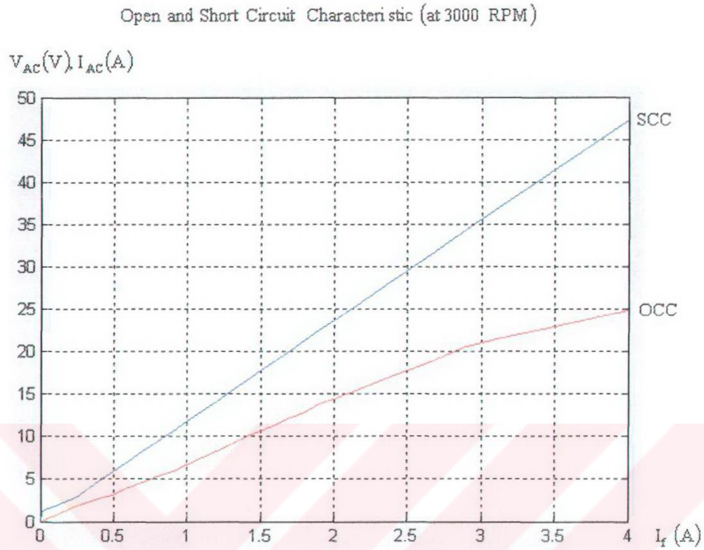

Figure 3.6 Open-Short circuit characteristics

3.1.3. Experimental Determination of Inertia

The moment of inertia of a complex inhomogeneous body, such as the rotor of an electrical machine, containing iron, copper and insulating material with complicated shapes can in practice be determined only by approximation. The problem is even more difficult with mechanical loads, the constructional details of which are usually unknown to the user. Sometimes the moment of inertia is not constant but changes periodically about a mean value, as in the case of a piston compressor with crankshaft and connecting rods. Therefore experimental tests are preferable; a very simple one, called the run-down test is described in the following. Its main advantage is that it can be conducted with the complete drive in place and operable, requiring no knowledge about details of the plant. The accuracy obtainable is adequate for most applications.

For the run-down test, the drive is now accelerated to some initial speed ω_0, where the drive power is switched off so that the plant is decelerating due to the loss torque with the speed measured as a function of time, $\omega(t)$. Solving the equation of motion for J result in

$$J = \frac{-m_L\omega}{\frac{d\omega}{dt}\omega} \tag{3.1}$$

Two special cases lead to particularly simple interpolations

a) Assuming the corrected the loss torque m_L to be approximately constant in a limited speed interval,

$$m_L \approx \text{const}, \quad \text{for} \quad \omega_1 \leq \omega \leq \omega_2 \tag{3.2}$$

Then $\omega(t)$ resembles a straight line; the inertia is determined from the slope of this line.

b) If the loss torque may be approximated by a straight line,

$$m_L \approx a + b\omega \quad \text{for} \quad \omega_1 < \omega < \omega_2 \tag{3.3}$$

a linear differential equation results,

$$J\frac{d\omega}{dt} + b\omega = -a \tag{3.4}$$

The solution is, with $\omega(t_2) = \omega_2$,

$$\omega(t) = -\frac{a}{b} + \left(\omega_2 + \frac{a}{b}\right)e^{-b(t-t_2)/J}, \qquad t \geq t_2 \tag{3.5}$$

Plotting this curve on semi-logarithmic paper yields a straight line with the slope –b/J, from which an approximate value of J is obtained.

From run-down test, as shown in Figure 3.7, phase voltage versus time values is obtained, while keeping field current constant, I_f ;

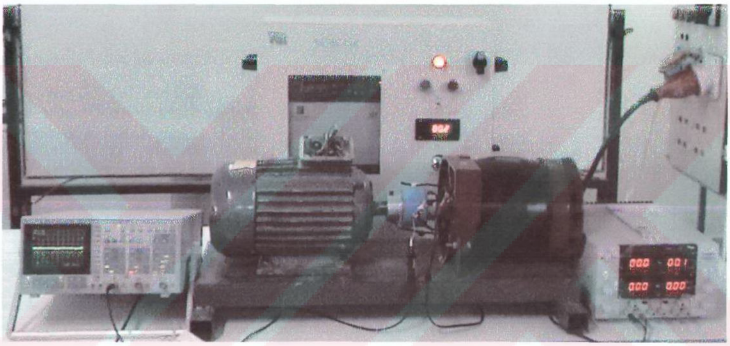

Figure 3.7 Run-down test

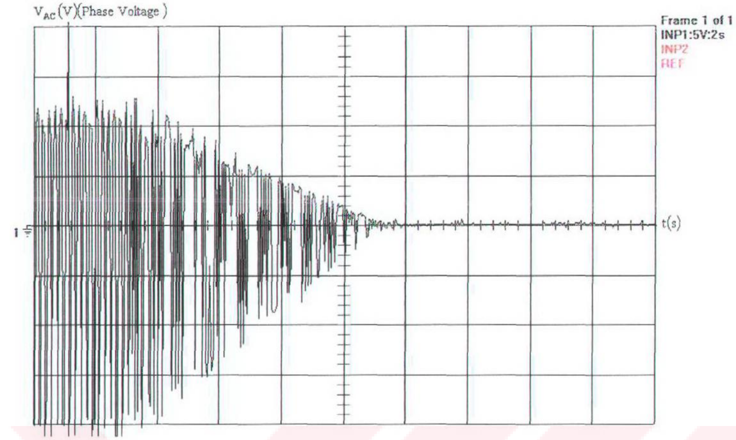

Figure 3.8 Run-down test result

Time/div = 2 sec.   Volt/div = ~5V

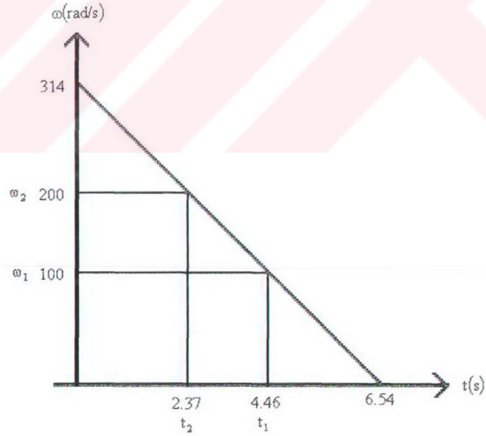

Figure 3.9 Graphic of moment of inertia versus time

Assume a=0 in equation (3.5)

$$\omega(t) = \omega_2 * e^{-b(t-t_2)/J} \qquad \omega_1 \le \omega \le \omega_2 \tag{3.6}$$

Measurement of friction and windage loss; b will be given in 3.2.7. All numeric values are obtained from Figure 3.9 and replaced to below equations.

$$\Rightarrow \quad 100 = 200 * e^{-\frac{1}{314}(4.46-2.37)/J} \tag{3.7}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-\frac{1}{314} * \frac{2.09}{J}} \tag{3.8}$$

$$\Rightarrow \quad \ln\left(\frac{1}{2}\right) = -\frac{1}{314} * \frac{2.09}{J} \tag{3.9}$$

$$\Rightarrow \quad -0.693 = -\frac{0.00665}{J} \tag{3.10}$$

$$\Rightarrow \quad J = 0.0096\ Kgm^2 \tag{3.11}$$

This moment of inertia value is total moment of inertia of the brushless synchronous generator and induction machine. It is assumed that both machines have the same amount of inertia, therefore, one of them has

$$\Rightarrow \quad J = \frac{0.0096}{2} \Rightarrow \quad J = 0.0048\ Kgm^2 \tag{3.12}$$

3.2. Calculation and Measurement of Parameters

3.2.1. Stator Winding Resistance Measurement

In order to measure stator winding resistance, DC power supply and current and voltage probe of scope are connected to the A-C phase terminals of brushless synchronous generator. The phase current and voltage values are read on the scope monitor. Then stator winding resistance values are calculated from following values and equations;

$$V = 0.2V$$
$$I = 3.30A$$

$$\Rightarrow \quad R_{S_{S\text{-}C}} = \frac{V}{I} = 0.0606\Omega \tag{3.13}$$

$$\Rightarrow \quad R_S = \frac{0.0606}{2} \Rightarrow \quad R_S = 0.0303\Omega \tag{3.14}$$

3.2.2. Calculation of Stator Winding Self Inductance

This machine feeds power into the rectifier. The rated output voltage of the rectifier is 28V. Therefore, the rated per phase voltage is computed from the reluctance below;

$$V_{dc} = \frac{3\sqrt{3}}{\pi} V_p \Rightarrow \quad 28 = \frac{3\sqrt{3}}{\pi} V_p \tag{3.15}$$

$$\Rightarrow \quad V_p = 17\,V \Rightarrow \quad V_{rms} = \frac{17}{\sqrt{2}} = 12\,V \tag{3.16}$$

At this rated voltage level 12 V, the short circuit current is obtained from the S.C.C.. The field current corresponding to the operating point is also obtained from O.C.C..

$$E_f = 12V \Rightarrow \quad I_f = 1.7A \text{ (from O.C.C.)}$$
$$I_{a,sc} = 20A \text{ (from S.C.C.)}$$

then;

$$Z_s = \sqrt{R_s^2 + X_s^2} \quad \text{and} \quad Z_s = \frac{E_f}{I_{a,sc}} \tag{3.17}$$

$$\Rightarrow \quad Z_s = \frac{E_f}{I_{a,sc}} = \frac{12}{20} \Rightarrow \quad Z_s = 0.6 \tag{3.18}$$

Stator resistance value is taken from the previous section in order to calculate the saturated value of synchronous reactance using saturated impedance method.

$$R_s = 0.0303\ \Omega/\text{phase} \Rightarrow \quad X_s = 0.6\ \Omega/\text{phase}$$

Since

$$X_s = 2\pi f L_s \tag{3.19}$$

and

$$n = \frac{120f}{p} \Rightarrow \quad 3000 = \frac{120f}{12} \Rightarrow \quad f = 300\text{Hz} \tag{3.20}$$

$$\Rightarrow \quad X_s = 2\pi f L_s \Rightarrow \quad 3.6 = 2\pi 300 L_s \tag{3.21}$$

$$\Rightarrow \quad L_s = 0.318\ \text{mH}$$

3.2.3. Calculation Mutual Inductance between Stator Winding and Field Winding

In order to model brushless synchronous generator completely, the mutual inductance is calculated. The relation between open circuit voltage and field current is obtained from equation (2.128), after substitution of I_{qs}=0 and I_{ds}=0. Here, V_{qs}^{r} is the peak line to neutral voltage.

$$V_{qs}^{r} = E_f = \omega L_m I_f' \tag{3.22}$$

$I_f = 1.7A \qquad V_0 = 12V$

$\omega = 2\pi f = 2\pi 300 \Rightarrow \qquad \omega = 1885\ \text{rad./s.}$

Turns ratio value are used in order to calculate referred field current;

$N_r = 400 \qquad N_s = 12$

Field current referred to stator side;

$$I_f' = \frac{2}{3}\left(\frac{N_r}{N_s}\right) I_f \tag{3.23}$$

$$\Rightarrow \quad I_f' = \frac{2}{3}\left(\frac{400}{9}\right) 1.7 \Rightarrow \quad I_f' = 37.77A \tag{3.24}$$

$$V_{qs}^{r} = E_f = \omega L_m I_f' \Rightarrow \quad \sqrt{2} * 12 = 1885 L_m (37.77) \tag{3.25}$$

$\Rightarrow \quad L_m = 0.237\ \text{mH}$

3.2.4. Field Winding Resistance Measurement

The test mechanism, as shown in the below, are established in order to calculate field winding resistance. And current probe are adjust 100mV/A, then voltage and current values are read on the scope monitor as a 3.9Vand 1.65A respectively (is shown in Figure 3.11).

$$R_f = \frac{V}{I} \tag{3.26}$$

$$R_f = \frac{3.9}{1.65} \Rightarrow R_f = 2.36\,\Omega \tag{3.27}$$

$$R'_f = \left(\frac{3}{2}\right)\left(\frac{N_s}{N_f}\right)^2 R_f \tag{3.28}$$

$$R'_f = \left(\frac{3}{2}\right)\left(\frac{12}{400}\right)^2 2.36 \tag{3.29}$$

$$R'_f = 0.00318\,\Omega$$

3.2.5. Calculation of Field Winding Inductance

In order to calculate field winding inductance, the test mechanism, as shown in Figure 3.10, is established in the laboratory;

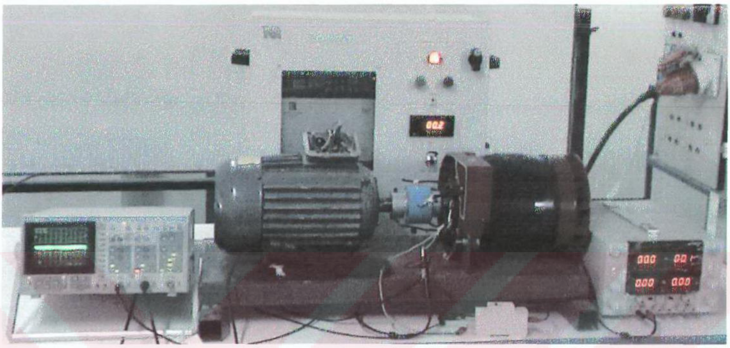

Figure 3.10 Field winding inductance mechanism

The below test result are obtained after doing this test;

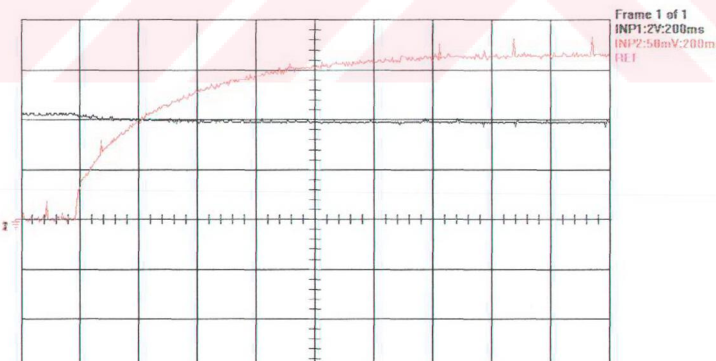

Figure 3.11 Test result

The current is reached steady state value, which is shown above figure, at 1.6 seconds. Then this current value is multiplied by 0.632 in order to calculate τ value, which is time constant.

$$\tau = \frac{L}{R} \tag{3.30}$$

$$1.6\,\mathrm{s} \rightarrow 1.65\mathrm{A}$$

$$\Rightarrow \quad 1.65 * 0.632 = 1.0428\mathrm{A}$$

$$\Rightarrow \quad 1.0428\mathrm{A} \rightarrow 228\,\mathrm{ms}$$

$$\tau = \frac{L}{R} \Rightarrow \quad 228\,\mathrm{ms} - 0\,\mathrm{ms} = \frac{L}{R} \tag{3.31}$$

$$\Rightarrow \quad 228 * 10^{-3} = \frac{L}{2.36}$$

$$\Rightarrow \quad L_f = 538\,\mathrm{mH}$$

The circuit, as shown in Figure 3.12, is established in Matlab program in order to prove field winding inductance value.

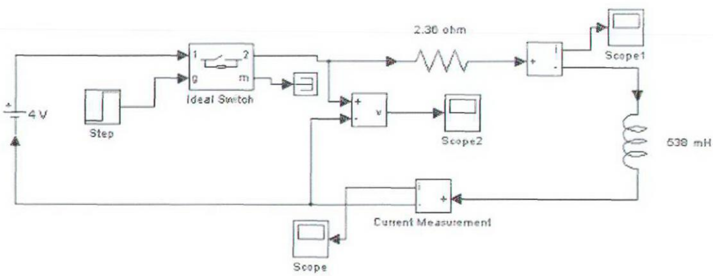

Figure 3.12 Matlab circuit

The field winding inductance current graphic is shown in the below figure.

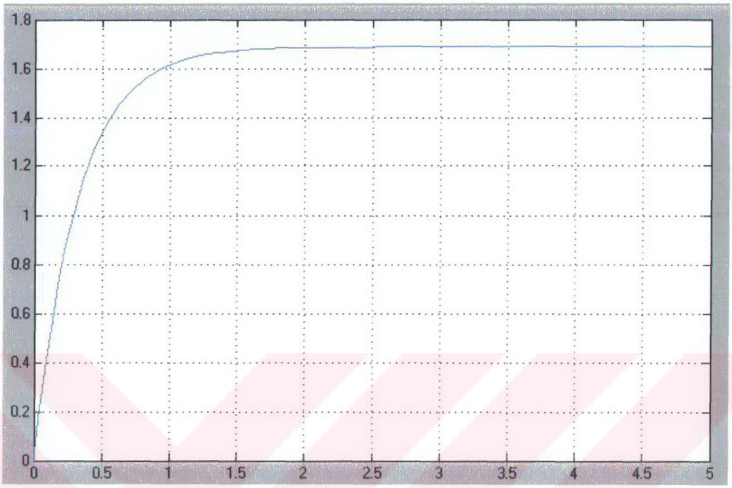

Fig. 3.13 Field Winding inductance current graphic

As shown in Figure 3.13, the field winding inductance current is reached steady state value at 1.6 seconds. So L=538 mH, which is calculated in the above.

As shown in the below, the field winding self inductance value is calculated like this;

$$L_f = L_{lf} + L_{mf} \quad \text{and} \quad L_{lf} + L_{mf} = 538\,\text{mH} \tag{3.32}$$

Field winding inductance value referred to stator value;

$$X'_f = X'_{lf} + X_m \Rightarrow \quad L'_f = L'_{lf} + L_m \tag{3.33}$$

$$L'_{lf} = \left(\frac{3}{2}\right)\left(\frac{N_s}{N_r}\right)^2 L_{lf} \tag{3.34}$$

$$L_m = \left(\frac{N_s}{N_r}\right)^2 \left(\frac{3}{2}\right) L_{mf} \tag{3.35}$$

$$\Rightarrow \quad L'_f = \left(\frac{3}{2}\right)\left(\frac{N_s}{N_r}\right)^2 \left(L_{lf} + L_{mf}\right) \tag{3.36}$$

$$\Rightarrow \quad L'_f = \left(\frac{3}{2}\right)\left(\frac{12}{400}\right)^2 \left(538.08 * 10^{-3}\right) \tag{3.37}$$

$$\Rightarrow \quad L'_f = 0.726\,\text{mH}$$

3.2.6. Calculation of Rated Torque

The purpose of this section is to derive the rated torque value for the brushless synchronous generator.

$P_{out} = 2.5\,\text{kW} = 2500\,\text{W}$ and $n_s = 3000\text{rev./min.}$

$$T_L = \frac{P_{out}}{\omega_r} \Rightarrow \quad T_L = \frac{P_{out}}{2\pi \frac{n_s}{60}} \tag{3.38}$$

$$\Rightarrow \quad T_L = \frac{2500}{2\pi \frac{3000}{60}} \Rightarrow \quad T_L = \frac{2500}{314}$$

$$\Rightarrow \quad T_L = 7.96\,\text{Nm}$$

3.2.7. Measurement of Friction & Windage Losses

Firstly, friction and windage losses are measured in order to calculate moment of inertia. The inverter provides the dc link current in proportional to the electromagnetic torque developed by the induction machine. Therefore, the calibration of current to torque is carried out as follows;

1. ABC phase points of PWM inverter are connected with abc phase points of induction machine.
2. TP3 test point of PWM inverter is connected to one channel of scope.
3. Induction motor is rotated at 3000 RPM by PWM inverter.
4. 0.7 volt (DC Voltage) is read on the scope from TP3 test point, When 1 N.m. load is coupled to the shaft of the induction machine.
5. The test machine is coupled to the shaft of induction machine and it is driven at no-load. The torque is measured 1 N.m.

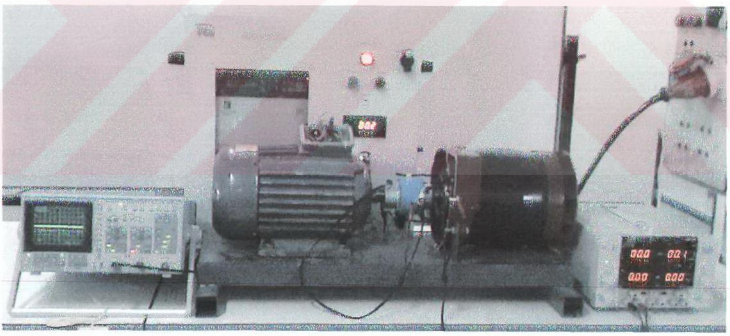

Figure 3.14 Friction & Windage loss

The test mechanism is shown in the below;

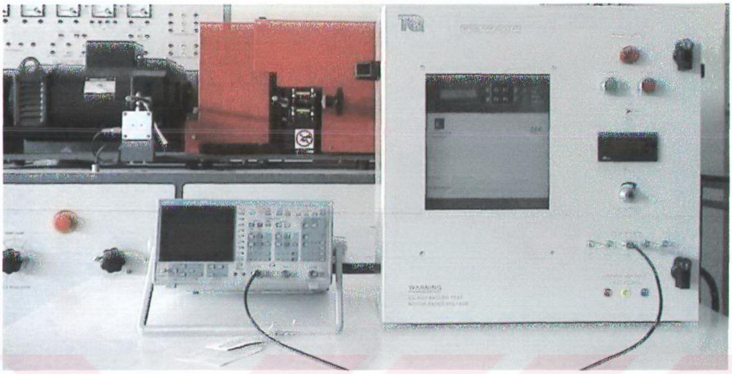

Figure 3.15 Friction & Windage loss

$$P_{F\&W} = T_L \omega_m \Rightarrow \qquad P_{F\&W} = T_L \, 2\pi \frac{n_s}{60} \tag{3.39}$$

$$\Rightarrow \qquad P_{F\&W} = 1 * 2\pi \frac{3000}{60}$$

$$\Rightarrow \qquad P_{F\&W} = 314 \text{ Watt}$$

CHAPTER FOUR

SIMULATION PROGRAM AND SIMULATION RESULT

4. Simulation Program and Simulation Result

4.1. Simulation Program

In this chapter, the dedicated simulation program for a brushless synchronous generator will be introduced. The mathematical model of a brushless synchronous generator has been given in chapter II. Considering mathematical model, the simulation program, which refers to the system given in Chapter II, has been written. The outputs of the program were stored in data files and using the m file of the MATLAB program, graphical illustrations were obtained. These will be presented in this Chapter. The list of the program is given in Appendix A.

4.1.1. Program Algorithm and Flow-Chart

The flow-chart of the simulation program is given in Figure 4.1

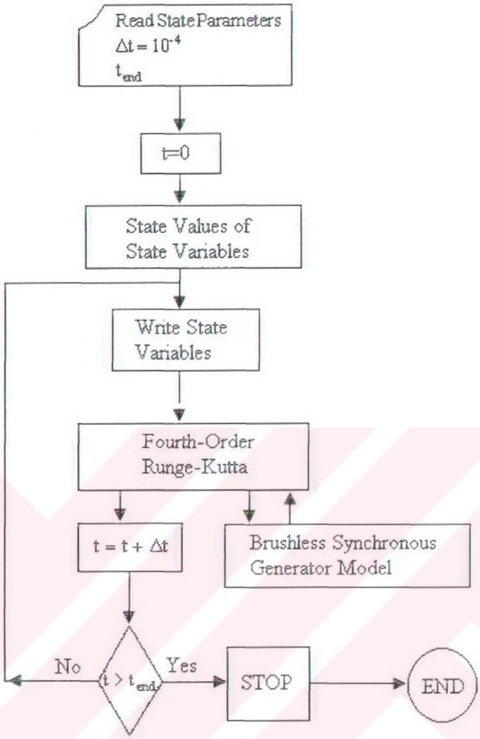

Figure 4.1 Flow-Chart of the simulation program

At first stage, initial conditions are substituted into the differential equations. At the second stage, the differential equation system, Equation (2.158, 2.159, 2.160, and 2.164) in Chapter II, is solved using four step Runge-Kutta algorithm, which will be introduced in the next section, and then the outputs are handled from the differential equation. While using Runge-Kutta algorithm, step size for the iteration was chosen as $1*10^{-4}$ seconds.

As mentioned earlier, at the final stage outputs are stored in data files and are to be drawn using a MATLAB m file and graphical editor. So that, desired waveforms are monitored by making use of this program. The source codes of the simulation and graphics programs are given in Appendix A.

4.1.2. Runge-Kutta Algorithm

Numerical methods are used in the solution of differential equations which cannot be solved in closed form and are also applied to the equations of which solution is very complicated in such form. As can be seen from the mathematical model of the brushless synchronous generator in Chapter II, the equations are difficult to solve in closed-form since they are time variant. To avoid this difficulty, the equations were solved numerically applying the fourth-order Runge-Kutta algorithm.

Runge-Kutta algorithm is applied to the initial value problems of the first-order differential equations which are in the form;

$$y' = f(x, y) \qquad , \; y(x_0) = y_0 \tag{4.1}$$

Where x_0 and y_0 are initial values.

Let the initial of the function (y) be y_n at the point x_n. The struggle of this method is to calculate the value of y at the next step, i.e. $y_1 = y(x_1) = y(x_0 + h)$, where h is the step size as shown in Figure 4.2 To achieve that, four quantities k_1, k_2, k_3, k_4 are computed first and then the y_{n+1} value at the point x_{n+1} is obtained following the procedure below (Kreyszig, E. , 1993);

Table 4.1 Fourth-Order Runge-Kutta Method

ALGORITHM RUNGE-KUTTA (f, x_0, y_0, h, N)

Input: Initial values x_0, y_0 step size h, number of step N

Output: Approximation y_{n+1} to the solution $y(x_{n+1})$ at $x_{n+1} = x_0 + (n+1)h$, where n=0,1,...(N-1)

For n=0,1,...(n-1) do:

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

End

Stop

END RUNGE-KUTTA

This procedure is repeated as much as it is desired. The previous outputs are used at each time step is the initial values and the computation goes on in the same way.

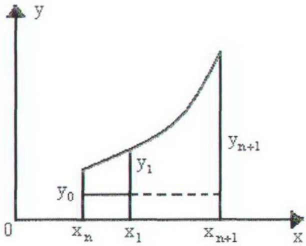

Figure 4.2 Runge-Kutta method

4.2. Experimental Result

Modern generators and also power components are designed and analyzed by using computer simulation as a tool for conducting transients and control studies. Simulation can be especially helpful about the dynamic behavior and interactions that are often not readily apparent from reading theory. Simulation is often chosen by engineers to study transient and control performance or to test conceptual designs, before the experiment of actual system.

It is instructive to observe the variables of the brushless synchronous generator during dynamic and steady-state operation. For this purpose, the differential equations which describe the brushless synchronous generator were programmed on a computer and a study was performed. In this section, our purpose is to understand the theory and principles of a brushless synchronous generator. Brushless synchronous generator is considered, information regarding brushless synchronous generator is given in Table 4.2.

Table 4.2 Brushless Synchronous Generator

Rating: 2.5 Kw

Adjustable Voltage Setting: 28 V

Poles: 12

Speed: 3000 RPM

Inertia of generator J= 0.048 $J.s^2$

Machine Parameters:

R_s= 0.0303 Ω L_s= 0.000318 H L_m= 0.000237 H

R_f' = 0.00318 Ω L_f' = 0.000726 H

4.2.1. Starting Transients

During this step, the value of load torque is 8.0 N.m. and the rotor speed is set to 3000 r.p.m..

Figure 4.3, 4.4, and 4.5 show the sinusoidal stator a, b, and c phase currents versus time waveforms of the brushless synchronous generator in rotor reference frame, respectively. The stator phase currents are balanced. Maximum values of stator phase currents are 42.7 A and -42.7 A for positive and negative cycles at steady state, respectively. And the frequency value of stator phase currents is 300 Hz.

Figure 4.6, 4.7 show the quadrature and direct axis currents versus time waveforms in qd forms in rotor reference frame, the value of quadrature and direct axis currents are 35,76 A and 23.34 A at steady state, respectively. And figure 4.8 shows the field winding current versus time waveform in rotor reference frame and the steady-state field winding current value is 104.85 A, this value is the referred value to the stator.

Figure 4.9 illustrates the angular velocity of rotor versus time waveform. And the steady state value of angular velocity of rotor is 1885 rad./sec..

Figure 4.10 and 4.11 show the input and output power versus time waveforms and the steady state value of input and output power are 2513 Watt and 2430 Watt, respectively. Figure 4.12 only shows the efficiency versus time waveform at steady state and the steady state efficiency value is %96.6, which is calculated from input and output power values.

Figure 4.13 and 4.14 illustrates electromagnetic torque and rotor angle versus time waveforms, respectively. And while the electromagnetic torque value is 8 N.m., the rotor angle value is 33.13° at steady state. Additionally, while maximum rotor angle is 45°, the maximum electromagnetic torque value is 8.74 N.m.

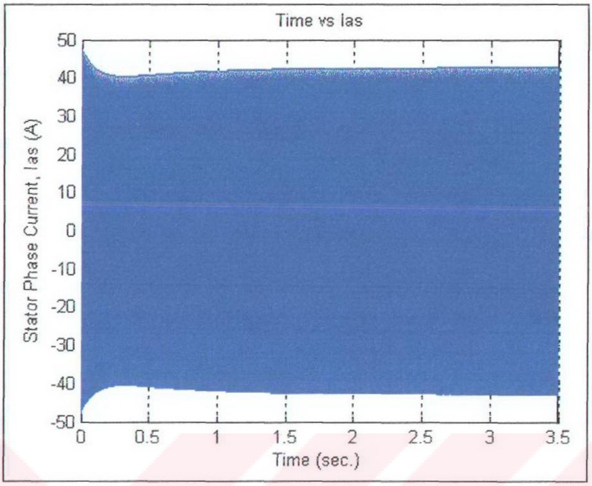

Figure 4.3 Stator phase current, Ias

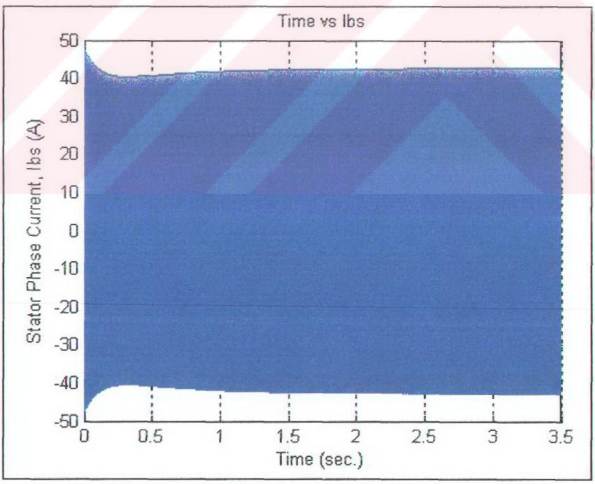

Figure 4.4 Stator phase current, Ibs

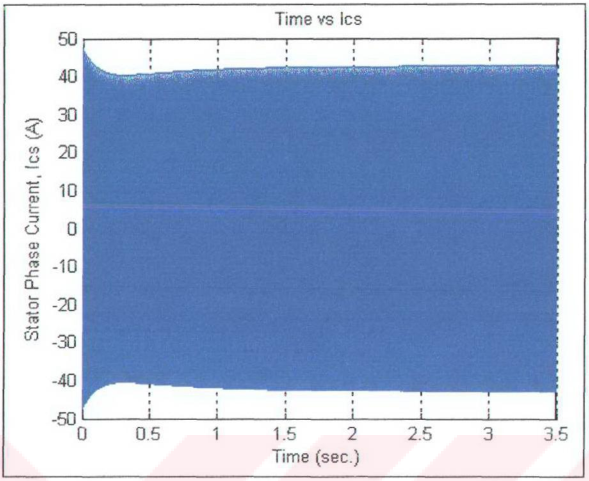

Figure 4.5 Stator phase current, Ics

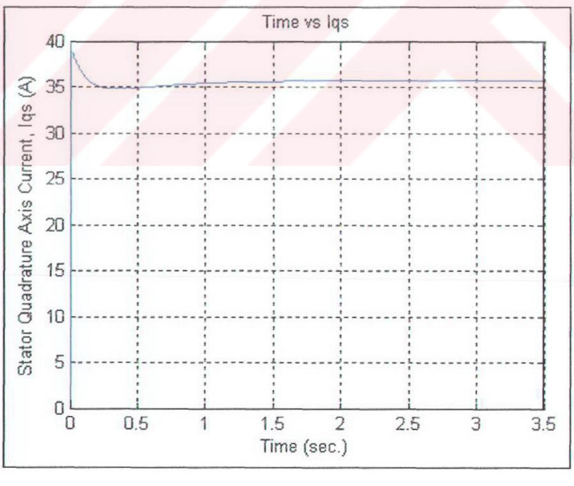

Figure 4.6 Stator quadrature axis curent, Iqs

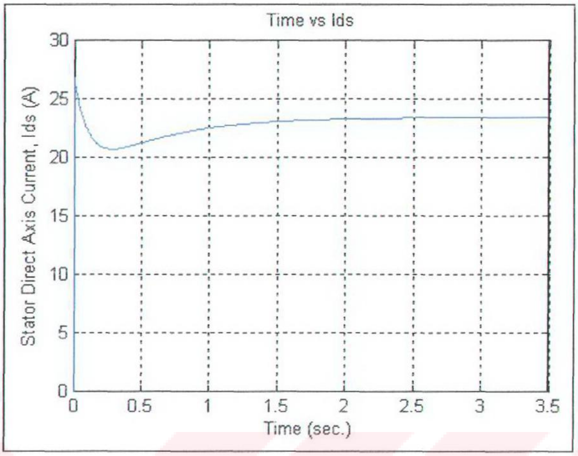

Figure 4.7 Stator direct axis current, Ids

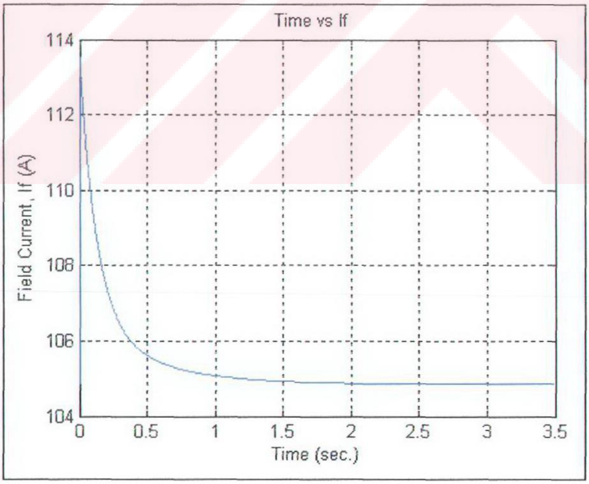

Figure 4.8 Field Current, If

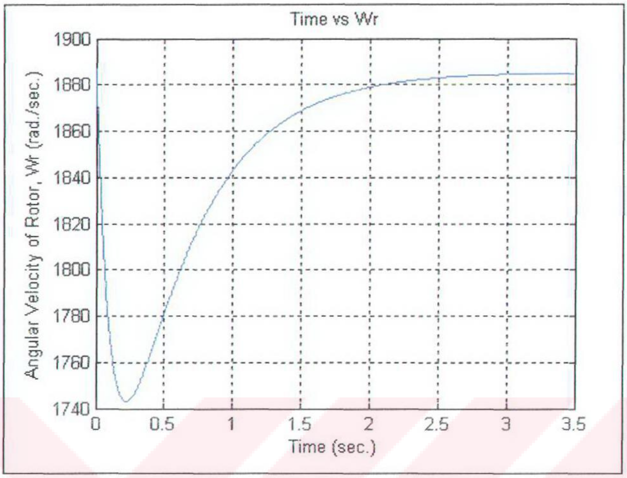

Figure 4.9 Angular velocity of rotor, Wr

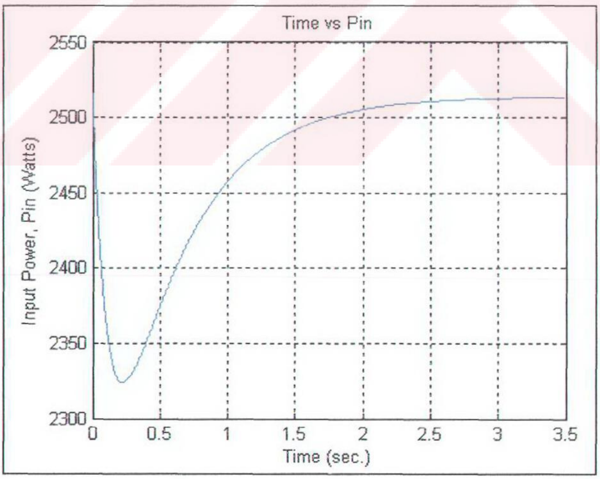

Figure 4.10 Input power, Pin

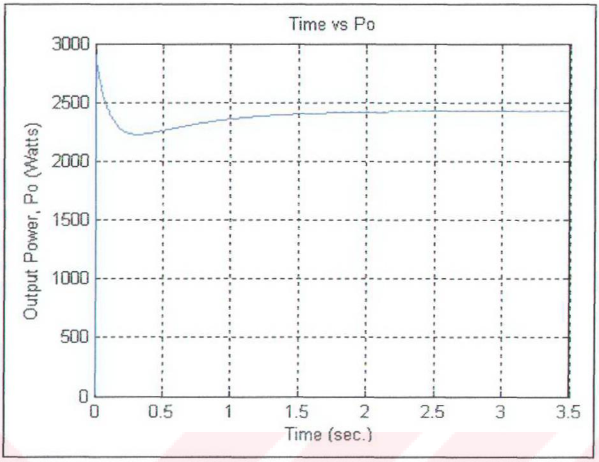

Figure 4.11 Output power, Po

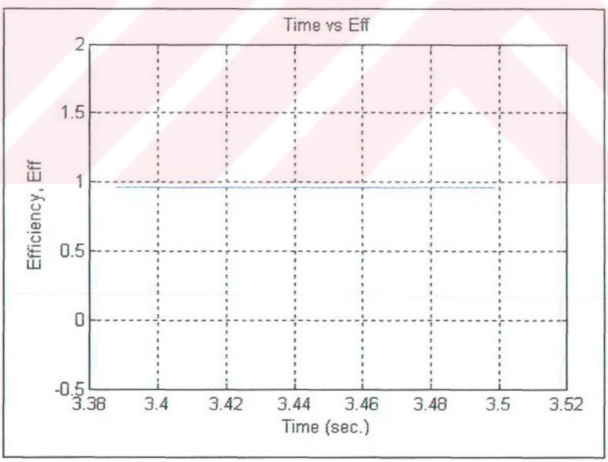

Figure 4.12 Efficiency (at steady state), Eff

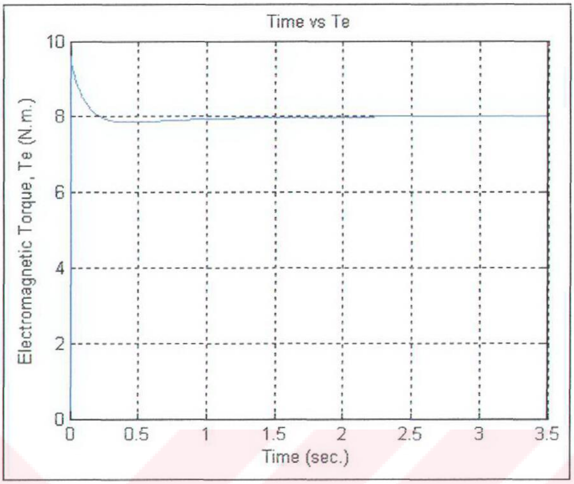

Figure 4.13 Electromagnetic torque, Te

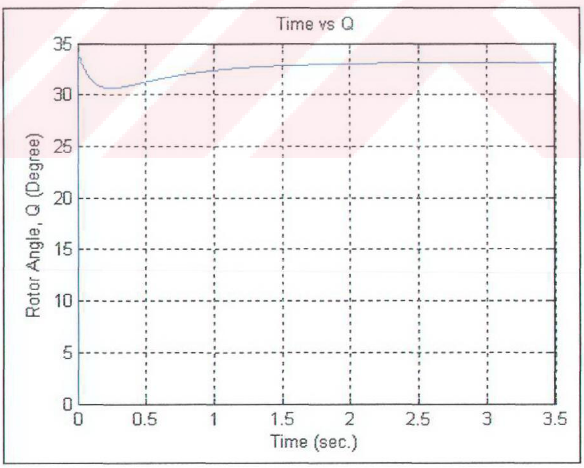

Figure 4.14 Rotor Angle, Q

4.2.2. Step Change on Load Torque

In the above part, the machine is operating with constant load torque. In particular, in this section, 8.0 N.m. is applied to the brushless synchronous generator as a load torque during the first time interval, which is between 0.0 second with 3.0 second, and 4.0 N.m. is applied to the machine during the second time interval, which is between 3.0 second and 6.0 second.

Figure 4.15, 4.16, and 4.17 show the sinusoidal stator a, b, and c phase currents versus time waveforms of the brushless synchronous generator in rotor reference frame, respectively. And the stator phase currents are balanced. Maximum values of stator phase currents are ±42.7 A and ±18.4 A for positive and negative cycles at steady state for the first and second time intervals, respectively. And the stator phase currents frequency is 300 Hz.

Figure 4.18 shows the quadrature axis current versus time waveform, the steady state value of quadrature axis current are 35.76 A and 17.89 A for the first and second time intervals, respectively. Figure 4.19 shows the direct axis current versus time waveform, the steady state values of direct axis current are 23.34 A and 4.33 A for the first and second time intervals, respectively. And Figure 4.20 shows the field winding current versus time waveform, the steady state value of field winding current is 104.85 A for both the first and second time intervals, respectively, this value is the referred value to the stator.

Figure 4.21 shows the angular velocity of rotor versus time waveform, the steady state values of angular velocity of rotor are 1885 rad./sec. and 1042 rad./sec. for the first and second time intervals, respectively.

Figure 4.22 shows the input power versus time waveform, the steady state values of input power are 2513 Watt and 695 Watt for the first and second time intervals, respectively. Figure 4.23 shows the output power versus time waveform, the steady state values of output power are 2430 Watt and 679.5 Watt for the first and second time intervals, respectively.

Figure 4.24 illustrates the electromagnetic torque versus time waveform, the steady state values of electromagnetic torque are 8.0 N.m. and 4.0 N.m. for the first and second time intervals, respectively. Figure 4.25 illustrates the rotor angle versus time waveform, the steady state values of rotor angle are 33.13° and 13.63° for the first and second time intervals, respectively.

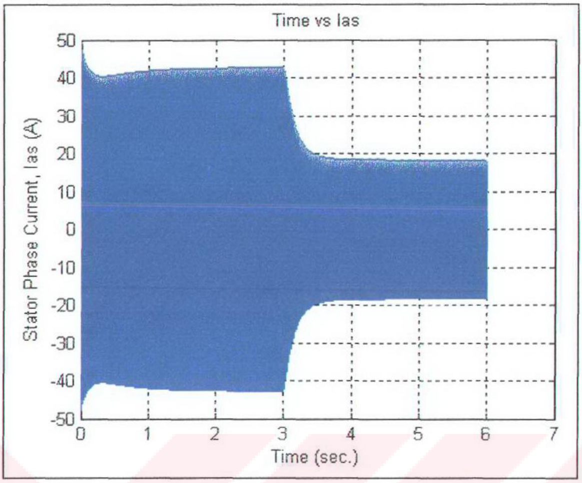

Figure 4.15 Stator phase current, Ias

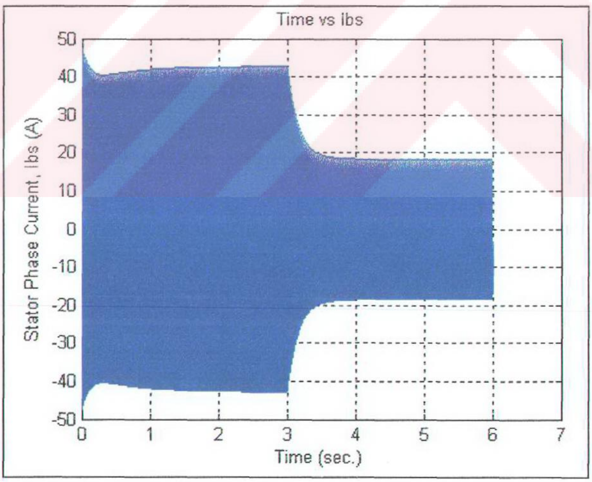

Figure 4.16 Stator phase current, Ibs

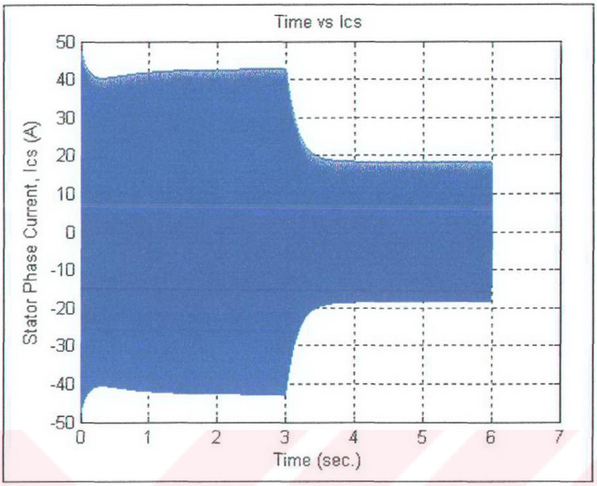

Figure 4.17 Stator phase current, Ics

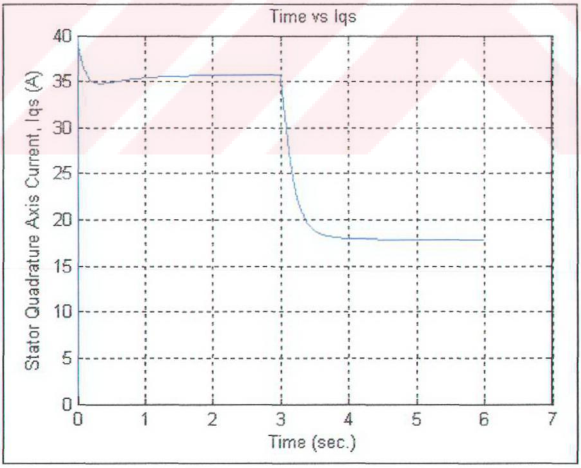

Figure 4.18 Stator quadrature axis current, Iqs

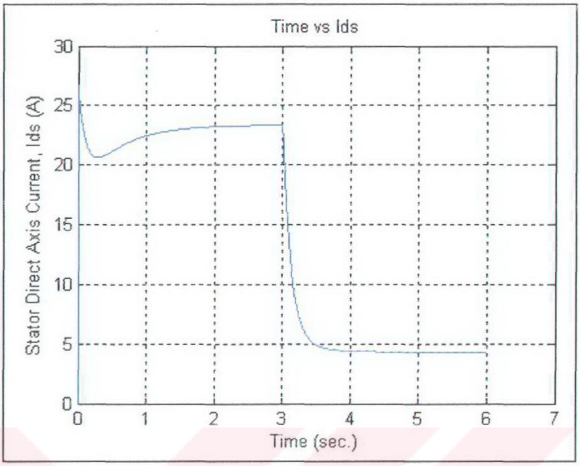

Figure 4.19 Stator direct axis current, Ids

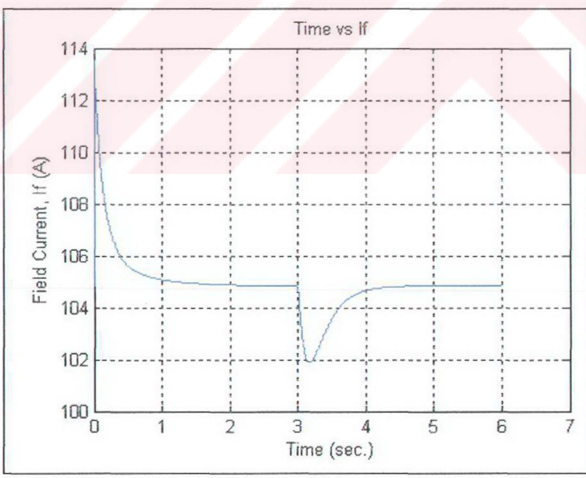

Figure 4.20 Field current, If

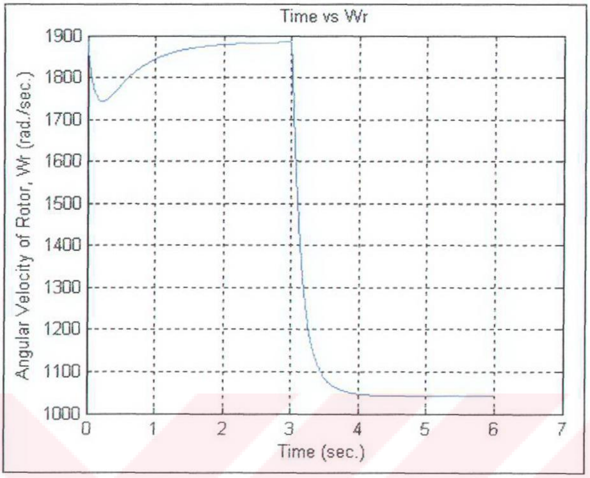

Figure 4.21 Angular velocity of rotor, Wr

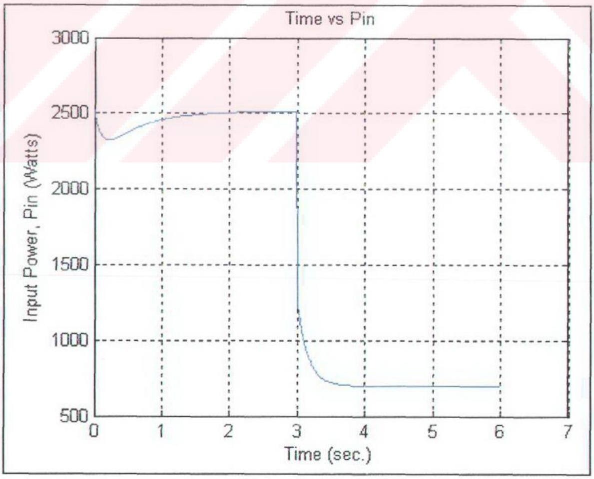

Figure 4.22 Input power, Pin

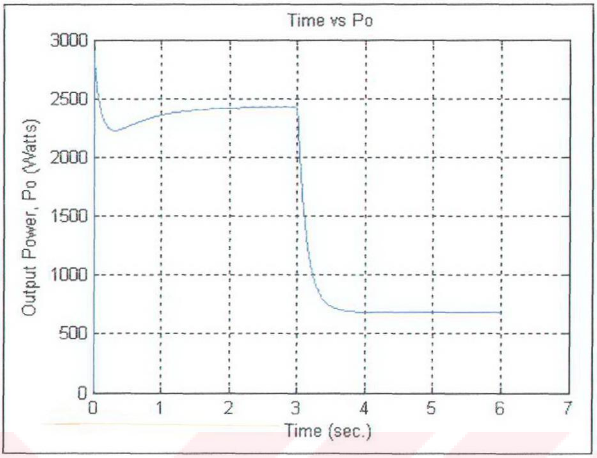

Figure 4.23 Output power, Po

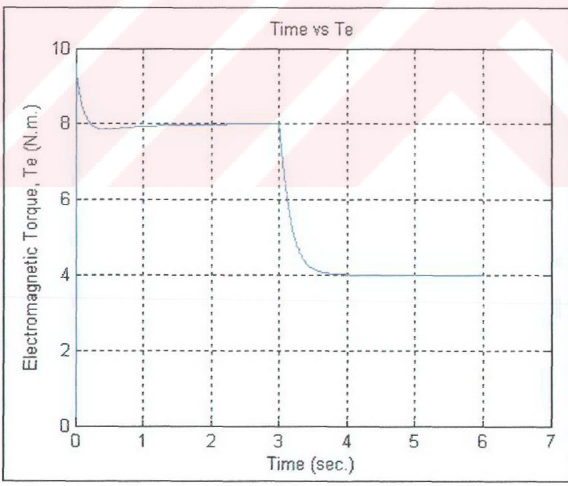

Figure 4.24 Electromagnetic Torque, Te

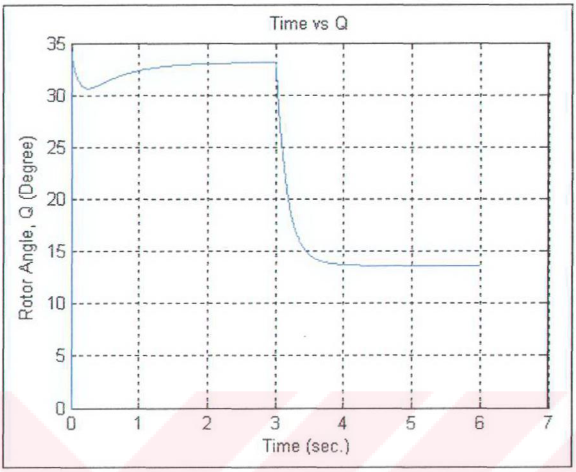

Figure 4.25 Rotor angle, Q

CHAPTER FIVE

CONCLUSIONS

In this thesis, a brushless synchronous generator with a wound field has been theoretically modeled and analyzed. Computer simulation program is prepared in the FORTRAN program using mathematical model of the brushless synchronous generator. Also the machine parameters have been experimentally measured in the laboratory.

In order to model brushless synchronous machine, the standard abc/qd and qd/abc models for synchronous machine in reference frame theory have been used. And the mathematical model of the rectifier have been derived and combined with the brushless synchronous machine model for computer simulation. The simulation results during the transient and steady-state operations have been obtained from the combined machine model.

The parameters of a brushless synchronous machine are obtained by means of realizing open-circuit, short-circuit, run-down and f&w losses tests. In order to do this tests, the induction machine / brushless synchronous machine coupling mechanism is established in the laboratory. And induction machine is driven by the PWM inverter. Consequently, the obtained parameters are referred to the stator in order to substitute in the combined machine model.

The differential equations, which have been obtained from the combined machine model, of the state variables (I_{qs}, I_{ds}, I_{fd} ...etc) of a brushless synchronous machine are solved with FORTRAN program by using fourth-order Runge-Kutta algorithm. And the graphics of state variables are drawn by the MATLAB program. So, the dynamic performance of the brushless synchronous machine is observed by means of this graphics.

REFERENCES

Krause, P.C.,& Wasynczuk, O.,& Sudhoff, S.C. (1994). Analysis of electric machinery. New York: McGraw-Hill Book Company: IEEE Press.

Krause, P.C.,& Wasynczuk, O. (1989). Electromechanical motion devices. New York: McGraw-Hill Book Company.

Tsao, P.,& Senesky, M.,& Sanders, S.R. (2002). A homopolar machine for high Speed applications. Conference Record of the IEEE IAS Annual Meeting, 30, 110-116.

Synchronous generator and motor. (1986). Unified machine theory, pp.120-150

He, J.,& Lin, F. (1995). A high frequency high power IGBT inverter drive for 45 HP /16,000 RPM brushless homopolar inductor motor. Conference Record of the IEEE IAS Annual Meeting, 45, 9-15

Siegl, M.,& Kotrba, V. (1991). Losses and cooling of a high-speed high output power homopolar inductor alternator. IEE Fifth International Conference on Electrical Machines and Drives, 341, 295-299

U.S. Patent. (2000). Hybrid Permanent Magnet/Homopolar Generator and Motor. Washington: Rao, G.P.,& Kirtley, J.L.,& Meeker, D.C.,& Donegan, K.J.

Ichikawa, O.,& Chiba, A.,& Fukao, T. (1999). Development of homo-polar type bearingless motors. Conference Record of the IEEE IAS Annual Meeting, 16, 1223-1228

Hippner, M.,& Harley, R.G. (1992). High speed synchronous homopolar and permanent magnet machines comparative study. Conference Record of the IEEE Industry Applications Society Annual Meeting, 49, 74-78

Akpınar, E. (2002). Elektrik makinelerinin temel ilkeleri. DEÜ Müh. Fak. Yayını.

Ellis, G., (1996). Advances in Brushless DC Motor Technology, Control, and Manufacture. Kollmorgen Corporation, Radford, Virginia, USA.

Kreyszig, E. (1993). Advanced engineering mathematics. (7th ed.). London: John Wiley & Sons.

APPENDIX A

```
c
c
      Real*4  x(6)

      Open(8,FILE='Te1.plo')
      Open(11,File='kr1.plo')
      Open(14,File='kr2.plo')
      Open(17,File='kr3.plo')
      Open(20,File='kr4.plo')
c
      if(t.eq.0.) then
      x(1)=1.0
      x(2)=0.0
      x(3)= 46.8367/0.4467
      x(4)=1885.0
      x(5)=0.0

      endif

      DO 101 t=0.0, 3.5, 1e-4
c

c     write(11,*) t,x(1),x(2),x(3)
c     write(8,*)t,x(4)
c     write(*,*)t,x(4)
```

```
      CALL RKSYST(x,t)

101   CONTINUE

      END

CC    ---------------------------------------------------------------
      Subroutine   RKSYST(x,t)
CC    ---------------------------------------------------------------

      Real*4       x(6),xdot(6),xend(6),xwrk(5,6),h,t
      Integer i

      h=1e-4

      CALL         Derivatives(t,x,xdot)
      Do 10 i=1,6
      xwrk(1,i)=h*xdot(i)
      xend(i)=x(i)+xwrk(1,i)/2.0
10    Continue

      CALL         Derivatives(t+h/2.0,xend,xdot)
      Do 20 i=1,6
      xwrk(2,i)=h*xdot(i)
      xend(i)=x(i)+xwrk(2,i)/2.0
20    Continue

      CALL         Derivatives(t+h/2.0,xend,xdot)
      Do 30 i=1,6
      xwrk(3,i)=h*xdot(i)
      xend(i)=x(i)+xwrk(3,i)
30    Continue
```

```
 CALL        Derivatives(t+h,xend,xdot)
      Do 40 i=1,6
      xwrk(4,i)=h*xdot(i)
40    Continue

      Do 50 i=1,6
      xend(i)=x(i)+(xwrk(1,i)+2.0*xwrk(2,i)+2.0*xwrk(3,i)+xwrk(4,i))/6.0
      x(i)=xend(i)
50    Continue

      Return
      End

CC    ---------------------------------------------------------------
      Subroutine    Derivatives(t,x,xdot)
CC    ---------------------------------------------------------------

      Real*4   x(6),xdot(6),t

      Real*4   wb,pi,R1,R2,R,uqs,uds,det,b11,b22,b33
      Real*4   b23,b32,A11,A22,A33,P,Tl,Te
      Real*4   Rs,Xs,Ls,Rf,Xm,Lm,Lf,rf,Vi,Exf
      Real*4   R3,R4,R5,R6,A221,If,J,Q,Ia,Ib,Ic

      pi=4*atan(1.0)
      P=12.

        IF (t>3.0) then
        Tl=8.0
        else
        Tl=8.0
        endif
```

```
c       Ti=8.0
        wb=2.0*pi*300.
        J=0.0048
        Rs=0.0303
        Xs=0.000318*wb
        Ls=(Xs/wb)
        Rf=0.00318
        Xm=0.000237*wb
        Lm=Xmd/wb
        Lf=0.000726
        rf=1.0
        lf=0.0
        Vi=24.0
        Exf=46.8367

        R1=(pi*pi/18.)*rf
        R2=((pi*Vi*(3**0.5))/(9.0))*(1.0/((x(1)**2)+(x(2)**2))**0.5)
        R3=(pi*pi*lf/(18.0))*(1.0/((x(1)**2)+(x(2)**2)))
        R4=(-x(1)/Ls)*(Rs*x(1)+Ls*x(4)*x(2)-Lm*x(4)*x(3))
        A221=(-Ls*x(4)*x(1)+Rs*x(2))
        A33=(Exf-Xm*x(3))

        det=(Xm/Rf)*(Lm*Lm-Lf*Ls)

        b22= Xm*Lf/(Rf*det)
        b23= -Lm/det
        R5= b22*A221*x(2)+b23*A33*x(2)
        R6= R3*(R4+R5)

         R= (R1+R2+R6)/(1+(pi*pi*lf/(18.0))*(1.0/((x(1)**2)+(x(2)**2)))
        * *(x(1)*x(1)/Ls)-b22*x(2)*x(2)
```

```
 * *(pi*pi*lf/(18.0))*(1.0/((x(1)**2)
     * +(x(2)**2))))
      uqs=R*x(1)
      uds=R*x(2)

c      write(*,*)R
c   inductance matrix tersi

       det=(Xm/Rf)*(Lm*Lm-Lf*Ls)
       b11=-1/Ls
       b22=Xm*Lf/(Rf*det)
       b33=-Ls/det
       b23=-Lm/det
       b32=Xm*Lm/(Rf*det)
c
   A11=(uqs+Rs*x(1)+Ls*x(4)*x(2)-Lm*x(4)*x(3))
   A22=(uds-Ls*x(4)*x(1)+Rs*x(2))
   A33=(Exf-Xm*x(3))
       XDOT(1)=b11*A11
       XDOT(2)=b22*A22+b23*A33
       XDOT(3)=b32*A22+b33*A33
       XDOT(4)=-(((3/2.)*(P/2.)*Lm*x(1)*x(3))-Tl)/(J*2./P)
       XDOT(5)=x(4)
       Ia=x(1)*cos(x(5))+x(2)*sin(x(5))
       Ib=x(1)*cos(x(5)-(2*pi/3))+x(2)*sin(x(5)- (2*pi/3))
       Ic=x(1)*cos(x(5)+ (2*pi/3))+x(2)*sin(x(5)+ (2*pi/3))
       Q=atand(uds/uqs)
c      write(*,*)t,Q
c      write(*,*)t,Ia,Ib,Ic
       write(20,*)t,Ia,Ib,Ic
       Pout=1.5*(uqs*x(1)+uds*x(2))
       Pin=Tl*x(4)*2./pole
```

```
 Eff=Pout/Pin
        Te=(3.0/2.0)*(pole/2.0)*(Lm*x(3)*x(1))


        write(14,*)t,Pin,Pout,Eff
c       write(*,*)t,Pin,Pout,Eff
        write(17,*)t,Te,Q


c       write(*,*)t,Te
c       write(*,*)t,Eff
      Return
      End
```

APPENDIX B

```
load fakim.txt -ascii
time = fakim(:,1);
Iqs = fakim(:,2);
Ids = fakim(:,3);
If = fakim(:,4);

figure(1)
plot(time,Iqs)
title('Time vs Iqs')
grid on

figure(2)
plot(time,Ids)
title('Time vs Ids')
grid on

figure(3)
plot(time,If)
title('Time vs If')
grid on

-----------------------------------------------------------------------------------------------------

load fguc.txt -ascii
time = fguc(:,1);
Pin = fguc(:,2);
```

```
Pout = fguc(:,3);
Eff = fguc(:,4);

figure(1)
plot(time,Pin)
title('Time vs Pin')
grid on

figure(2)
plot(time,Pout)
title('Time vs Pout')
grid on

figure(3)
plot(time,Eff)
title('Time vs Eff')
grid on
```

```
load fhiz.txt -ascii
time = fhiz(:,1);
Wr = fhiz(:,2);

figure(1)
plot(time,Wr)
title('Time vs Wr')
grid on
```

```
load fIaIbIc.txt -ascii
```

```
time = fIaIbIc(:,1);
Ias = fIaIbIc(:,2);
Ibs = fIaIbIc(:,3);
Ics = fIaIbIc(:,4);

figure(1)
plot(time,Ias)
title('Time vs Ias')
grid on

figure(2)
plot(time,Ibs)
title('Time vs Ibs')
grid on

figure(3)
plot(time,Ics)
title('Time vs Ics')
grid on
```

```
load ftorque.txt -ascii
time = ftorque(:,1);
Te = ftorque(:,2);

figure(1)
plot(time,Te)
title('Time vs Te')
grid on
```

```
load faci.txt -ascii
time = faci(:,1);
Q = faci(:,2);

figure(1)
plot(time,Q)
title('Time vs Q')
grid on
```